Lecture Notes On Physics

For class XI and XII Students

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LECTURE NOTES ON PHYSICS

For class XI and XII Students

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PREFACE

I feel extremely happy in presenting this book titled " Lecture Notes On Physics" to the students of Class XI and XII Science of Meghalaya, following the curriculum of the Meghalaya Board Of School Education system. I have developed this book from the lecture notes prepared for teaching the Higher Secondary level Courses for the last two years i.e., 2013 and 2014 and the comments and suggestions provided by the students.

I feel highly obliged to numerous authors and publishers of various books which I have often referred for preparing the notes.

I express my gratitude to my family members, friends, my colleagues in the Department and well-wishers for their love, cooperation and constant encouragement without which this work would have never been accomplished.

Above all I thank God, the Almighty who has given me the strength and guidance all through my life.

M. J. S. Rynjah

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PHYSICAL WORLD AND MEASUREMENT

What is physics?
Physics is a branch of science which deals with the study of nature and its natural phenomena.

Measurement: Measurement is the comparison of unknown physical quantities with a known fixed unit quantity.

Force: Force is the external agency applied on a body to change its state of rest and motion.

Forces in nature: There are four basic forces in nature. They are gravitational force, electromagnetic force, strong nuclear force and weak nuclear force.

Fundamental and derived quantities: Physical quantities can be classified into two categories namely, fundamental quantities and derived quantities.

(1) Fundamental quantities are quantities which cannot be expressed in terms of any other physical quantity. For example, quantities like length, mass, time, temperature are fundamental quantities.

(2) Derived quantities are quantities that can be expressed in terms of fundamental quantities. For example, quantities like area, velocity, acceleration etc. are derived quantities.

Unit: The quantity used as a standard of measurement is called the unit.

Or

Unit of a physical quantity is defined as the established standard used for comparison of the given physical quantity.

Fundamental and derived units: The units in which the fundamental quantities are measured are called fundamental units and the units used to measure derived quantities are called derived units.

Or

Fundamental units: The units selected for measuring mass, length, time, temperature, current, luminous intensity, quantity of matter, plane angle and solid angle are called fundamental units. For examples, metre (m), kilogram (kg), time (s) etc are fundamental units.

Derived units: The unit of physical quantities which can be expressed in terms of fundamental units (mass, length and time) are called derived units. For examples, metre per second (ms^{-1}), metre per second^2 (ms^{-2}), newton (N) etc are derived units.

Systems of units: There are three main systems of units. They are The British system of foot-pound-second or fps system, the Gaussian system of centimetre-gram-second or cgs system and the metre-kilogram-second or the mks system. The SI unit is essentially a modification of the mks system.

SI Units: This system is essentially a modification of the mks system and is, therefore rationalised mksA (metre kilogram second ampere) system.

In the SI system of units there are seven fundamental quantities and two supplementary quantities. They are presented in the Table below.

SI system of units

Physical quantity	Unit	Symbol
	(Fundamental quantities)	
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Current	ampere	A
Luminous intensity	candela	cd
Quantity of matter	mole	mol
	(Supplementary quantities)	
Plane angle	radian	rad
Solid angle	steradian	sr

Note:

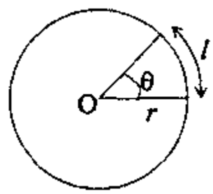
(1) The fundamental units are not definable in terms of other quantities. For this reason they are called base or fundamental units.

(2) To express the unit of physical quantities in mechanics, the unit of mass, length and time are adequate.

Supplementary units:

Radian: The angle in radians θ is defined to be the ratio of the arc length l to the radius r.

i.e., $\theta = \frac{l}{r}$

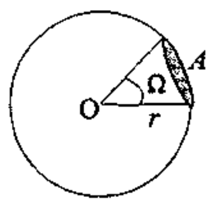


Note: $\frac{\theta}{2\pi} = \frac{l}{2\pi r} \qquad \Rightarrow \theta = \frac{l}{r}$

One radian: One radian is defined as the angle subtended by an arc whose length is equal to the radius.

$$\because \theta = \frac{l}{r} = \frac{1}{1} = 1\, rad$$

Steradian: It is the measure of solid angle Ω, and is defined as the ratio of the subtended area A to radius squared r^2. i.e., $\Omega = \frac{A}{r^2}$



Note: $\frac{\Omega}{4\pi} = \frac{A}{4\pi r^2} \qquad \Rightarrow \Omega = \frac{A}{r^2}$

One steradian: One steradian is defined as the angle subtended at the centre of a sphere of radius $1m$ by a part of its surface having an area of $1m^2$.

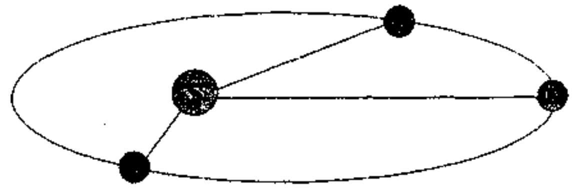
$$\because \Omega = \frac{A}{r^2} = \frac{1m^2}{(1m)^2} = 1\, sr$$

Practical units for large distances: In order to measure very large distances, we use the following three units.

(1) Astronomical Unit (*A.U*): It is the mean distance between the centres of the earth and the sun.

$1\,A.U = 1.496\times10^{11}\,m$

$1\,A.U \approx 1.5\times10^{11}\,m$



(2) Light year (ly): It is the distance travelled by light in vacuum in one year.

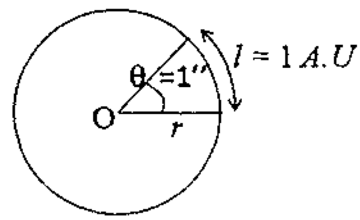
In 1 second light travelled $3\times10^{8}\,m$

In 1 year or $1\times365\times24\times60\times60$ seconds, light travelled

$$3\times10^{8}\times1\times365\times24\times60\times60\,m = 9.46\times10^{15}m$$

$$\therefore 1\,ly = 9.46\times10^{15}\,m$$

(3) Parsec: It is the distance at which an arc of 1 *A.U* subtends an angle of 1"



$$\because \theta = \frac{l}{r}$$

$$\text{Or}\quad r = \frac{l}{\theta} = \frac{1\,A.U}{1''} = \frac{1.5\times10^{11}}{\frac{\pi}{180\times60\times60}}\,m$$

$$\text{Or}\quad r = \frac{1.5\times10^{11}(180\times60\times60)}{\pi}\,m = 3.08\times10^{16}\,m$$

$$\therefore 1\,parsec = 3.08\times10^{16}\,m$$

Note: $180^0 = \pi\ rad$ $\qquad \Rightarrow 1^0 = \frac{\pi}{180}\ rad$ $\qquad$ Or $60' = \frac{\pi}{180^0}\ rad$

$1' = \frac{\pi}{180\times60}\ rad$ $\qquad$ Or $60'' = \frac{\pi}{180\times60}\ rad$

$1'' = \frac{\pi}{180\times60\times60}\ rad$

Practical units for very small distances:

(1) Angstrom (Å): Diameter of an atom (1 Å = $10^{-10}m$)

(2) Fermi (fm): Diameter of the nucleus of an atom (1fm = $10^{-15}m$)

Expressing larger and smaller physical quantities

Power of ten	Prefix	Abbreviation
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Measurement of large distances (parallax method)

(1) Measurement of size of heavenly bodies:

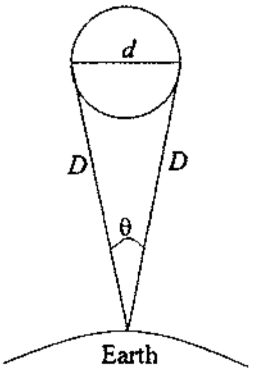
$$\because \theta = \frac{d}{D} \qquad \Rightarrow d = D\theta$$

If D is known, θ is known, then d is known

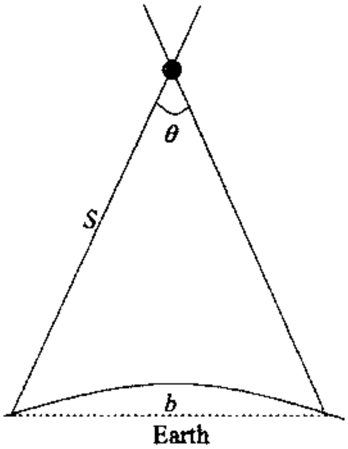
(2) Measurement of distances of heavenly bodies:

$$\because \theta = \frac{b}{S} \qquad \Rightarrow S = \frac{b}{\theta}$$

If b is known, θ is known, then S is known.



(1)



(2)

Measurement of mass:

(1) Inertial mass: When a body is in translational motion, the ratio of resultant force applied on the body and the acceleration produced in it is called inertial mass (m_i) of the body.

$$F = m_i a$$

$$\Rightarrow m_i = \frac{F}{a}$$

(2) Gravitational mass: When the mass of a body is measured under the effect of gravity, and the body is not in motion, it is called gravitational mass (m_G).

$$W = m_G g \qquad \Rightarrow m_G = \frac{W}{g}$$

Note: $m_i \approx m_G$. But if the body moves with a very high speed (compare to the speed of light), $m_i > m_G$

Measurement of gravitational mass:

Weight: The weight of the body is defined as the gravitational force exerted on it due to the Earth.

$$\because W = mg$$

Thus if a body has a mass of 10 kg, then its weight is $W = 10 \times 9.8N = 98N$.

Note:
(1) The mass of the body is the same whether it is on the Earth, on the Moon or somewhere in space. But the weight of the body depends on the acceleration due to gravity.
(2) The mass has magnitude only (scalar quantity), whereas weight has magnitude as well as direction (vector quantity).
(3) The mass is a measure of body's inertia, however weight of a body is the gravitational force.
(4) The SI unit of mass is kg, whereas that of the weight is newton (N).

Dimensions of a physical quantity: The dimensions of a physical quantity are the powers to which the fundamental units are raised in order to obtain the units of that quantity.

$$\because \text{Unit of density} = \frac{\text{Unit of mass}}{\text{Unit of volume}}$$

Thus in order to get the unit of density, we raise the unit of mass to the power 1 and the unit of length to the power −3. These powers are called the "dimensions of density". In other words dimensions of density are 1 in Mass and −3 in Length i.e., $[M^1L^{-3}]$

Dimensions of fundamental quantities

Fundamental quantities	Dimension
Length	$[L]$
Mass	$[M]$
Time	$[T]$
Temperature	$[K]$
Current	$[A]$
Luminous intensity	$[cd]$
Quantity of matter	$[mol]$

Example:

We know that velocity is given by $v = \frac{S}{t}$

The dimensional formula of velocity is $[v] = \frac{[L]}{[T]} = [L][T]^{-1} = [LT^{-1}]$

Or $[v] = [M^0L^1T^{-1}]$

Where $[M]$, $[L]$ and $[T]$ are the dimensions of the fundamental quantities mass, length and time respectively.

Therefore velocity has (0) dimension in mass, (1) dimension in length and (−1) dimension in time. Thus the dimensional formula for velocity is $[M^0L^1T^{-1}]$ or simply $[LT^{-1}]$.

Dimensional formulae of some derived quantities

Physical quantity	Expression	Dimensional formula
Area	length × breadth	$[L][L] = [L^2]$
Volume	length × breadth × height	$[L][L][L] = [L^3]$
Density	mass/volume	$[ML^{-3}T^0]$
speed or velocity	distance/time	$[M^0LT^{-1}]$
Acceleration	velocity/time	$[M^0LT^{-2}]$
Force	mass × acceleration	$[MLT^{-2}]$
Momentum	mass × velocity	$[MLT^{-1}]$
Work	force × displacement	$[ML^2T^{-2}]$

Four categories of physical quantities:

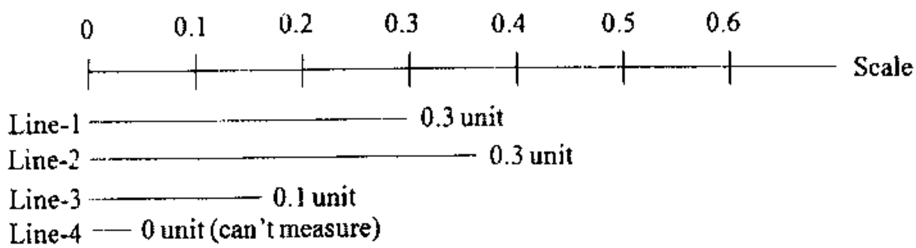
(1) Dimensional variables: Physical quantities which have dimensions but do not have fixed value are called dimensional variables. Examples force, velocity etc.
(2) Dimensional constants: Physical quantities which have dimensions and a fixed value are called dimensional constants. Examples velocity of light, planck's constant etc.
(3) Dimensionless variables: Physical quantities which have neither dimensions nor fixed value are called dimensionless variables. Examples angle, specific gravity etc.
(4) Dimensionless constants: Physical quantities which have no dimensions but have fixed value are called dimensionless constants. Examples 1, 2, π etc.

Principle of homogeneity of dimensions: An equation is dimensionally correct if the dimensions of the various terms on either side of the equation are the same. This is called the principle of homogeneity of dimensions.
The equation A + B = C is valid only if the dimensions of A, B and C are the same.

Uses of dimensional analysis: The method of dimensional analysis is used to
(1) convert a physical quantity from one system of units to another.
(2) check the dimensional correctness of a given equation.
(3) establish a relationship between different physical quantities in an equation.

Limitations of Dimensional Analysis:
(1) The value of dimensionless constants cannot be determined by this method.
(2) This method cannot be applied to equations involving exponential and trigonometric functions.
(3) It cannot be applied to an equation involving more than three physical quantities.
(4) It can check only whether a physical relation is dimensionally correct or not. It cannot tell whether the relation is absolutely correct or not. For example, applying this technique $S = ut + \frac{3}{4} at^2$ is dimensionally correct whereas the correct relation is $S = ut + \frac{1}{2} at^2$.

Least count: The least count of a measuring device is the smallest value that can be measured accurately.
For example,



The least count of the given scale is 0.1 unit.

Significant figures: The number of digits in a measurement about which we are certain plus one additional digit which is uncertain are known as significant figures.
Example:
23.1m ----- 3 significant figures
2310 cm--- 3 significant figures
23100mm- 3 significant figures
0.0231km- 3 significant figures

Note:
(1) If we expressed the length of the table as 52.37 cm, four significant figures are there. Three certain and one estimated. It may be noted that in recording a measurement or calculation, one and only one doubtful digit is retained.
(2) If the distance between two objects is 3050 m then 3050 m or 305000 cm or 3.050 km has four significant figures.

Example

Number	Number of significant figures	Rules
1.31	3	All nonzero digits are significant
13.1	3	"
231	3	"
450076	6	All zeros between two nonzero digits are significant
21.05	4	All zeros to the right of a decimal point and to the right of a nonzero digit are significant
0.10004	5	All zeros between two nonzero digits are significant
1.001	4	All zeros to the right of a decimal point and to the right of a nonzero digit are significant
1.000	4	"
1.310	4	"
0.310	3	"
0.0310	3	"
2.0310	5	"
2310	3	All zeros to the left of an understood decimal point but to the right of a nonzero digit are not significant
2310.	4	All zeros to the left of an expressed decimal point and to the right of a nonzero digit are significant
0.1000	4	All zeros to the right of a decimal point and to the right of a nonzero digit are significant
0.0001	1	All zeros to the right of a decimal point but to the left of a nonzero digit are not significant
1.0001	5	All zeros to the right of a decimal point and to the right of a nonzero digit are significant
6200	2	All zeros to the left of an understood decimal point but to the right of a nonzero digit are not significant
62×10^2	2	
6.2×10^3	2	
6.200×10^3	4	
6200.	4	All zeros to the left of an expressed decimal point and to the right of a nonzero digit are significant
8.26×10^{-3}	3	

Significant figures in algebraic operation:
(1) In addition or subtraction: In addition or subtraction of the numbers, the final result should retain the least decimal place as in the various numbers.

Example: If $a = 1.234\ m$ and $b = 1.20\ m$

Then $a + b = (1.234 + 1.20)\ m = 2.434\ m$

As b has measured upto two decimal places

$\therefore\ a + b = 2.43\ m$

(2) In multiplication or division: In Multiplication or division of the numbers, the final result should retain the least significant figures as in the various numbers.

Example: If $a = 1.234\ m$ and $b = 1.20\ m$

Then $a \times b = 1.234 \times 1.20\ m^2 = 1.4808\ m^2$

As b has only 3 significant figures,

$\therefore\ a \times b = 1.48\ m^2$

Rounding off:

(1) If the digit to be dropped is less than 5, the preceding digit is left unchanged. Example

Number	Rounding off to three significant figures
1.870	1.87
1.871	1.87
1.872	1.87
1.873	1.87
1.874	1.87

(2) If the digit to be dropped is more than 5, the preceding digit is raised by one. Example

Number	Rounding off to three significant figures
1.875	1.88
1.876	1.88
1.877	1.88
1.878	1.88
1.879	1.88

Special cases:

(1) If the digit to be dropped is 5 followed by digits other than zero, the preceding digit is raised by one. Example

Number	Rounding off to three significant figures
1.8051	1.81
1.8151	1.82
1.8252	1.83
1.8351	1.84
1.8453	1.85
1.8555	1.86
1.8656	1.87
1.8758	1.88
1.8851	1.89
1.8953	1.90

(2) If the digit to be dropped is 5 or 5 followed by zeros, the preceding digit, if it is even is left unchanged. Example

Number	Rounding off to three significant figures
1.8050*	1.81* (zero is not an even or an odd number)
1.8250	1.82
1.8450	1.84
1.8650	1.86
1.8850	1.88

(3) If the digit to be dropped is 5 or 5 followed by zeros, the preceding digit, if it is odd is raised by one. Example

Number	Rounding off to three significant figures
1.8150	1.82
1.8350	1.84
1.8550	1.86
1.8750	1.88
1.8950	1.90

Errors of measurement: The difference in the true value and the measured value of a quantity is called error of measurement.

Types of error:

(1) Systematic errors: Systematic errors are errors whose causes are known. Such errors can, therefore be minimised. For examples
(i) Instrumental errors
(ii) Personal errors
(iii) Errors due to imperfection arise on account of ignoring certain facts
(iv) Errors due to external causes arises due to changes in temperature, pressure, humidity etc.

(2) Random errors: These errors may arise due to a large variety of factors. The causes of such errors are, therefore not known precisely.

Commonly used terms in errors analysis:

(1) True value: The true value of a physical quantity is the arithmetic mean of a large number of readings of that quantity.
If $a_1, a_2, a_3, ...a_n$ are the n different readings of a physical quantity in an experiment, then true value of that quantity is

$$\bar{a} = \frac{a_1 + a_2 + a_3 + ... + a_n}{n} = \frac{1}{n}\sum_{i=1}^{i=n} a_i$$

Example: The period of oscillation of a simple pendulum in an experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. Find (1) True value (mean time period) (2) Absolute error (3) Mean absolute error (4) Relative error and percentage error.

Solution (1) $\because \overline{T} = \dfrac{T_1 + T_2 + T_3 + T_4 + T_5}{5}$

Mean time period $\overline{T} = \dfrac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5} = \dfrac{13.12}{5} = 2.62s$

(2) Absolute error: The difference in the magnitudes of the true value and the measured value of a physical quantity is called absolute error.

$$\because \overline{a} = \frac{a_1 + a_2 + a_3 + ... + a_n}{n}$$

The absolute errors in the various measured values are

$\Delta a_1 = \overline{a} - a_1$

$\Delta a_2 = \overline{a} - a_2$

$\Delta a_3 = \overline{a} - a_3$

$\Delta a_n = \overline{a} - a_n$

Solution (2) Absolute error in each observation is

$\Delta T_1 = \overline{T} - T_1 = 2.62 - 2.63 = -0.01s$ $\Delta T_2 = \overline{T} - T_2 = 2.62 - 2.56 = 0.06s$

$\Delta T_3 = \overline{T} - T_3 = 2.62 - 2.42 = 0.20s$ $\Delta T_4 = \overline{T} - T_4 = 2.62 - 2.71 = -0.09s$

$\Delta T_5 = \overline{T} - T_5 = 2.62 - 2.80 = -0.18s$

(3) Mean absolute error: The arithmetic mean of all the absolute errors in the measured values is called mean absolute error.

The mean absolute error or the final absolute error is

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ... + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{i=n} |\Delta a_i|$$

Solution (3) The mean absolute error is

$$\overline{\Delta T} = \frac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4| + |\Delta T_5|}{5} = \frac{0.01 + 0.06 + 0.20 + 0.09 + 0.18}{5} = 0.11s$$

Then the time period of the pendulum is recorded as $T = \overline{T} \pm \overline{\Delta T} = 2.62 \pm 0.11s$

(4) Relative error and percentage error: The ratio of mean absolute error and mean value or true value of the quantity being measured is called relative error or fractional error.

Relative error is $\dfrac{\overline{\Delta a}}{\overline{a}}$

Percentage error is $\dfrac{\overline{\Delta a}}{\overline{a}} \times 100\%$

Solution (4) The percentage error is $\dfrac{\overline{\Delta T}}{\overline{T}} \times 100\% = \dfrac{0.11}{2.62} \times 100\% = 4.2\%$

Propagation or combination of errors:

(1) Error in sum:

Let $Q \pm \Delta Q = (a \pm \Delta a) + (b \pm \Delta b)$

$Q \pm \Delta Q = a \pm \Delta a + b \pm \Delta b$

$Q \pm \Delta Q = a + b \pm \Delta a \pm \Delta b$

Here $Q = a + b$

And $\pm \Delta Q = \pm \Delta a \pm \Delta b$

The four possible values of ΔQ are $(+\Delta a + \Delta b), (+\Delta a - \Delta b), (-\Delta a + \Delta b)$ and $(-\Delta a - \Delta b)$

Therefore the maximum absolute error in Q is $\Delta Q = \pm(\Delta a + \Delta b)$

(2) Error in difference:

Let $Q \pm \Delta Q = (a \pm \Delta a) - (b \pm \Delta b)$

$Q \pm \Delta Q = a \pm \Delta a - b \mp \Delta b$

$Q \pm \Delta Q = a - b \pm \Delta a \mp \Delta b$

Here $Q = a - b$

And $\pm \Delta Q = \pm \Delta a \mp \Delta b$

The four possible values of ΔQ are $(+\Delta a - \Delta b), (+\Delta a + \Delta b), (-\Delta a - \Delta b)$ and $(-\Delta a + \Delta b)$

Therefore the maximum absolute error in Q is $\Delta Q = \pm(\Delta a + \Delta b)$

(3) Error in product:

Let $Q \pm \Delta Q = (a \pm \Delta a) \times (b \pm \Delta b)$

$$Q\left(1 \pm \frac{\Delta Q}{Q}\right) = a\left(1 \pm \frac{\Delta a}{a}\right) \times b\left(1 \pm \frac{\Delta b}{b}\right)$$

$$Q\left(1 \pm \frac{\Delta Q}{Q}\right) = ab\left(1 \pm \frac{\Delta a}{a}\right) \times \left(1 \pm \frac{\Delta b}{b}\right)$$

$$Q\left(1 \pm \frac{\Delta Q}{Q}\right) = ab\left(1 \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}\right)$$

Here $Q = ab$

And $1 \pm \frac{\Delta Q}{Q} = 1 \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$

Or $\pm \frac{\Delta Q}{Q} = \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$

As $\left(\frac{\Delta a}{a}\right)$ and $\left(\frac{\Delta b}{b}\right)$ both are small, their product is still smaller and can be neglected.

$\therefore \quad \pm \frac{\Delta Q}{Q} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b}$

Four possible values of $\frac{\Delta Q}{Q}$ are $\left(+\frac{\Delta a}{a} + \frac{\Delta b}{b}\right), \left(+\frac{\Delta a}{a} - \frac{\Delta b}{b}\right), \left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$

Hence maximum possible value of $\frac{\Delta Q}{Q} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

(4) Error in division:

$$\text{Let } Q \pm \Delta Q = \frac{(a \pm \Delta a)}{(b \pm \Delta b)}$$

$$Q\left(1 \pm \frac{\Delta Q}{Q}\right) = \frac{a\left(1 \pm \frac{\Delta a}{a}\right)}{b\left(1 \pm \frac{\Delta b}{b}\right)} = \frac{a}{b}\left(1 \pm \frac{\Delta a}{a}\right)\cdot\left(1 \pm \frac{\Delta b}{b}\right)^{-1}$$

As $\frac{\Delta b}{b} << 1$, therefore expanding Binomially, we get

$$Q\left(1 \pm \frac{\Delta Q}{Q}\right) = \frac{a}{b}\left(1 \pm \frac{\Delta a}{a}\right)\cdot\left(1 \mp \frac{\Delta b}{b}\right)$$

$$Q\left(1 \pm \frac{\Delta Q}{Q}\right) = \frac{a}{b}\left(1 \mp \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a}\frac{\Delta b}{b}\right)$$

Here $Q = \frac{a}{b}$

And $1 \pm \frac{\Delta Q}{Q} = 1 \mp \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a}\frac{\Delta b}{b}$

Or $\pm \frac{\Delta Q}{Q} = \mp \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a}\frac{\Delta b}{b}$

As $\left(\frac{\Delta a}{a}\right)$ and $\left(\frac{\Delta b}{b}\right)$ both are small, their product is still smaller and can be neglected.

$\therefore \quad \pm \frac{\Delta Q}{Q} = \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b}$

Four possible values of $\frac{\Delta Q}{Q}$ are $\left(+\frac{\Delta a}{a} - \frac{\Delta b}{b}\right), \left(+\frac{\Delta a}{a} + \frac{\Delta b}{b}\right), \left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Hence maximum possible value of $\frac{\Delta Q}{Q} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

(4) Error in power of a quantity:

$$\text{Let } Q \pm \Delta Q = \frac{(a \pm \Delta a)^n}{(b \pm \Delta b)^m}$$

$$\text{Or } Q = \frac{a^n}{b^m} \text{ -------- (1)}$$

Suppose Δa is the absolute error in the measurement of a, Δb is the absolute error in the measurement of b and ΔQ is the absolute error in calculation of Q.

Taking log of both sides of equation (1) gives

$$\log Q = \log a^n - \log b^m$$
$$\log Q = n\log a - m\log b$$

Differentiating both sides with respect to Q, we get

$$\frac{1}{Q} = n\frac{1}{a}\frac{da}{dQ} - m\frac{1}{b}\frac{db}{dQ}$$
$$\frac{dQ}{Q} = n\frac{da}{a} - m\frac{db}{b}$$

In terms of fractional error, we may rewrite this equation as

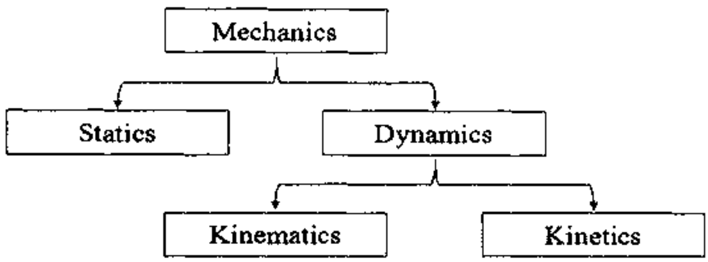
$$\pm\frac{\Delta Q}{Q} = \pm n\frac{\Delta a}{a} \mp m\frac{\Delta b}{b}$$

Hence maximum value of $\frac{\Delta Q}{Q} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$

KINEMATICS

Mechanics: Mechanics deals with the study of particles or bodies when they are at rest or in motion.
Mechanics can be divided into two branches, statics and dynamics.
(1) Statics is the study of objects at rest, this requires the idea of forces in equilibrium.
(2) Dynamics is the study of moving objects.

Dynamics is further subdivided into kinematics and kinetics.
(1) Kinematics is the study of the relationship between displacement, velocity, acceleration and time of a given motion, without considering the forces that cause the motion.
(2) Kinetics deals with the relationship between the motion of bodies and forces acting on them.



Particle: A particle is ideally just a piece or a quantity of matter, having practically no linear dimensions but only a position.

Or

A particle is defined as an isolated point mass having no size and incapable of rotation.

Rest: When a body does not change its position with respect to time, then it is said to be at rest.

Motion: Motion is the change of position of the body with respect to time.

Types of motion of a body:
(1) Rectilinear motion and translatory motion: Rectilinear motion is that motion in which a particle or point mass body is moving along a straight line. Translatory motion is that motion in which a body, which is not a point mass body is moving such that all its constituent particles moves simultaneously along parallel straight lines and shift through equal distance in a given interval of time.

(2) Circular motion and rotatory motion: A circular motion is that motion in which a particle or a point mass body is moving on a circle. A rotatory motion is that motion in which a body, which is not a point mass body, is moving such that all its constituent particles move simultaneously along concentric circles, whose centres lie on a line, called axis of rotation and shift through equal angle in a given time.

(3) Oscillatory motion and vibratory motion: Oscillatory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point (mean position) in a definite interval of time. If in the oscillatory motion, the amplitude is very small i.e., microscopic, the motion of the body is said to be a vibratory motion.

Frame of reference: The place from which motion is observed and measured is called frame of reference.

Frames of reference can be of two types,

(1) Inertial frame of reference: Inertial frame of reference is one in which Newton's laws of motion hold good

(2) Non-inertial frame of reference: Non-inertial frame of reference is one in which Newton's laws of motion do not hold good.

Motion in one, two and three dimensions:

(1) Motion in one dimension: Motion of an object is said to be one dimensional, if only one of the three coordinates specifying the position of the object changes with respect to time. For example, an ant moving in a straight line etc.

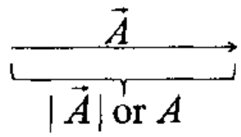
(2) Motion in two dimensions: In this type, the motion is represented by any two of the three coordinates. For example, a body moving in a plane.

(3) Motion in three dimensions: Motion of a body is said to be three dimensional, if all the three coordinates of the position of the body change with respect to time. For examples, the motion of a flying bird, motion of a kite in the sky, motion of a molecule, etc.

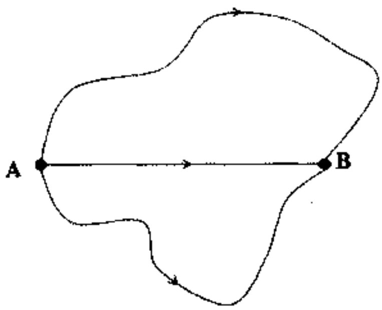
Scalar quantities or scalars: Physical quantities which have only magnitude but no direction, are called scalar quantities or scalars. Mass, length, time, temperature, speed, work, distance covered etc are examples of scalars.

Vector quantities or vectors: Physical quantities which have both magnitude and direction are called vector quantities or vectors. Velocity, displacement, acceleration, force etc are examples of vectors.

A vector can be represented by a single letter with an arrow head on it, for example $\vec{A}$ is a vector whose magnitude is represented by A or $|\vec{A}|$



Distance and Displacement: The total length of the path travelled by the particle is called the distance, and the shortest distance between the initial and final position of the particle is the displacement.



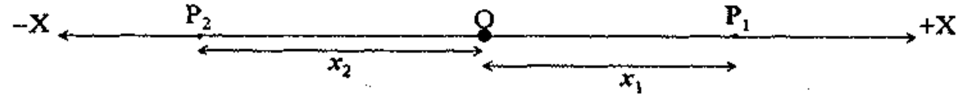
Note: The distance travelled is a scalar quantity and the displacement is a vector quantity.

Example: The distance travelled by a particle, is different from its displacement from the origin. For example, if the particle moves from a point O to position P_1 and then to position P_2, its displacement at the position P_2 from the origin is

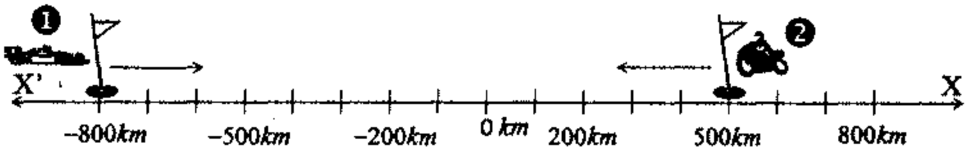
$$\Delta x = -x_2 - 0 = -x_2$$

but, the distance travelled by the particle is

$$x_1 + x_1 + x_2 = (2x_1 + x_2)$$



Note: The displacement can be positive, zero or negative. The distance is always positive.



In ❶, the displacement and also the velocity are positive i.e.,

$$\Delta x = \{0-(-800)\}+\{500-0\} = 800+500 = +1300km$$

$$\text{Or } \Delta x = 500-(-800) = +1300km$$

In ❷, the displacement and also the velocity are negative i.e.,

$$\Delta x = \{0-500\}+\{-800-0\} = -500-800 = -1300km$$

$$\text{Or } \Delta x = -800-500 = -1300km$$

Speed: Speed is defined as the time rate of change of distance. It is a scalar quantity. The SI unit of speed is ms^{-1}. The dimensional formula of speed is $[M^0L^1T^{-1}]$.

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time}}$$

Uniform speed: An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, however small these intervals may be.

Variable speed or non uniform speed: An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time, or unequal distances in equal intervals of time, however small these intervals may be.

Average speed: Average speed for the given motion is defined as the ratio of the total distance travelled to the total time taken i.e.,

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Instantaneous speed: The speed of an object at a given instant of time is called its instantaneous speed.

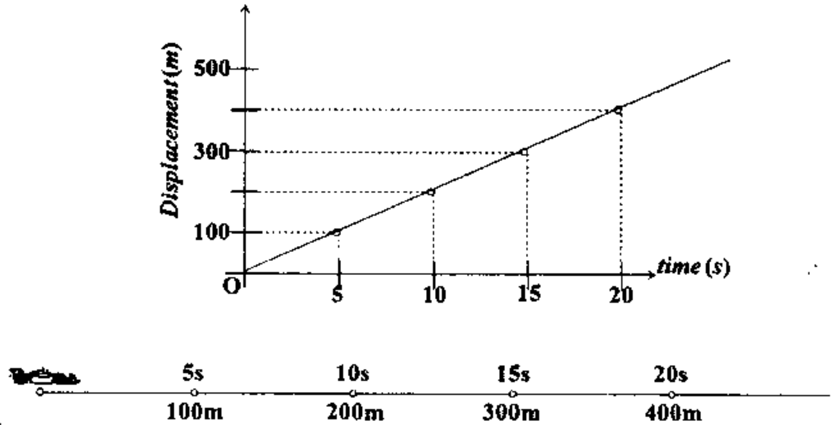
Let at an instant of time t, an object while moving covers a distance Δs in a small interval of time Δt around t, so that $\Delta t \to 0$, then

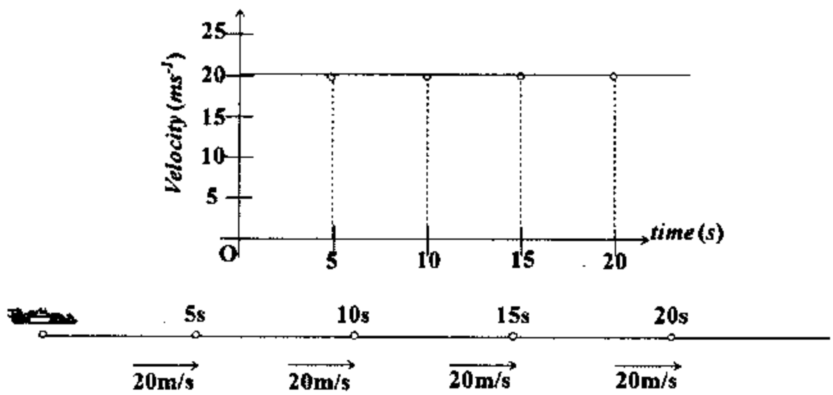
$$\text{Instantaneous speed} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Velocity: Velocity is defined as the time rate of change of displacement. It is a vector quantity. The SI unit of velocity is ms^{-1}. The dimensional formula of velocity is $[M^0L^1T^{-1}]$.

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

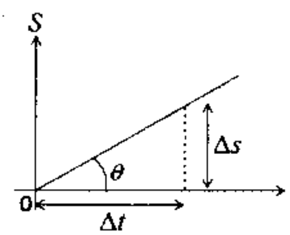
Uniform velocity: An object is said to be moving with a uniform velocity, if it covers equal displacements in equal intervals of time, however small these intervals may be.





Note:

(1) In the figure, the slope of the displacement-time graph represents the uniform velocity. If θ is different for different interval of time, then the velocity is non uniform.

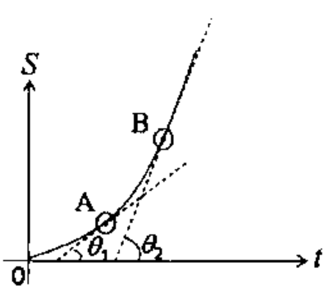


$$\tan\theta = \frac{\text{opp side}}{\text{adj side}} = \frac{\Delta s}{\Delta t} = v \qquad \Rightarrow v = \tan\theta$$

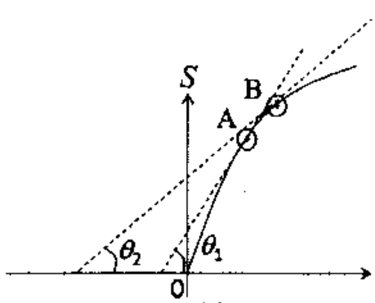
(2) In figure (1) below, $\theta_2 > \theta_1$ the velocity increases with time. So we have acceleration.

(3) In figure (2) below, $\theta_2 < \theta_1$ the velocity decreases with time. So we have retardation.

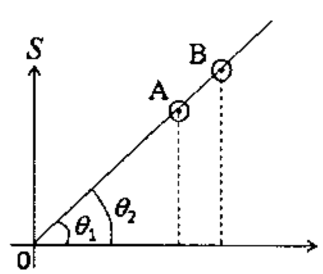
(4) In figure (3) below, $\theta_2 = \theta_1$ the velocity is constant with time. So there is no acceleration.



(1)

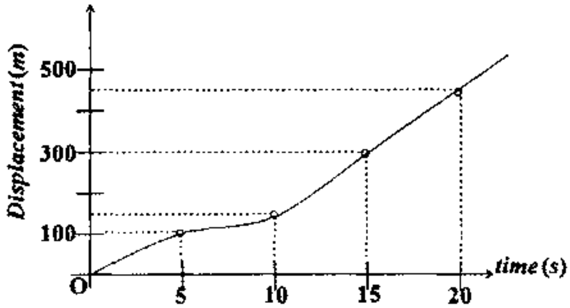


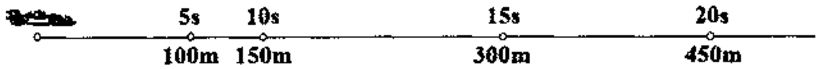
(2)



(3)

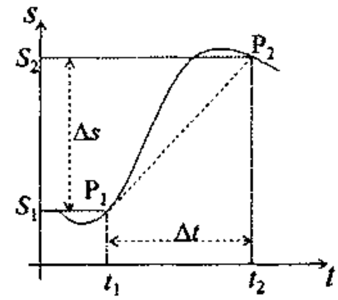
Variable velocity or non uniform velocity: An object is said to be moving with a variable velocity, if it covers equal displacements in unequal intervals of time, or unequal displacements in equal intervals of time, however small these intervals may be.





Average velocity: Average velocity of an object is defined as the ratio of the displacement to the time interval for which the motion takes place i.e.,

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time taken}}$$

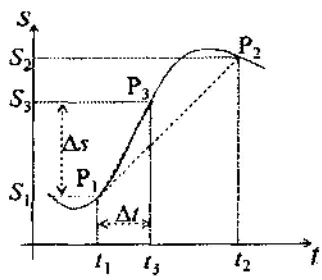


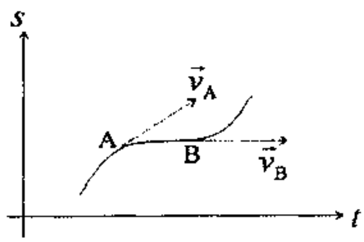
Let S_1 be the displacement of a body in time t_1 and S_2 be its displacement in time t_2. The change in displacement is $(S_2 - S_1)$. The average velocity during the time interval $(t_2 - t_1)$ is defined as

$$v_{av} = \frac{S_2 - S_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$

Instantaneous velocity: It is the velocity at any given instant of time or at any given point of its path. The instantaneous velocity $\vec{v}$ is given by

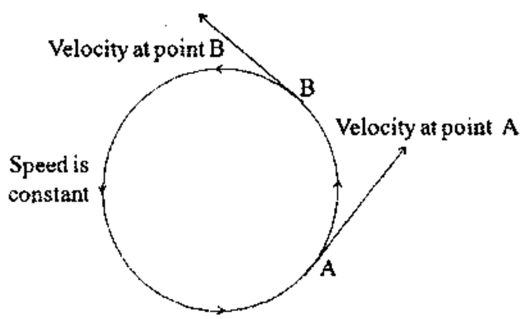
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$$





Note: The magnitude of the instantaneous velocity is called the instantaneous speed.

Speed versus velocity: When a particle moves on a circular path with a constant speed, then at every point on the path, the speed will be the same, but the velocity is different.

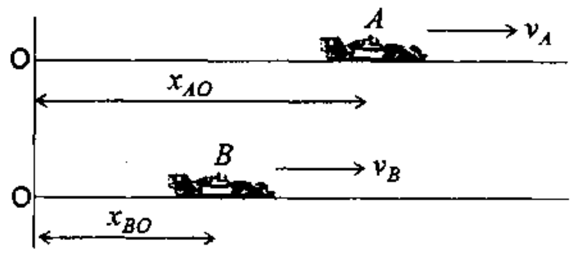


Note:

(1) Speed is a scalar quantity, velocity is a vector quantity.

(2) The magnitude of velocity represents the speed.

Relative velocity in one dimension: The relative velocity of a body A w.r.t. a body B is the time rate at which a body A changes its position w.r.t. a body B.



Let x_{AO} and x_{BO} be the displacements of the bodies A and B from the origin O at $t = 0$. Clearly, $(x_{AO} - x_{BO})$ is the displacement between the bodies A and B at $t = 0$. If x_A and x_B are their position coordinates in time t, then

$$x_A = x_{AO} + v_A t$$

and $$x_B = x_{BO} + v_B t$$

$$\therefore x_A - x_B = (x_{AO} - x_{BO}) + (v_A - v_B)t$$

Increase in displacement between the two bodies in time t

= Final displacement – Original displacement

$= [(x_{AO} - x_{BO}) + (v_A - v_B)t] - [(x_{AO} - x_{BO})]$

$= (v_A - v_B)t$

Therefore, relative velocity of A w.r.t. B is

$$v_{AB} = \frac{\text{displacement}}{\text{time}} = \frac{(v_A - v_B)t}{t} = v_A - v_B$$

Note:

(1) Relative velocity of A w.r.t. B is $v_{AB} = v_A - v_B$ (i.e., B is the observer)

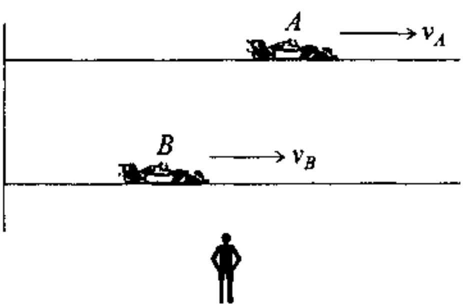
(2) Relative velocity of B w.r.t. A is $v_{BA} = v_B - v_A$ (i.e., A is the observer)

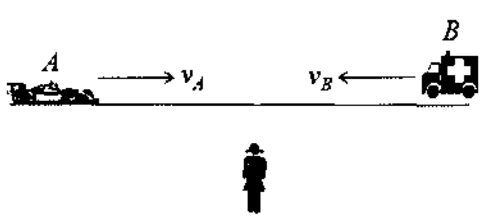
(3) If $v_A > v_B$, v_{AB} is positive i.e., B will see that A moves to the right with velocity v_{AB}

(4) If $v_A > v_B$, v_{BA} is negative i.e., A will see that B moves to the left with velocity v_{BA}

(5) To an observer on the ground, A and B move to the right with velocity v_A and v_B respectively. So v_A and v_B are both positive

i.e., $v_{AG} = v_A - v_G = v_A - 0 = v_A$ and $v_{BG} = v_B - v_G = v_B - 0 = v_B$



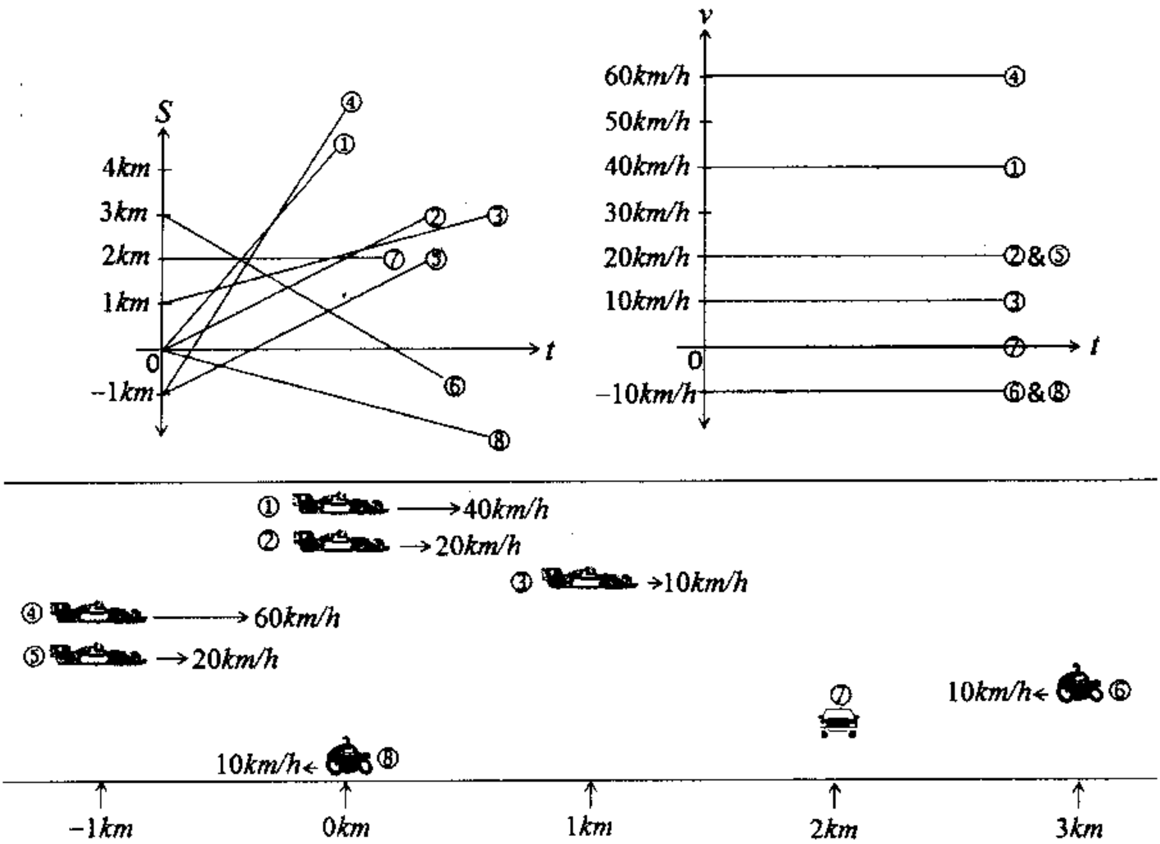


(6) If A and B are moving in opposite direction, then $v_{AB} = v_A - (-v_B) = v_A + v_B$ also $v_{BA} = -v_B - v_A = -(v_B + v_A)$. In this case $v_{AB} = v_{BA}$ in magnitude. v_{AB} is positive, while v_{BA} is negative.

(7) To an observer on the ground, A moves to the right, so v_A is positive while B moves to the left, so v_B is negative

i.e., $v_{AG} = v_A - v_G = v_A - 0 = v_A$ and $v_{BG} = -v_B - v_G = -v_B - 0 = -v_B$

Graphical representation of uniform motion



Acceleration: Acceleration of a particle is defined as the time rate of change of velocity.

Or

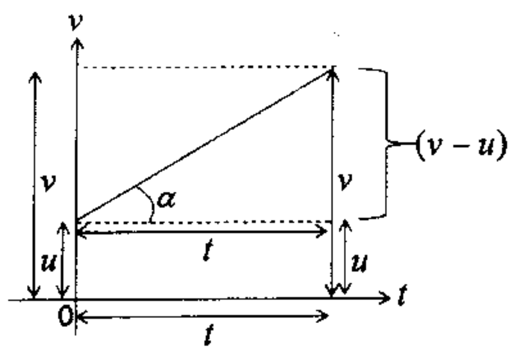
If the magnitude or direction or both of the velocity changes with respect to time, the particle is said to be under acceleration.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

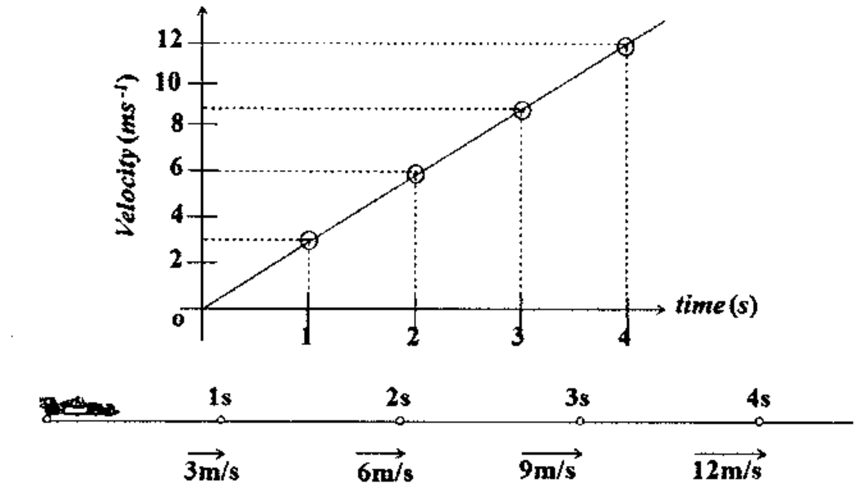
$$\text{Or} \quad a = \frac{v-u}{t}$$

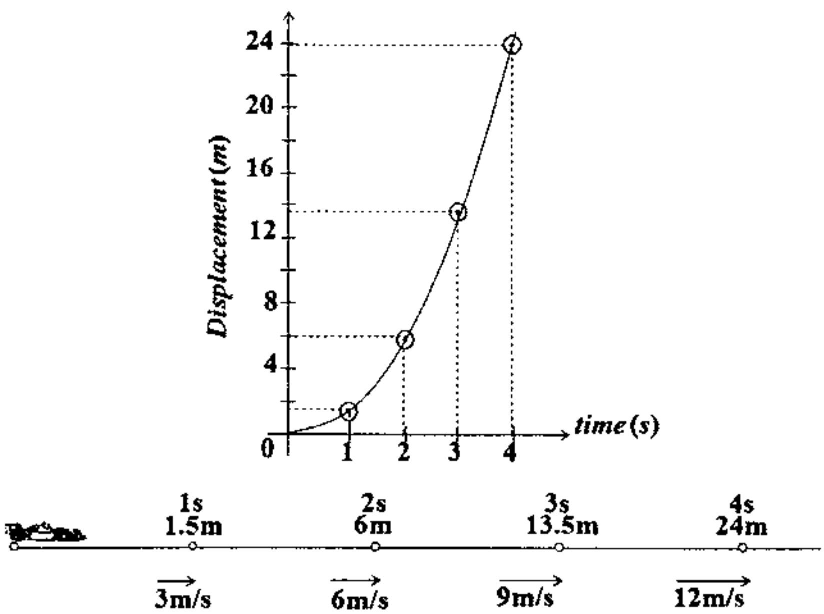
Where u is the initial velocity, v is the final velocity and t is the time taken.

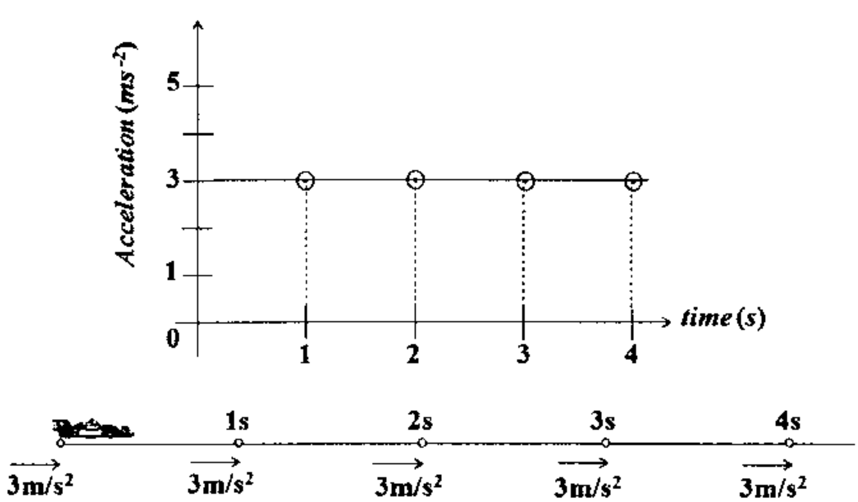
Acceleration is a vector quantity. The SI unit of acceleration is ms^{-2}. The dimensional formula of acceleration is $[M^0L^1T^{-2}]$.



Uniform acceleration: An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time.

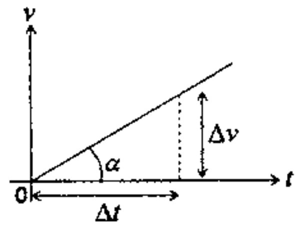






Note:

(1) In the figure, the slope of the velocity-time graph represents the uniform acceleration. If α is different for different interval of time, then the acceleration is non uniform.



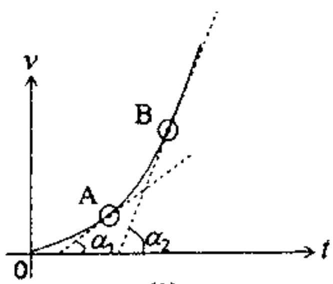
$$\tan\alpha = \frac{\text{opp side}}{\text{adj side}} = \frac{\Delta v}{\Delta t} = a \qquad \Rightarrow a = \tan\alpha$$

(2) In figure (1) below, $\alpha_2 > \alpha_1$ the acceleration increases with time.

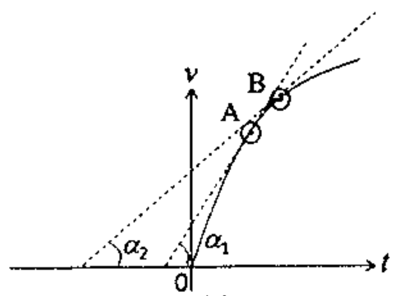
(3) In figure (2) below, $\alpha_2 < \alpha_1$ the acceleration decreases with time.

(4) In figure (3) below, $\alpha_2 = \alpha_1$ the acceleration is constant with time i.e, we have uniform acceleration.

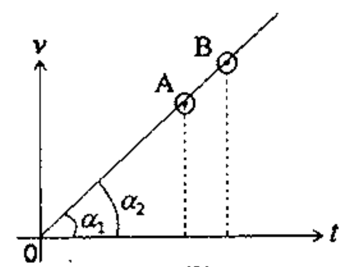
(5) If the acceleration increases uniformly, we have an acceleration (variable acceleration). An example is given below in figure (4).



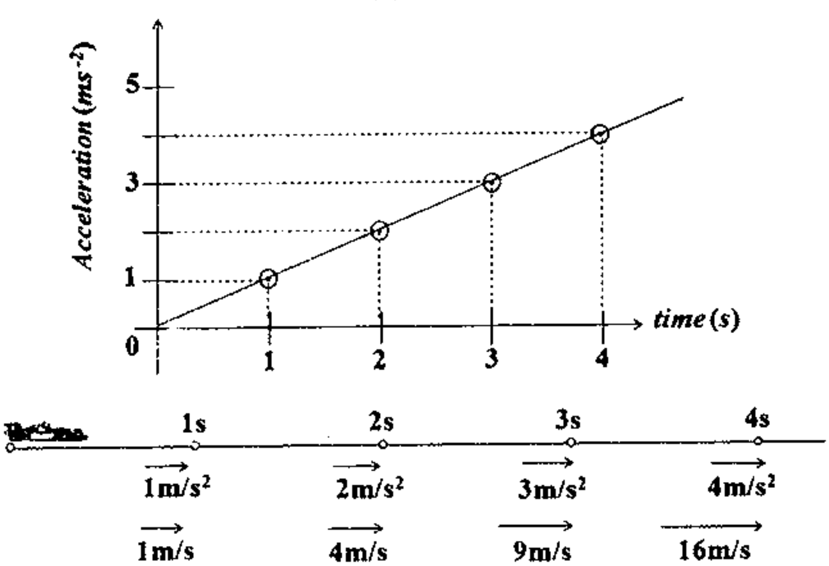
(1)



(2)

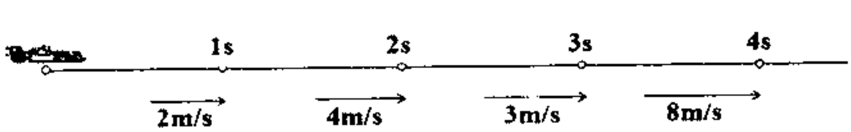


(3)



(4)

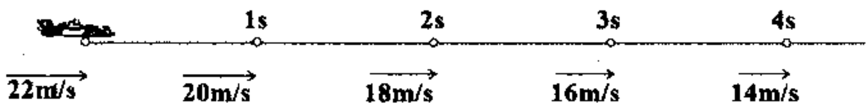
Variable acceleration: An object is said to be moving with a variable acceleration if its velocity changes by unequal amounts in equal intervals of time.



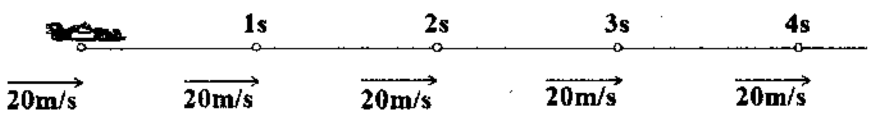
Average acceleration: When an object is moving with a variable acceleration, then the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken i.e.,

$$a_{av} = \frac{\text{total change in velocity}}{\text{total time taken}}$$

Retardation or deceleration: If the velocity decreases with time, the acceleration is negative. The negative acceleration is called retardation or deceleration.



Uniform motion: When a particle is in uniform motion, its velocity is constant, so the acceleration is zero.



Instantaneous acceleration or acceleration: Instantaneous acceleration is an acceleration of an object at any given instant of time.

Instantaneous acceleration can be expressed in terms of average acceleration as

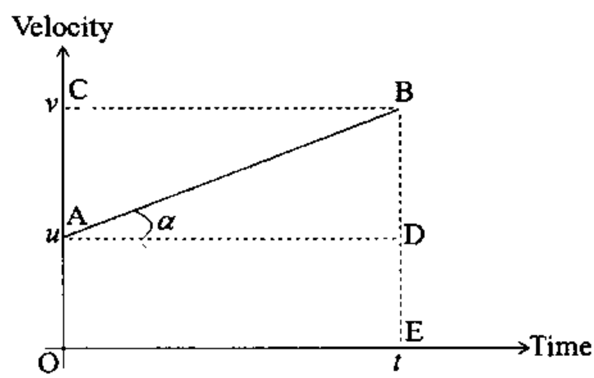
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} (a_{av}) = \frac{dv}{dt}$$

Instantaneous acceleration can be expressed in terms of position as

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2 s}{dt^2}$$

Derivation of equations of uniform accelerated motion from velocity-time graph:

Consider an object moving along a straight line with uniform acceleration a. Let u be the initial velocity of the object at time $t = 0$, v be the final velocity of the object after time t and S is the distance travelled by object in time t. The velocity-time graph of this motion is a straight line AB, as shown in the figure.



Let OA = ED = u,

OC = EB = v,

OE = t = AD.

(1) Prove that $v = u + at$

Acceleration = slope of the graph AB

$$a = \tan\alpha = \frac{DB}{AD} = \frac{EB - ED}{OE} = \frac{v-u}{t}$$

Or $v-u=at$
Or $v=u+at$ Proved

(2) Prove that $S=ut+\frac{1}{2}at^2$

$S=$ Area OADE + Area ABD

$S=OA\times OE+\frac{1}{2}\times AD\times DB$

$S=u\times t+\frac{1}{2}\times t\times(v-u)$

$S=u\times t+\frac{1}{2}\times t\times at \qquad [\because v-u=at]$

$S=ut+\frac{1}{2}at^2$ Proved

(3) Prove that $v^2-u^2=2aS$

$S=$ Area OABE

$S=\frac{1}{2}(EB+OA)\times OE$

$S=\frac{1}{2}(EB+ED)\times OE$ ------- (1)

$\because a=\frac{DB}{AD}=\frac{EB-ED}{OE}$

Or $OE=\frac{EB-ED}{a}$ ------- (2)

Putting equation (2) in equation (1) we have

$S=\frac{1}{2}(EB+ED)\times\frac{EB-ED}{a}$

$S=\frac{1}{2a}(EB^2-ED^2)$

$S=\frac{1}{2a}(v^2-u^2)$

$v^2-u^2=2aS$ Proved

Derivation of equations of uniform accelerated motion by calculus method:

Consider an object moving in a straight line with uniform acceleration a. Let v_1, v_2 are the velocities of the object in time t_1, t_2 respectively.

(1) Prove that $v=u+at$

$\because a=\frac{dv}{dt}$

Or $dv=a\,dt$

Integrating the above equation we have

$\int_{v_1}^{v_2}dv=a\int_{t_1}^{t_2}dt$

$$v_2 - v_1 = a(t_2 - t_1)$$

If $t_1 = 0$, $v_1 = u$, $v_2 = v$ and $t_2 = t$ then we have

$$v - u = a(t - 0) = at$$

$$v = u + at$$ Proved

(2) Prove that $S = ut + \frac{1}{2}at^2$

Consider an object moving in a straight line with uniform acceleration a. Let at instant t, dx be the displacement of the object in time interval dt. then

$$v = \frac{dx}{dt}$$

$$dx = v\,dt$$

Or $dx = (u + at)\,dt$ $\quad [\because v - u = at]$

Or $dx = u\,dt + at\,dt$

Integrating the above equation we have

$$\int_{x_o}^{x} dx = u\int_0^t dt + a\int_0^t t\,dt$$

$$x - x_o = u(t - 0) + a\left[\frac{t^2}{2} - \frac{0}{2}\right]$$

$$x - x_o = u\,t + a\frac{t^2}{2}$$

If $x = S$, $x_o = 0$ then we have

$$S = ut + \frac{1}{2}at^2$$ Proved

(3) Prove that $v^2 - u^2 = 2aS$

Consider a particle moving in a straight line with initial velocity u and uniform acceleration a. Then

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v$$

$$v\,dv = a\,dx$$

Let u and v be the velocity of the particle at position given by displacement x_o and x.

Integrating the above equation we have

$$\int_u^v v\,dv = a\int_{x_o}^{x} dx$$

$$\left[\frac{v^2}{2}\right]_u^v = a\left[x\right]_{x_o}^{x}$$

$$\frac{v^2}{2} - \frac{u^2}{2} = a(x - x_o)$$

$$v^2 - u^2 = 2a(x - x_o)$$

If $x = S$, $x_o = 0$ then we have

$$v^2 - u^2 = 2aS$$ Proved

Prove that distance travelled during the n^{th} second is $S_{nth} = u + \frac{a}{2}(2n-1)$

Distance travelled during n seconds is $S_n = un + \frac{1}{2}an^2$

Distance travelled during (n -1) seconds is $S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$

$\therefore$ the distance travelled in the n^{th} second is

$$S_{nth} = S_n - S_{n-1}$$

$$S_{nth} = un + \frac{1}{2}an^2 - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$S_{nth} = un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2$$

$$S_{nth} = un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n^2 + 1 - 2n)$$

$$S_{nth} = un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 - \frac{1}{2}a + an$$

$$S_{nth} = u - \frac{1}{2}a + an$$

$$S_{nth} = u - \frac{a}{2}(1-2n)$$

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

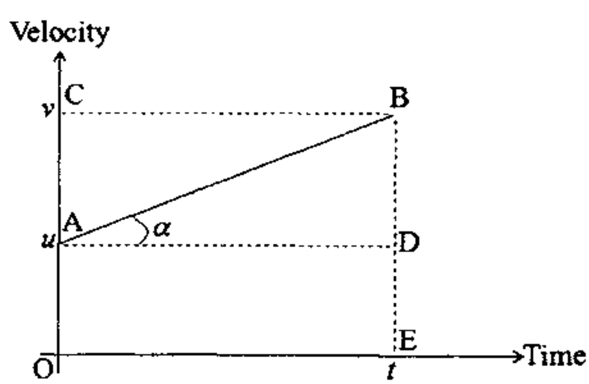
Special cases:

(1) Prove that $\bar{v} = \frac{u+v}{2}$

Given $\bar{v}$ is the average velocity, u is the initial velocity and v is the final velocity.

S = Area OADE + Area ABD

$$S = OA \times OE + \frac{1}{2} \times AD \times DB$$



$$S = u \times t + \frac{1}{2} \times t \times (v-u)$$

$$S = ut + \frac{1}{2} \times tv - \frac{1}{2} \times tu$$

$$S = \frac{1}{2}ut + \frac{1}{2} \times tv$$

$$S = \left(\frac{u}{2} + \frac{v}{2}\right)t = \left(\frac{u+v}{2}\right)t$$

$$\bar{v} = \frac{S}{t} = \left(\frac{u+v}{2}\right) \text{ Proved}$$

Note: $S = \left(\frac{v+u}{2}\right)t = \left(\frac{v+u}{2}\right)t \times \frac{(v-u)}{(v-u)} = \frac{v^2 - u^2}{2} \times \frac{t}{v-u} = \frac{v^2-u^2}{2} \times \frac{1}{a} = \frac{v^2-u^2}{2a}$

$\therefore v^2 - u^2 = 2aS$

(2) Prove that $\bar{v} = u + \frac{1}{2}at$

Consider a body moves with a constant acceleration a in time t and let u be the initial velocity.

$$\because S = ut + \frac{1}{2}at^2$$

$$\bar{v} = \frac{S}{t} = \frac{ut}{t} + \frac{1}{2t}at^2$$

$$\bar{v} = u + \frac{1}{2}at \quad \text{Proved}$$

Vertical motion under gravity:

(1) For downward motion: For a particle moving downwards, $a = g$, since the particle moves in the direction of gravity.
(2) For a freely falling body: For a freely falling body, $a = g$ and $u = 0$, since it starts from rest.
(3) For upward motion: For a particle moving upwards, $a = -g$, since the particle moves against the gravity.

Equations of motion for a freely falling body are:

(1) $v = u + gt$ (2) $h = ut + \frac{1}{2}gt^2$ (3) $v^2 - u^2 = 2gh$

(4) $h_{nth} = u + \frac{g}{2}(2n-1)$ (5) $h = \left(\frac{u+v}{2}\right)t$ (6) $\bar{v} = u + \frac{1}{2}gt$

Equations of motion for a body moving upward are:

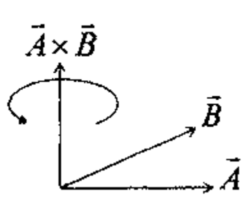
(1) $v = u - gt$ (2) $h = ut - \frac{1}{2}gt^2$ (3) $v^2 - u^2 = -2gh$

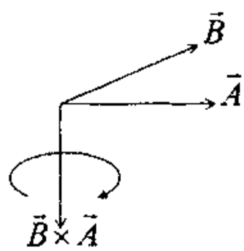
(4) $h_{nth} = u - \frac{g}{2}(2n-1)$ (5) $h = \left(\frac{u+v}{2}\right)t$ (6) $\bar{v} = u - \frac{1}{2}gt$

Types of vectors:

(1) Polar vector: Vectors which have a starting point or a point of application are polar vectors. Examples are displacement, force etc

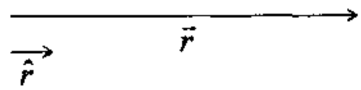
(2) Axial vectors: Vectors which represent rotational effect and act along the axis of rotation in accordance with the right hand screw rule are axial vectors. Examples are angular velocity, torque, angular momentum etc





A few definitions in vector algebra:

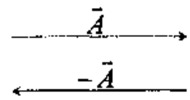
(1) Unit vector: A vector having unit magnitude is called a unit vector. It is also defined as a vector divided by its own magnitude.



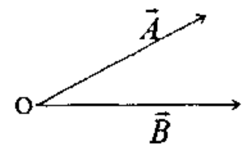
$$\hat{r} = \frac{\vec{r}}{|\vec{r}\,|} = \frac{\vec{r}}{r}$$

(2) Equal vectors: Two vectors are said to be equal if they have the same magnitude and same direction.

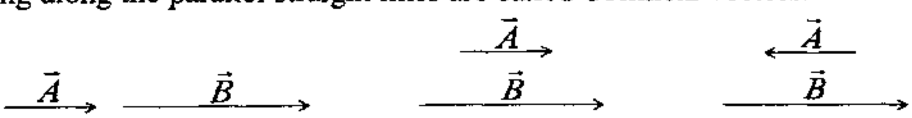
(3) Negative vectors or opposite vectors: The vectors of same magnitude but opposite in direction are called negative vectors or opposite vectors.



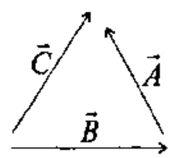
(4) Co-initial vectors: Vectors having the same starting point are called co-initial vectors.



(5) Collinear vectors: Vectors having equal or unequal magnitudes and are acting along the parallel straight lines are called Collinear vectors.



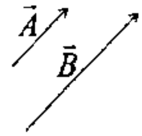
(6) Coplanar vectors: Vectors lying in the same plane are called coplanar vectors and the plane in which the vectors lie is called plane of vectors.



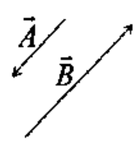
(7) Localised vector: It is a vector whose initial point is fixed. It is also called fixed vector.

(8) Non-localised vector: It is a vector whose initial point is not fixed. It is also called a free vector.

(9) Like vectors: Two vectors are said to be like vectors, if they have same direction but different magnitudes.



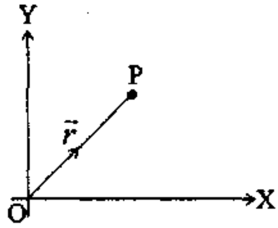
(10) Unlike vectors: The vectors of different magnitude acting in opposite directions are called unlike vectors.



(11) Null vector or zero vector: A vector whose magnitude is zero, is called a null vector or zero vector. It is represented by $\vec{0}$.

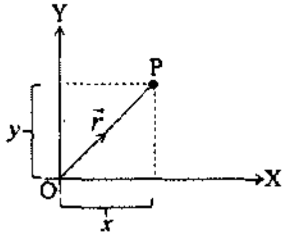
(12) Proper vector: All the non-zero vectors are called proper vectors.

(13) Position vector: The position vector of a particle is the vector from the origin of the coordinate system to the position of the particle.



In the figure $\vec{r}$ is the position vector joining the origin O of the coordinate axes X, Y and the position P of the particle.

Note: The position vector in terms of coordinates x and y is given by



$$\vec{r} = \hat{i}x + \hat{j}y$$

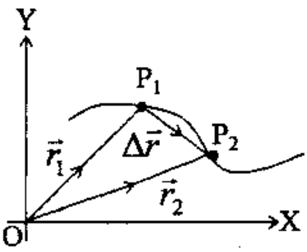
$$\vec{r}\cdot\vec{r} = (\hat{i}x + \hat{j}y)\cdot(\hat{i}x + \hat{j}y)$$

$$r^2 = (\hat{i}x\cdot\hat{i}x) + (\hat{i}x\cdot\hat{j}y) + (\hat{j}y\cdot\hat{i}x) + (\hat{j}y\cdot\hat{j}y)$$

$$r^2 = x^2 + (0) + (0) + y^2$$

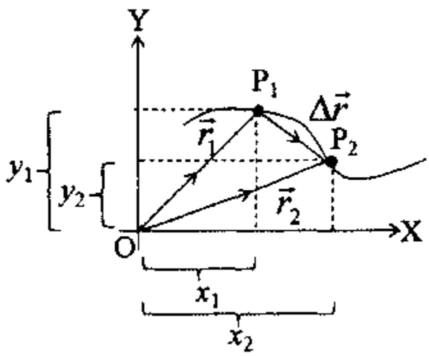
$$r^2 = x^2 + y^2 \qquad \Rightarrow r = \sqrt{x^2 + y^2}$$

(14) Displacement vector: When a particle moves, the position vector changes from $\vec{r}_1$ to $\vec{r}_2$ during a certain time interval. Then the particle's displacement $\Delta\vec{r}$ during that time interval is called the displacement vector.



In the figure $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ is called the displacement vector.

Note: The displacement vector in terms of coordinates (x_1, y_1) and (x_2, y_2) is given by



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta\vec{r} = (\hat{i}x_2 + \hat{j}y_2) - (\hat{i}x_1 + \hat{j}y_1)$$

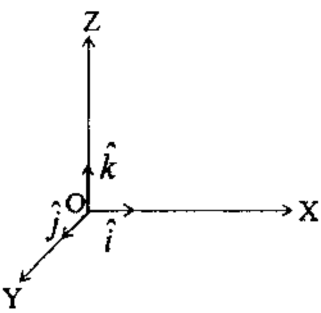
$$\Delta\vec{r} = \hat{i}x_2 + \hat{j}y_2 - \hat{i}x_1 - \hat{j}y_1$$

$$\Delta\vec{r} = \hat{i}x_2 - \hat{i}x_1 + \hat{j}y_2 - \hat{j}y_1$$

$$\Delta\vec{r} = \hat{i}(x_2 - x_1) + \hat{j}(y_2 - y_1)$$

$$\Delta\vec{r} = \hat{i}\Delta x + \hat{j}\Delta y$$

(15) Orthogonal vectors: If two or three vectors are perpendicular to each other, they are known as orthogonal vectors. For example, the cartesian coordinate axes are orthogonal vectors.



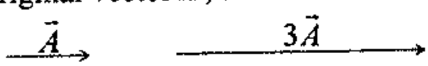
(16) Orthogonal unit vectors: There are three most common unit vectors in the positive directions of X, Y and Z axes of Cartesian coordinate system, denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively. Since they are along the mutually perpendicular directions, they are called orthogonal unit vectors.

Multiplication of a vector by a real number: Multiplication of a vector $\vec{A}$ by a real number n becomes another vector $n\vec{A}$. Its magnitude becomes n times the magnitude of the given vector. Its direction is the same or opposite as that of $\vec{A}$, according to n is a positive or negative real number. Thus

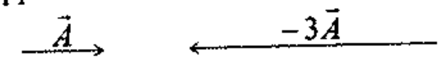
If n is positive real number, $+n(\vec{A}) = +n\vec{A}$

If n is negative real number, $-n(\vec{A}) = -n\vec{A}$

Note: If a vector $\vec{A}$ is multiply by 3, we have a new vector $3\vec{A}$ whose magnitude is three times the original vector $\vec{A}$, and the direction is in the direction of $\vec{A}$ as shown in figure (1). But if a vector $\vec{A}$ is multiply by −3, we have a new vector $-3\vec{A}$ whose magnitude is three times the original vector $\vec{A}$, and the direction is opposite to $\vec{A}$ as shown in figure (2).

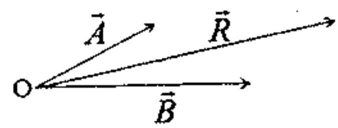


(1)



(2)

Resultant vector: The resultant vector of two or more vectors is defined as that single vector which produces the same effect as is produced by the individual vectors together.



In the figure, $\vec{R}$ is the resultant vector of two vectors $\vec{A}$ and $\vec{B}$.

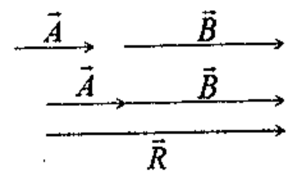
Addition of vectors:

(1) When the two vectors are acting in the same direction:

The resultant vector is $\vec{R} = \vec{A} + \vec{B}$

The magnitude of $\vec{R}$ is $|\vec{R}| = |\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$

Or $R = A + B$

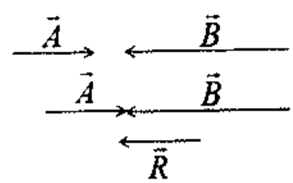


(2) When two vectors are acting in opposite directions:

The resultant vector is $\vec{R} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$

The magnitude of $\vec{R}$ is $|\vec{R}| = |\vec{A} - \vec{B}| = |\vec{A}| - |\vec{B}|$

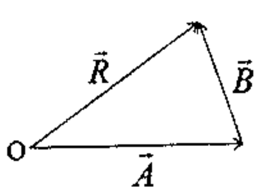
Or $R = A - B$



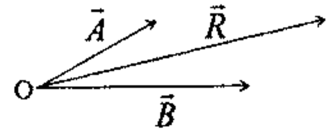
(3) When two vectors act at some angle:

The resultant vector in both the cases is $\vec{R} = \vec{A} + \vec{B}$

But the magnitude of $\vec{R}$ is not $R = A + B$.

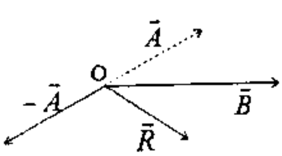


(1)

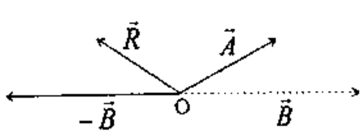


(2)

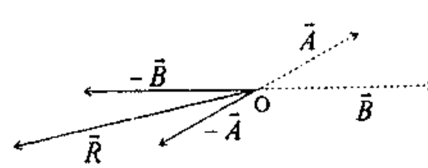
Note: In figure (2) we can have three more direction of $\vec{R}$ as shown below



(1)



(2)



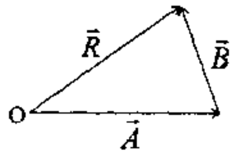
(3)

Figure (1) $\vec{R} = \vec{B} - \vec{A}$

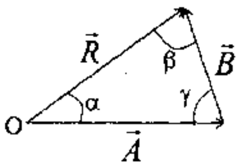
Figure (2) $\vec{R} = \vec{A} - \vec{B}$

Figure (3) $\vec{R} = -\vec{A} - \vec{B} = -(\vec{A} + \vec{B})$

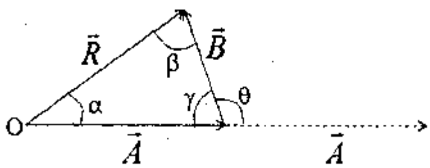
Triangular law of vectors: If two vectors are represented in magnitude and direction by the two adjacent sides of a triangle taken in order, then their resultant is the closing side of the triangle taken in the reverse order.



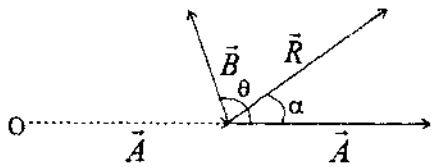
Note: In the figures below we have



(1)



(2)



(3)

Figure (1) $\dfrac{A}{\sin\beta} = \dfrac{B}{\sin\alpha} = \dfrac{R}{\sin\gamma}$

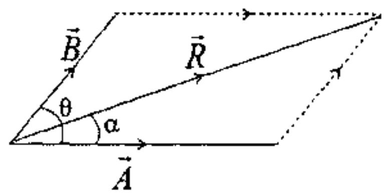
Figure (2) $\dfrac{A}{\sin\beta} = \dfrac{B}{\sin\alpha} = \dfrac{R}{\sin(180^0 - \theta)}$

$$\frac{A}{\sin\beta} = \frac{B}{\sin\alpha} = \frac{R}{\sin\theta}$$

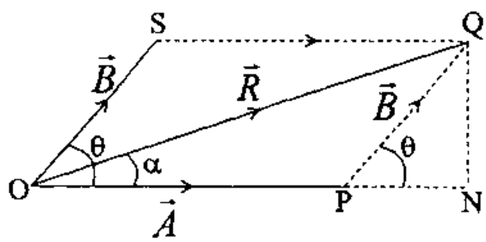
Figure (3) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Parallelogram law of vectors: If two vectors acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal passing through the common tail of the two vectors.



Note: In the right angled triangle OQN we have



$$OQ^2 = ON^2 + QN^2$$
$$OQ^2 = (OP + PN)^2 + QN^2$$
$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$
$$R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$
$$R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$
$$R^2 = A^2 + B^2 + 2AB\cos\theta$$
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{QN}{ON} = \frac{QN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\alpha = \tan^{-1}\frac{B\sin\theta}{A+B\cos\theta}$$

Special cases:

(1) When two vectors $\vec{A}$ and $\vec{B}$ are acting in the same direction,

then $\theta = 0^0$, $\cos\theta = 1$ and $\sin\theta = 0$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 + 2AB} = \sqrt{(A+B)^2}$$

$$\therefore R = A + B$$

$$\alpha = \tan^{-1}\frac{B\sin\theta}{A+B\cos\theta} = \tan^{-1}\frac{0}{A+B} = \tan^{-1}0$$

$$\therefore \alpha = 0^0$$

(2) When two vectors $\vec{A}$ and $\vec{B}$ are acting in opposite direction,

then $\theta = 180^0$, $\cos\theta = -1$ and $\sin\theta = 0$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 - 2AB} = \sqrt{(A-B)^2}$$

$$\therefore R = A - B$$

$$\alpha = \tan^{-1}\frac{B\sin\theta}{A+B\cos\theta} = \tan^{-1}\frac{0}{A+B} = \tan^{-1}0$$

$$\therefore \alpha = 0^0 \text{or } 180^0$$

(3) When two vectors $\vec{A}$ and $\vec{B}$ act at right angle to each other,

then $\theta = 90^0$, $\cos\theta = 0$ and $\sin\theta = 1$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2}$$

$$\therefore R^2 = A^2 + B^2$$

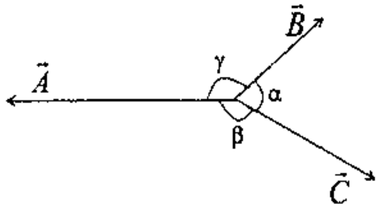
$$\alpha = \tan^{-1}\frac{B\sin\theta}{A+B\cos\theta} = \tan^{-1}\frac{B}{A}$$

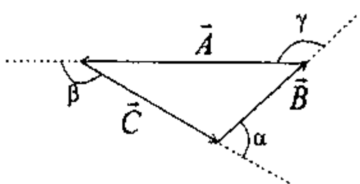
$$\therefore \alpha = \tan^{-1}\left(\frac{B}{A}\right)$$

Lami's theorem: If three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces i.e.,

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

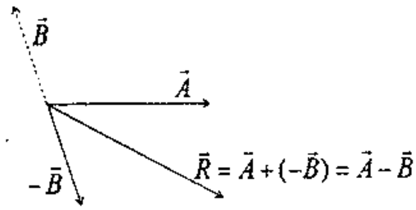
Where $\vec{A}$, $\vec{B}$ and $\vec{C}$ are the three forces and α, β and γ are the angles between forces $\vec{B}$ and $\vec{C}$, $\vec{C}$ and $\vec{A}$ and $\vec{A}$ and $\vec{B}$ respectively.



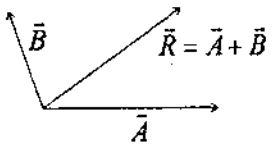


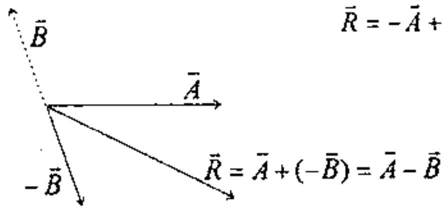
Subtraction of vectors: Subtraction of a vector $\vec{B}$ from a vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ (negative of vector $\vec{B}$) to a vector $\vec{A}$. Thus

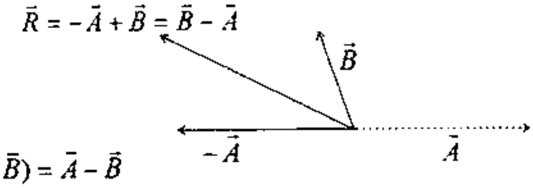
$$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$$



Note: Figures show the addition of a vector $\vec{A}$ and a vector $\vec{B}$, subtraction of a vector $\vec{B}$ from a vector $\vec{A}$ and subtraction of a vector $\vec{A}$ from a vector $\vec{B}$.







Relative velocity in two dimensions: Let two objects A and B are moving in a plane with velocities $\vec{v}_A$ and $\vec{v}_B$, and θ be the angle between their directions.

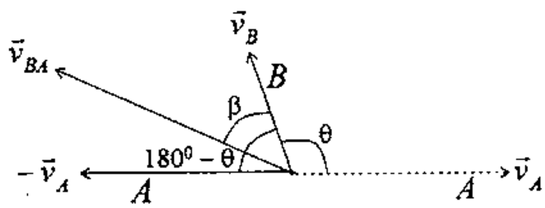
(1) The relative velocity of B w.r.t A is shown in figure (1)

$$\vec{v}_{BA}=\sqrt{v_A^2+v_B^2+2v_Av_B\cos(180^0-\theta)}$$

$$\vec{v}_{BA}=\sqrt{v_A^2+v_B^2-2v_Av_B\cos\theta}$$

$$\tan\beta=\frac{v_A\sin(180^0-\theta)}{v_B+v_A\cos(180^0-\theta)}$$

$$\tan\beta=\frac{v_A\sin\theta}{v_B-v_A\cos\theta}$$



(1)

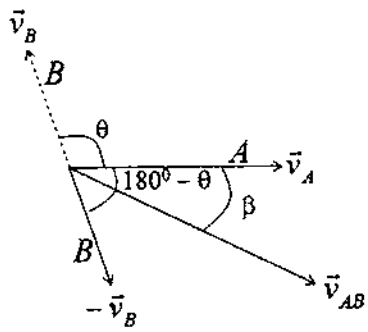
(2) The relative velocity of A w.r.t B is shown in figure (2)

$$\vec{v}_{AB}=\sqrt{v_B^2+v_A^2+2v_Bv_A\cos(180^0-\theta)}$$

$$\vec{v}_{BA}=\sqrt{v_A^2+v_B^2-2v_Av_B\cos\theta}$$

$$\tan\beta=\frac{v_B\sin(180^0-\theta)}{v_A+v_B\cos(180^0-\theta)}$$

$$\tan\beta=\frac{v_B\sin\theta}{v_A-v_B\cos\theta}$$

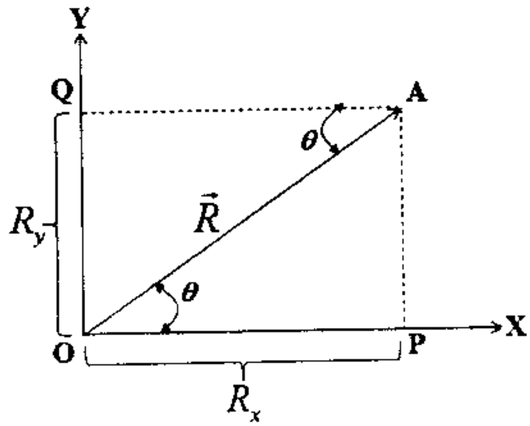


(2)

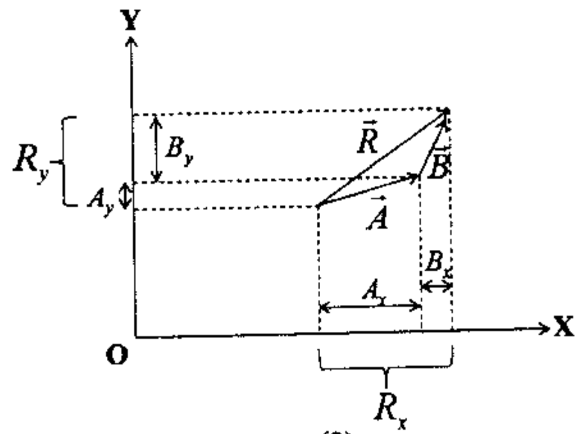
Resolution of vectors and rectangular components: A vector directed at an angle with the co-ordinate axis, can be resolved into its components along the axes. This process of splitting a vector into its components is known as resolution of a vector.

Note: Any number of vectors can be combined to give a single equivalent vector, therefore a vector can be resolved into any number of components.

Rectangular components of a vector in two dimensions (2D):



(1)



(2)

Consider a vector $\vec{R}$ in the X-Y plane, this vector can be resolved into two rectangular components R_x and R_y along the X-axis and Y-axis as shown in the figure (1).

$$\vec{R} = \vec{R}_x + \vec{R}_y$$

$$\vec{R} = \hat{i}R_x + \hat{j}R_y$$

In ΔAOP and ΔAOQ we have

$$\frac{R_x}{R} = \cos\theta \qquad \Rightarrow R_x = R\cos\theta \text{ ------- } (1)$$

and $$\frac{R_y}{R} = \sin\theta \qquad \Rightarrow R_y = R\sin\theta \text{ ------- } (2)$$

A vector $\vec{R}$ can also be written as $\vec{R} = \hat{i}R\cos\theta + \hat{j}R\sin\theta = R\,(\hat{i}\cos\theta + \hat{j}\sin\theta)$

Squaring and adding equation (1) and equation (2) we have

$$(R\cos\theta)^2 + (R\sin\theta)^2 = (R_x)^2 + (R_y)^2$$

$$R^2(\cos^2\theta + \sin^2\theta) = R_x^2 + R_y^2$$

$$R^2 = R_x^2 + R_y^2$$

$$R = \sqrt{R_x^2 + R_y^2}$$

Dividing equation (2) by equation (1) we have

$$\frac{R\sin\theta}{R\cos\theta} = \frac{R_y}{R_x} \qquad \Rightarrow \tan\theta = \frac{R_y}{R_x}$$

$$\text{Or } \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

In the figure (2) we have

$$R_x = A_x + B_x \quad \text{and} \quad R_y = A_y + B_y$$

$$\because \vec{R} = \hat{i}R_x + \hat{j}R_y$$

$$\therefore \vec{R} = \hat{i}(A_x + B_x) + \hat{j}(A_y + B_y)$$

Note:

(1) By using the dot product

$\because \vec{R} = \hat{i}R_x + \hat{j}R_y$

$$\vec{R}\cdot\vec{R}=(\hat{i}R_x+\hat{j}R_y)\cdot(\hat{i}R_x+\hat{j}R_y)$$

$$R^2=(\hat{i}R_x\cdot\hat{i}R_x)+(\hat{i}R_x\cdot\hat{j}R_y)+(\hat{j}R_y\cdot\hat{i}R_x)+(\hat{j}R_y\cdot\hat{j}R_y)$$

$$R^2=R_x^2+R_y^2$$

$$R=\sqrt{R_x^2+R_y^2}$$

(2) By using the Pythagoras theprem
In the right angled triangle AOP, we have

$$R^2=R_x^2+R_y^2$$

$$\therefore R=\sqrt{R_x^2+R_y^2}$$

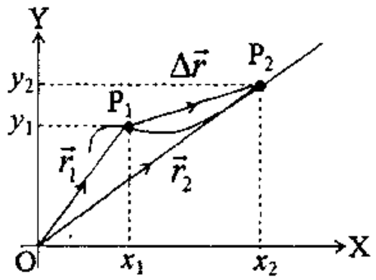
(3) In three dimensions (3D) $\vec{R}$ can be written as

$$\vec{R}=\hat{i}R_x+\hat{j}R_y+\hat{k}R_z$$

$$R^2=R_x^2+R_y^2+R_z^2$$

$$R=\sqrt{R_x^2+R_y^2+R_z^2}$$

The velocity vector: Consider a particle moving along a curved path in the XY plane as shown in the figure. At time t_1 the particle is at point P_1 and at time t_2 the particle is at point P_2. The vector $\vec{r}_1$ is the position vector of the particle at time t_1 and vector $\vec{r}_2$ is the position vector of the particle at time t_2.



Displacement $\Delta\vec{r}=\vec{r}_2-\vec{r}_1=\hat{i}\Delta x+\hat{j}\Delta y$

Time interval $\Delta t=t_2-t_1$

Or $\vec{v}_{av}=\frac{\Delta\vec{r}}{\Delta t}=\frac{\hat{i}\Delta x+\hat{j}\Delta y}{\Delta t}$

Or $\vec{v}_{av}=\hat{i}\frac{\Delta x}{\Delta t}+\hat{j}\frac{\Delta y}{\Delta t}$

Or $\vec{v}_{av}=\hat{i}v_x+\hat{j}v_y$

Where $v_x=\frac{\Delta x}{\Delta t}$ and $v_y=\frac{\Delta y}{\Delta t}$

Instantaneous velocity vector: The instantaneous velocity vector is given by

$$\vec{v}=\lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t}=\frac{d\vec{r}}{dt}$$

$$\vec{v}=\frac{\hat{i}dx+\hat{j}dy}{dt}=\hat{i}\frac{dx}{dt}+\hat{j}\frac{dy}{dt}$$

$$\vec{v}=\hat{i}v_x+\hat{j}v_y$$

Note :

(1) The magnitude of the velocity is

$$v=\sqrt{v_x^2+v_y^2}$$

(2) The direction of the particle at any time is described by an angle θ between the velocity vector and the X axis as

$$\tan\theta = \frac{v_y}{v_x} \qquad \Rightarrow \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

The acceleration vector: The acceleration of a body is given by

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

Instantaneous acceleration vector:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{\hat{i}dv_x + \hat{j}dv_y}{dt} = \hat{i}\frac{dv_x}{dt} + \hat{j}\frac{dv_y}{dt}$$

$$\vec{a} = \hat{i}a_x + \hat{j}a_y$$

Where $a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$ and $a_y = \frac{dv_y}{dt} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d^2y}{dt^2}$

Laws of vector algebra: If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are vectors and m and n are scalars, then

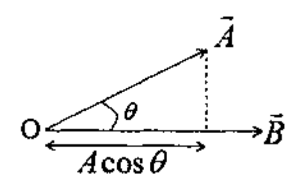
(1) $\vec{A}+\vec{B}=\vec{B}+\vec{A}$	Commutative law for addition
(2) $\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$	Associative law for addition
(3) $m\vec{A}=\vec{A}m$	Commutative law for multiplication
(4) $m(n\vec{A})=(mn)\vec{A}$	Associative law for multiplication
(5) $(m+n)\vec{A}=m\vec{A}+n\vec{A}$	Distributive law
(6) $m(\vec{A}+\vec{B})=m\vec{A}+m\vec{B}$	Distributive law

The dot or scalar product: The multiplication of one vector by another vector so as to produce a scalar is called scalar product or dot product of vectors.

If $\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, then

$$\vec{A}\cdot\vec{B} = A\cos\theta(B) = AB\cos\theta, \qquad 0 \le \theta \le \pi$$

Where $A\cos\theta$ is the projection of a vector $\vec{A}$ on a vector $\vec{B}$



The following laws are valid:

(1) $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$	Commutative law for dot product
(2) $\vec{A}\cdot(\vec{B}+\vec{C})=\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$	Distributive law
(3) $m(\vec{A}\cdot\vec{B})=(m\vec{A})\cdot\vec{B}=\vec{A}\cdot(m\vec{B})=(\vec{A}\cdot\vec{B})m$	Where m is a scalar
(4) $\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$	Where $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors

(5) $\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$

(6) If $\vec{A}=\hat{i}A_x+\hat{j}A_y+\hat{k}A_z$ and $\vec{B}=\hat{i}B_x+\hat{j}B_y+\hat{k}B_z$, then

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A}\cdot\vec{A} = A^2 = A_x^2 + A_y^2 + A_z^2$$

$$\vec{B}\cdot\vec{B} = B^2 = B_x^2 + B_y^2 + B_z^2$$

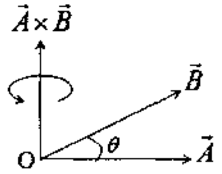
(7) If $\vec{A}\cdot\vec{B}=0$, $\vec{A}$ and $\vec{B}$ are not null vectors, then $\vec{A}$ and $\vec{B}$ are perpendicular.

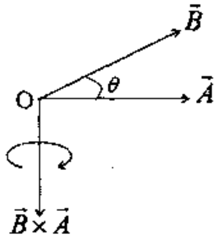
The cross or vector product: The multiplication of one vector by another vector so as to produce a vector is called vector product or cross product of vectors.

If $\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, then

$$\vec{A}\times\vec{B}=AB\sin\theta\,\hat{u}\,, \qquad 0\le\theta\le\pi$$

Where $\hat{u}$ is a unit vector indicating the direction of $\vec{A}\times\vec{B}$.





In the above figures, it is clear that the direction of $\vec{A}\times\vec{B}$ is opposite to that of $\vec{B}\times\vec{A}$

The following laws are valid:

(1) $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$ — Commutative law for cross product fails

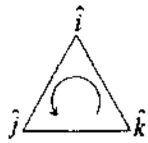
(2) $\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$ — Distributive law

(3) $m\,(\vec{A}\times\vec{B})=(m\vec{A})\times\vec{B}=\vec{A}\times(m\vec{B})=(\vec{A}\times\vec{B})m$ — Where m is a scalar

(4) $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$ — Where $\hat{i}\,,\hat{j}\,,\hat{k}$ are unit vectors

(5) $\hat{i}\times\hat{j}=\hat{k} \qquad \hat{j}\times\hat{k}=\hat{i} \qquad \hat{k}\times\hat{i}=\hat{j}$

(6) $\hat{j}\times\hat{i}=-\hat{k} \qquad \hat{k}\times\hat{j}=-\hat{i} \qquad \hat{i}\times\hat{k}=-\hat{j}$



(7) If $\vec{A}=\hat{i}A_x+\hat{j}A_y+\hat{k}A_z$ and $\vec{B}=\hat{i}B_x+\hat{j}B_y+\hat{k}B_z$, then

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

Or $$\vec{A}\times\vec{B}=\hat{i}(A_yB_z-B_yA_z)+\hat{j}(A_zB_x-B_zA_x)+\hat{k}(A_xB_y-B_xA_y)$$

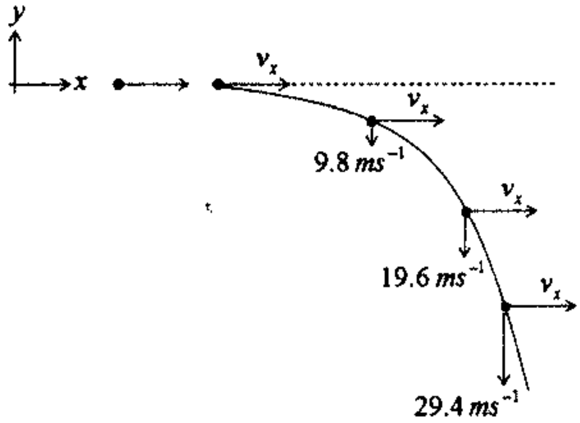
(8) The magnitude of $\vec{A}\times\vec{B}$ i.e., $|\,\vec{A}\times\vec{B}\,|$ is the same as the area of a parallelogram with sides $\vec{A}$ and $\vec{B}$.

(9) If $\vec{A}\times\vec{B}=0$, $\vec{A}$ and $\vec{B}$ are not null vectors, then $\vec{A}$ and $\vec{B}$ are parallel.

Projectile: A projectile is an object that is thrown into the air and moves under the influence of gravity alone. The path followed by the projectile is called trajectory. Examples of projectile are

(1) A body dropped from the window of a moving train.
(2) A bomb released from an aeroplane in flight.
(3) A bullet fired from a rifle.
(4) A piece of stone thrown in any direction.
(5) A javelin or hammer, thrown by an athlete.

A projectile moves under the combined effect of two velocities
(1) Uniform velocity in the horizontal direction, provided there is no air resistance.
(2) Uniformly changing velocity (increasing or decreasing) in the vertical direction due to gravity.

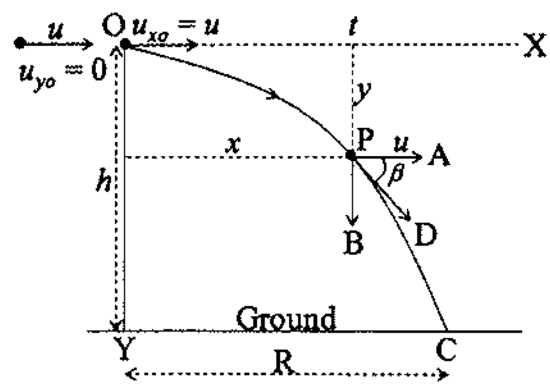


Two independent motion of a projectile

Motion	Forces	Velocity	Acceleration
Horizontal	No force acts	Constant	Zero
Vertical	The force of gravity acts downward	Changes (9.8 ms^{-1})	Downwards (9.8 ms^{-2})

Projectile given horizontal projection:

Motion of the projectile along the horizontal direction OX: Since the velocity of an object in the horizontal direction is constant, so the acceleration a_x along the horizontal direction is zero. The position of an object at any time t along the horizontal direction is given by



$$x = u_{xo}t + \frac{1}{2}a_x t^2$$

Or $x = ut + \frac{1}{2}(0)t^2$

Or $x = ut$

Motion of the projectile along the vertical direction OY: Since the vertical velocity of the object is increasing downwards due to gravity, hence the acceleration of the object a_y is + g. The position of the object at any time t along the vertical direction is given by

$$y = u_{yo}t + \frac{1}{2}a_y t^2$$

Or $y = (0)t + \frac{1}{2}gt^2$

Or $y = \frac{1}{2}gt^2$

$\because x = ut \qquad \Rightarrow t = \frac{x}{u}$

$$\therefore y=\frac{1}{2}g\left(\frac{x}{u}\right)^2=\left(\frac{1}{2}\frac{g}{u^2}\right)x^2$$

Or $y=k\,x^2$

Hence the path of the projectile projected horizontally from a certain height from the ground is a parabolic path.

Time of flight: It is the total time for which the projectile is in flight.
Let h be the vertical height of point of projection of the projectile from the ground, and the time taken by the projectile to hit the ground is T. thus

$$y=u_{yo}t+\frac{1}{2}a_yt^2$$

Or $h=(0)T+\frac{1}{2}gT^2$

Or $h=\frac{1}{2}gT^2$

Or $T=\sqrt{\frac{2h}{g}}$

Horizontal range: It is the horizontal distance travelled by the projectile during its flight. It is denoted by R.

$\because x=ut$

Or $R=uT$

$$\therefore R=u\sqrt{\frac{2h}{g}}$$

Velocity of the projectile at any instant: At any instant t, the projectile possesses two perpendicular velocities v_x and v_y along the X-axis and Y-axis respectively.

$v_x=u$

and $v_y=u_{yo}+a_yt$

Or $v_y=(0)+gt=gt$

The resultant velocity $\vec{v}$ of $\vec{v}_x$ and $\vec{v}_y$ is given by

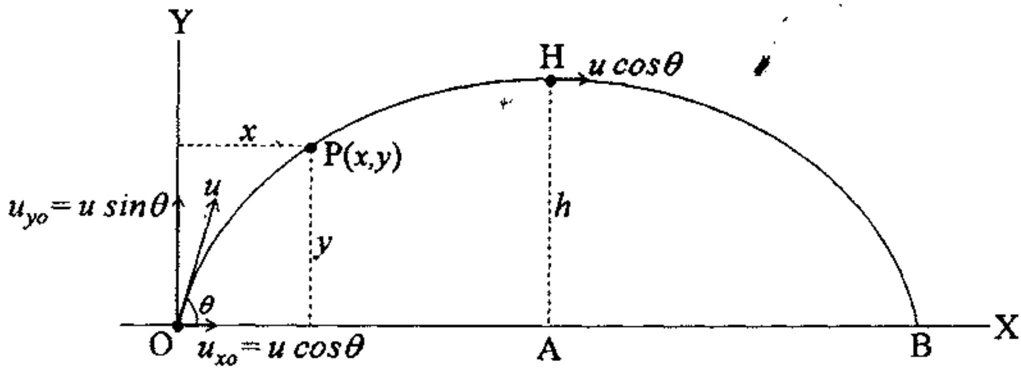
$$v=\sqrt{v_x^2+v_y^2}=\sqrt{u^2+g^2t^2}$$

Let $\vec{v}$ makes an angle β with the horizontal direction, then

$$\tan\beta=\frac{v_y}{v_x}=\frac{gt}{u}$$

Or $\beta=\tan^{-1}\left(\frac{gt}{u}\right)$

Projectile given angular projection:



Motion of the projectile along the horizontal direction OX: Since the velocity of an object in the horizontal direction is constant, so the acceleration a_x in the horizontal direction is zero. The position of the object at any time t along the horizontal direction is given by

$$x = u_{xo}t + \frac{1}{2}a_x t^2$$

$$x = u\cos\theta\, t + \frac{1}{2}a(0)t^2$$

$$x = u\cos\theta\, t$$

Motion of the projectile along the vertical direction OY: Since the vertical velocity of the object is decreasing from O to P due to gravity, hence acceleration a_y is $-g$. The position of the object at any time t along the vertical direction is given by

$$y = u_{yo}t + \frac{1}{2}a_y t^2$$

$$y = u\sin\theta\, t - \frac{1}{2}g\, t^2$$

$$\because x = u\cos\theta\, t \qquad \Rightarrow t = \frac{x}{u\cos\theta}$$

$$\therefore y = u\sin\theta\frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

Or $\quad y = x\dfrac{u\sin\theta}{u\cos\theta} - \dfrac{1}{2}g\dfrac{x^2}{u^2\cos^2\theta}$

Or $\quad y = x\tan\theta - \left(\dfrac{1}{2}\dfrac{g}{u^2\cos^2\theta}\right)x^2$

The path of a projectile projected at some angle with the horizontal direction from the ground is a parabolic path.

Time of flight or total time of flight: It is the total time for which the object is in flight. It is denoted by T.

Since time of ascent = time of decent = t (say)

As total time of flight = time of ascent + time of decent

$$T = t + t = 2t$$

$$\text{Or } t = \frac{T}{2}$$

At the highest point H, the vertical velocity v_y is zero. then

$$v_y = u_{yo} + a_y t$$

$$0 = u\sin\theta - g\left(\frac{T}{2}\right)$$

$$\text{Or } g\left(\frac{T}{2}\right) = u\sin\theta$$

$$\text{Or } T = \frac{2u\sin\theta}{g}$$

Time to reach the greatest height: If t be the time to reach the greatest height, then at the highest point the vertical velocity v_y is zero. Then

$$v_y = u_{yo} - a_y t \qquad \text{Or} \quad 0 = u\sin\theta - gt$$

$$\therefore t = \frac{u\sin\theta}{g}$$

Maximum height: It is the maximum vertical height attained by the object above the point of projection during its flight. It is denoted by h.

$$y = u\sin\theta\, t - \frac{1}{2}g t^2$$

$$h = u\sin\theta \frac{u\sin\theta}{g} - \frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2$$

$$h = \frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g}$$

$$h = \frac{u^2\sin^2\theta}{2g}$$

Note: At the highest point, the linear momentum is $mu\cos\theta$, and the kinetic energy is $\frac{1}{2}m(u\cos\theta)^2$.

Horizontal range: It is the horizontal distance covered by the object between its point of projection and the point of hitting the ground. It is denoted by R.

$$x = u\cos\theta\, t$$

$$R = u\cos\theta\, T$$

$$R = u\cos\theta \frac{2u\sin\theta}{g} = \frac{u^2(2\sin\theta\cos\theta)}{g}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

Note:

(1) The maximum horizontal range is when $\sin 2\theta$ is maximum i.e., $\sin 2\theta = 1$. Hence

$$R = \frac{u^2}{g}$$

Now $\sin 2\theta = 1$

Or $2\theta = \sin^{-1}(1)$

Or $2\theta = 90^0$

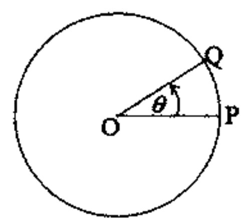
$\therefore \theta = 45^0$

(2) For the angle of projection θ and $(90^0 - \theta)$, the horizontal range is the same. So if the angle of projection is 15^0 or 75^0, the horizontal range is the same.

(3) The maximum height occurs when the projectile covers a horizontal distance equal to half the horizontal range i.e., $\frac{R}{2}$

(4) When the maximum range of the projectile is R, then its maximum height is $\frac{R}{4}$

Angular displacement: Angular displacement of an object moving around a circular path is defined as the angle traced out by the radius vector at the axis of the circular path in a given time.



Since angle = arc/radius,

$\therefore \theta = PQ/r$

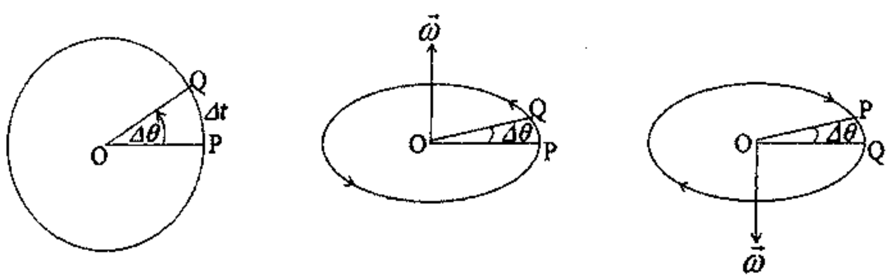
The angular displacement is expressed in radians (denoted by *rad*)

The angular displacement is a vector quantity.

Note: The angular displacement is a vector quantity provided θ is small because the commutative law of vector addition for large angles, is not valid, where as for small angles, the law is valid.

Angular velocity: Angular velocity ω of an object in circular motion is defined as the time rate of change of its angular displacement i.e.,

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$



Its SI unit is radian per second (denoted by *rad/s* or $rad\, s^{-1}$). Its dimensional formula is $[M^0L^0T^{-1}]$.

Angular velocity is a vector quantity, and its direction is the same as that of $\Delta\vec{\theta}$.

Note: It is important to note that nothing actually moves in the direction of the angular velocity vector $\vec{\omega}$. The direction of $\vec{\omega}$ simply represents that the rotational motion is taking place in a plane perpendicular to it.

Uniform circular motion: When a point object is moving in a circular path with a constant speed, then the motion of the object is said to be uniform circular motion.

(1) Time period: The time period is defined as the time taken by the object to complete one revolution on its circular path. It is denoted by T and is expressed in second.

(2) Frequency: The frequency is defined as the number of revolutions completed by the object on its circular path in a unit time. It is denoted by ν. Its unit is s^{-1} or hertz (Hz).

Relation between time period and frequency:

If ν revolutions, the time taken = 1 second

$\therefore$ in 1 revolution, the time taken = $\frac{1}{\nu}$ second

$$\text{Or} \quad T = \frac{1}{\nu}$$

$$\text{Or} \quad T\nu = 1$$

Relation between angular velocity, time period and frequency:

When time t = T, then $\theta = 2\pi$

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T}$$

Also $T = \frac{1}{\nu}$

$$\therefore \omega = 2\pi\nu$$

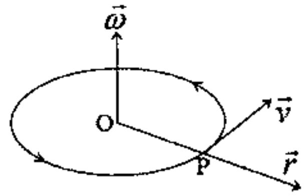
Relation between linear velocity and angular velocity:

When time t = T, then $\theta = 2\pi$

Angular velocity $\omega = \frac{\theta}{t} = \frac{2\pi}{T}$

Linear velocity $v = \frac{2\pi\, r}{T}$ where r is the radius of the circular path.

$$\text{Or} \quad v = \left(\frac{2\pi}{T}\right) r = \omega\, r$$



In vector notation we can write

(1) $\vec{v} = \vec{\omega} \times \vec{r}$

(2) $\vec{\omega} = \vec{r} \times \vec{v}$

(3) $\vec{r} = \vec{v} \times \vec{\omega}$

Angular acceleration: Angular acceleration α of an object in circular motion is defined as the time rate of change of its angular velocity i.e.,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Its SI unit is radian per second per second (denoted by *rad/s*^{2} or *rad s*$^{-2}$). Its dimensional formula is $[M^0L^0T^{-2}]$.

Relation between linear acceleration and angular acceleration: We know that the relation between the linear velocity and angular velocity is

$$v = \omega r$$

$$\therefore \frac{dv}{dt} = \frac{d\omega}{dt} r$$

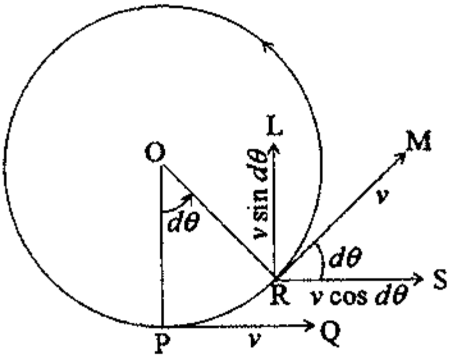
Or $a = \alpha r$

In vector notation we can write

$$\vec{a} = \vec{\alpha} \times \vec{r}$$

Centripetal acceleration: Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.

As shown in the figure, let an object moves from P to R with angular velocity ω, making a small angular displacement $d\theta$, and let the linear velocity at any point is v.
At P the horizontal component of v is PQ = $v \cos 0^0 = v$, and the vertical component is PO = $v \sin 0^0 = 0$.
At R the horizontal component of v is RS = $v \cos d\theta$, and the vertical component is RL = $v \sin d\theta$.



Change in velocity along the horizontal direction is

$$dv = v \cos d\theta - v = v - v = 0 \qquad [\because \cos d\theta \approx 1]$$

Change in velocity along the vertical direction is

$$dv = v \sin d\theta - 0 = v\, d\theta \qquad [\because \sin d\theta \approx d\theta]$$

Or $\frac{dv}{dt} = \frac{v\, d\theta}{dt} = v \frac{d\theta}{dt} = v\omega$

Or $a = v \frac{v}{r}$

Or $a = \frac{v^2}{r}$

This acceleration is directed towards the centre of the circle along the radius and perpendicular to the velocity of the particle.
This acceleration is known as centripetal or radial or normal acceleration.

LAWS OF MOTION

Force: A force is an agent that produces or tends to produce acceleration (or retardation) in a body. Force has magnitude and direction, so it is a vector quantity.

Or

Force is a push or a pull which changes or tries to change the state of rest, the state of uniform motion, the size or the shape of the body.
The SI unit of force is newton (N) and its dimensional formula is $[MLT^{-2}]$

Note: If several forces act simultaneously on the same object, it is the net force that determines the motion of the object. The net force is the vector sum of all the forces acting on the object. We often call the net force as the resultant force.

Inertia: The tendency of the body to maintain its state of rest or of uniform motion in a straight line is called inertia.
The inertia of a body depends upon its mass. The greater the mass of the body, the greater is its inertia. Both mass and inertia are measured in the same unit.

Types of inertia: Inertia of a body is of three types viz, (1) inertia of rest (2) inertia of motion (3) inertia of direction

(1) Inertia of rest: The resistance of a body to change its state of rest is called inertia of rest.
(2) Inertia of motion: The resistance of a body to change its state of motion is called inertia of motion.
(3) Inertia of direction: The resistance of a body to change its direction of motion is called inertia of direction.

Mass: The quantity of matter contained in the body is called mass.

Note: Mass = Density × Volume

Linear momentum: Linear momentum of a body is the quantity of motion contained in the body.

Or

Linear momentum $\vec{p}$ of a body with mass m travelling with velocity $\vec{v}$ is defined as the product of the mass and velocity i.e.,

$$\vec{p} = m\vec{v}$$

Its SI unit is $kg\ ms^{-1}$ and its dimensional formula is $[MLT^{-1}]$.

Note: The momentum associated with rotation is called angular momentum.

Newton's laws of motion:
(1) First law: A body continues to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change the state.

Note:
(1) Newton's first law of motion is also called law of inertia
(2) If no net force acts on the body, then the velocity of the body cannot change i.e., the body cannot accelerate

(2) Second law: The rate of change of linear momentum of a body is directly proportional to the external force applied on the body, and this change always takes place in the direction of the applied force.

If $\vec{p}$ is the momentum of a body and $\vec{F}$ the net external force applied on the body. Then according to Newton's second law of motion we have

$$\vec{F} \propto \frac{d\vec{p}}{dt}$$

$$\vec{F} = k\frac{d\vec{p}}{dt}$$

Where k is a constant of proportionality. In SI or cgs unit $k = 1$

$$\therefore \vec{F} = \frac{d\vec{p}}{dt}$$

Or $\vec{F} = \frac{d}{dt}(m\vec{v})$

Or $\vec{F} = m\frac{d\vec{v}}{dt}$

But $\frac{d\vec{v}}{dt} = \vec{a}$ the acceleration produced in the body

$$\therefore \vec{F} = m\vec{a}$$

If the acceleration produced is in three dimensions, having components a_x, a_y, a_z along X-axis, Y-axis, Z-axis respectively, then

$$\vec{a} = \hat{i}a_x + \hat{j}a_y + \hat{k}a_z$$

$$\therefore \vec{F} = m\vec{a} = m(\hat{i}a_x + \hat{j}a_y + \hat{k}a_z)$$

If F_x, F_y, F_z are the components of $\vec{F}$ along X-axis, Y-axis, Z-axis respectively, then

$$\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$$

Or $\hat{i}F_x + \hat{j}F_y + \hat{k}F_z = \hat{i}ma_x + \hat{j}ma_y + \hat{k}ma_z$

$$\therefore \quad F_x = ma_x, \qquad F_y = ma_y, \qquad F_z = ma_z$$

The SI unit of force is $kgms^{-2}$ or newton (N). Hence $1N$ is that force which produces an acceleration of $1ms^{-2}$ in a mass of $1kg$.
Its dimensional formula is $[MLT^{-2}]$.

Note:
(1) $1N = 10^5\ dyne$
(2) 1 *kg-wt* or 1 *kg-f* = $1\ kg \times 9.8\ ms^{-2} = 9.8\ N$
(3) Forces acting simultaneously at the same point on the body are called concurrent forces and forces acting in the same plane are called coplanar forces. Forces acting parallel to each other are called collinear forces.

(3) Third law: To every action, there is an equal and opposite reaction.

Note:
(1) Action and reaction forces act on different bodies
(2) Newton's second law of motion is called the real law of motion because the first and the third laws of motion can be obtained by it.
(3) If no external force acts on the system, then its total linear momentum remains conserved.
(4) Linear momentum depends on the frame of reference but the law of conservation of linear momentum is independent of the frame of reference.
(5) Newton's laws of motion are valid only in the inertial frame of reference.

Impulse or impulse of a force: Impulse of a force is a measure of total effect of the force. It is given by the product of force and the time for which the force acts on the body i.e.,

Impulse = Force × time

According to Newton's second law of motion,

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\therefore \vec{F}dt = d\vec{p}$$

Integrating both sides within the limit indicated we get

$$\int_0^t \vec{F}dt = \int_{\vec{P}_1}^{\vec{P}_2} d\vec{p}$$

If $\vec{F} = \vec{F}_{av}$ is constant during this time, then the above equation can be written as

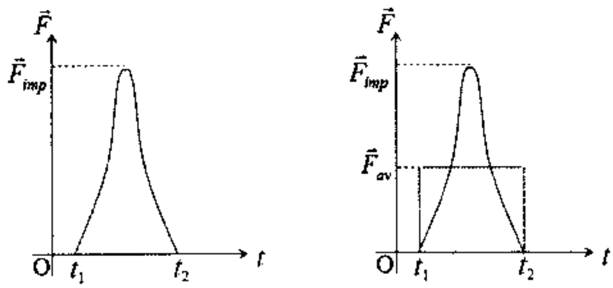
$$\vec{F}_{av}[t]_0^t = [\vec{p}]_{\vec{P}_1}^{\vec{P}_2}$$

$$\vec{F}_{av} \times t = \vec{P}_2 - \vec{P}_1$$

Or Impulse $\vec{I} = \vec{F}_{av} \times t = \vec{P}_2 - \vec{P}_1$

Impulse is a vector quantity. Its direction is the same as that of the constant force.

Impulsive force: Forces which are exerted over a short time interval are called impulsive force.

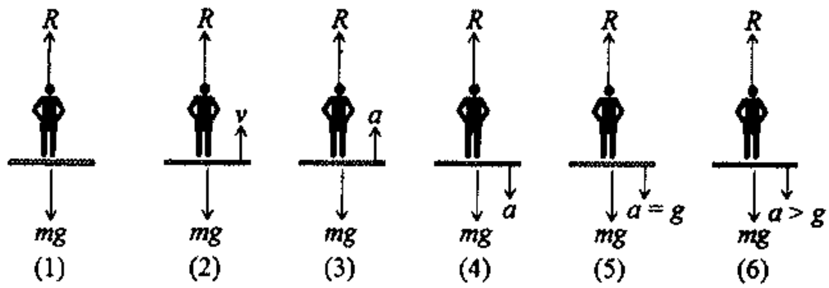


Note: The impulsive force is much larger than any other forces acting on the body. All other forces can be neglected. So change in momentum during a collision is due almost entirely to the impulsive force.

Impulse-momentum theorem: A given change in momentum can be produced by applying a larger force for a smaller time or by applying a smaller force for a larger time. This is called Impulse-momentum theorem.

Note: Change in momentum depends only on $\int \vec{F}dt$, and independent of the detailed time dependence of the force.

Apparent weight of a man in a lift:



(1) When the lift is at rest:
Acceleration of the person = 0
Net force on the person F = 0
i.e., $R = mg$
Apparent weight is equal to the actual weight of the person. Shown in figure (1).

(2) When the lift is moving uniformly in the upward or downward direction:
Acceleration of the person = 0
Net force on the person F = 0
i.e., $R = mg$
Apparent weight is equal to the actual weight of the person. Shown in figure (2).

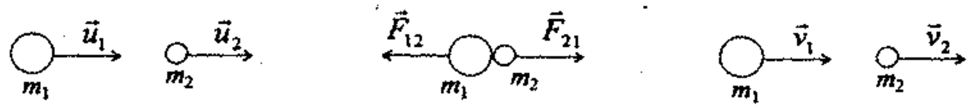
(3) When the lift is accelerating upwards:
Acceleration of the person = a
Net force on the person F = ma
i.e., $ma = R - mg$
Or $R = mg + ma = m(g + a)$
Thus $R > mg$
Apparent weight of the person becomes more than the actual weight, when the lift is accelerating upwards. Shown in figure (3).

(4) When the lift is accelerating downwards:
Acceleration of the person = a
Net force on the person F = ma
i.e., $ma = mg - R$
Or $R = mg - ma = m(g - a)$
Thus $R < mg$
Apparent weight of the person becomes less than the actual weight, when the lift is accelerating downwards. Shown in figure (4).

(5) In free fall of a body under gravity i.e., $a = g$
Acceleration of the person = g
Net force on the person F = mg
i.e., $mg = mg - R$
Or $R = mg - mg = m(g - g) = 0$
Thus $R = 0$
Apparent weight of the person becomes zero, or the body becomes weightless. Shown in figure (5).

(6) When the downward acceleration is greater than g
Acceleration of the person = a
Net force on the person F = ma
i.e., $ma = mg - R$
Or $R = mg - ma = m(g-a)$
Thus $R < 0$
Apparent weight of the person becomes negative. In that event, the person will rise from the floor of the lift and stick to the ceiling of the lift. Shown in figure (6).

Principle of conservation of linear momentum: If no external force acts on a system, the total momentum of the system remains constant.



(1) (2) (3)

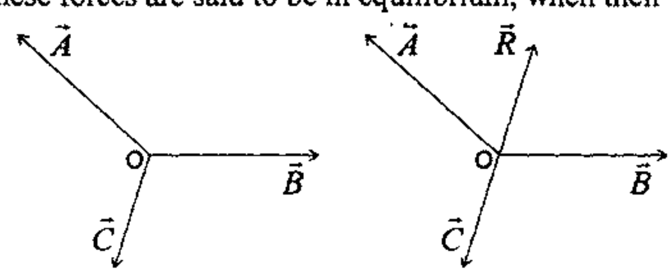
Consider the collision of two balls of masses m_1 and m_2 as shown in the figure. Before collision, their respective velocities are $\vec{u}_1$ and $\vec{u}_2$. After collision, the velocities of m_1 and m_2 are $\vec{v}_1$ and $\vec{v}_2$ respectively.
Impulse experienced by m_1 will be $\vec{F}_{12}\Delta t = m_1\vec{v}_1 - m_1\vec{u}_1$
Impulse experienced by m_2 will be $\vec{F}_{21}\Delta t = m_2\vec{v}_2 - m_2\vec{u}_2$
Here Δt is the time of contact of the two balls.
According to Newton's third law, $\vec{F}_{12} = -\vec{F}_{21}$

$$\text{Or } \vec{F}_{12}\Delta t = -\vec{F}_{21}\Delta t$$
$$\text{Or } m_1\vec{v}_1 - m_1\vec{u}_1 = -(m_2\vec{v}_2 - m_2\vec{u}_2)$$
$$\text{Or } m_1\vec{v}_1 - m_1\vec{u}_1 = -m_2\vec{v}_2 + m_2\vec{u}_2$$
$$\text{Or } m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

Total momentum before collision = Total momentum after collision
Thus the total momentum of the two balls is conserved.

Note:
(1) Momentum gained by one ball is lost by the other ball.
(2) The law of conservation of momentum is true for any number of objects in a system.
(3) Momentum is a vector quantity.

Equilibrium of concurrent forces: The forces which are acting at a point are called concurrent forces. These forces are said to be in equilibrium, when their resultant is zero.

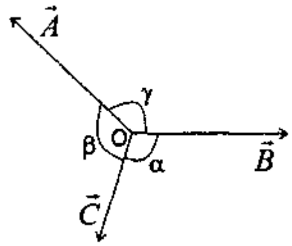


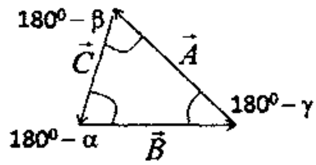
Suppose three concurrent forces $\vec{A}, \vec{B}$ and $\vec{C}$ acting on point O are in equilibrium, then

$$\vec{A}+\vec{B}+\vec{C}=0$$
$$\text{Or } \vec{A}+\vec{B}=-\vec{C}$$

It means that the Resultant force of $\vec{A}$ and $\vec{B}$ is equal in magnitude and opposite in direction to the force $\vec{C}$.

Lami's theorem: When three concurrent forces are in equilibrium, then according to Lami's theorem,





$$\frac{A}{\sin\alpha}=\frac{B}{\sin\beta}=\frac{C}{\sin\gamma}$$

Note: $\sin(180^0-\alpha)=\sin\alpha$, $\sin(180^0-\beta)=\sin\beta$, $\sin(180^0-\gamma)=\sin\gamma$

Friction: Friction is a force directed opposite to the direction of motion or attempted motion. The frictional force is always parallel to the surfaces in contact.

Types of friction:

(1) Internal friction: Internal friction arises on account of relative motion between two layers of a liquid. Internal friction is also referred to as viscosity of the liquid.

(2) External friction: External friction arises when two bodies in contact with each other, try to move, or there is an actual relative motion between the two. The external friction is also called contact friction. External friction is of three types.

(i) Static friction
(ii) Limiting friction
(iii) Kinetic friction

Static friction(f_S): The opposing force that comes into play when one body tends to move over the surface of another, but the actual motion has yet not started is called static friction.

Limiting friction(f_l): The maximum force of static friction which comes into play when one body is just at the verge of moving over the surface of the other body.

Kinetic friction(f_K): Kinetic friction or dynamic friction is the opposing force that comes into play when one body is actually moving over the surface of another body.

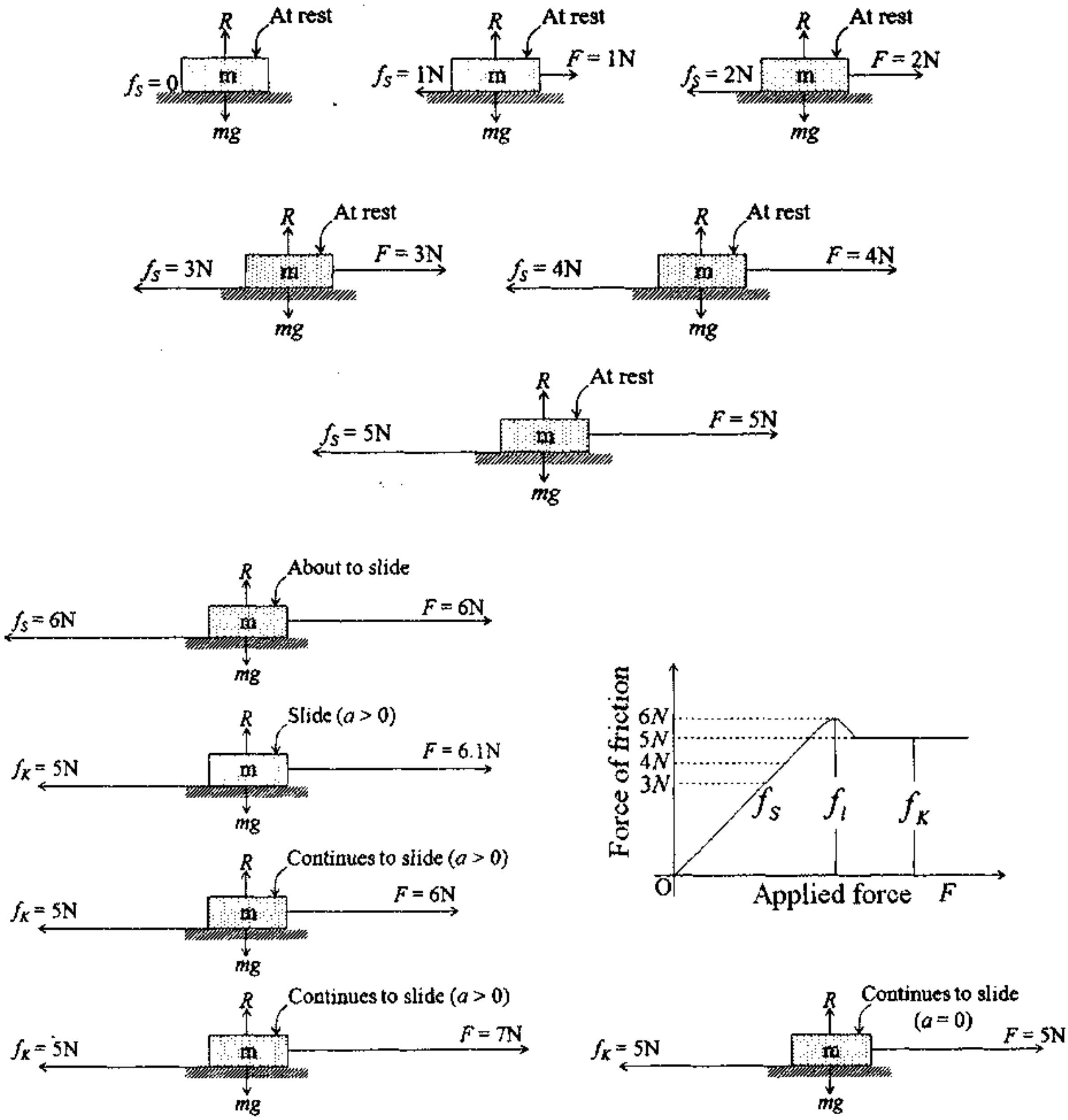
The kinetic friction or dynamic friction is of two types viz

(i) Sliding friction
(i) Rolling friction

When a flat block moves over the flat surface of a floor, the opposing force is the sliding friction. When a wheel rolls over a surface, the opposing force is the rolling friction.

Note: Rolling friction is quite small compared to sliding friction.

Illustration of Friction



Note:

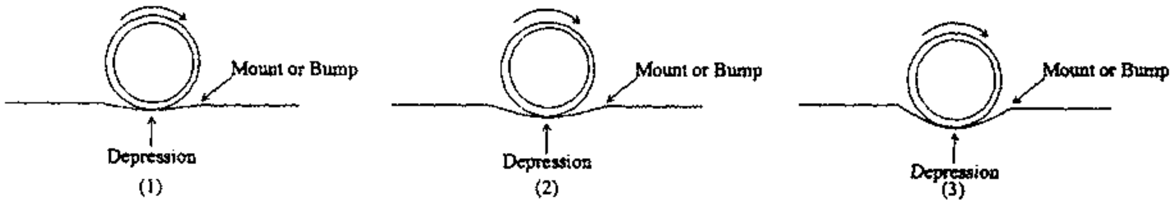
(1) The maximum static friction is called the limiting friction i.e., $f_S = f_l$

(2) The kinetic friction f_K remains constant even if the applied force increases. Here the body accelerates.

(3) To maintain the speed, i.e., for the body to move with constant velocity, F must equal to f_K.

(4) Friction arises only when body is actually sliding/rolling over the surface of another body or the body is simply trying to slide/roll over the surface of the other. Further, static friction alone is a self adjusting force.

Causes of rolling friction:



Rolling friction is minimum in figure (1) and maximum in figure (3).

Experiments show that the force of rolling friction f_r is directly proportional to the normal reaction R and inversely proportional to the radius r of the rolling cylinder or wheel. Thus

$$f_r = \mu_r \frac{R}{r}$$

Where μ_r is coefficient of rolling friction. It would have the dimensions of length and would be measured in meter.

Note: The velocity of the point of contact of the wheel with respect to the floor remains zero all the time, although the centre of the wheel moves forward.

The laws of friction:
First Law: The magnitude of the frictional force (kinetic and static) is directly proportional to the perpendicular force (normal reaction R) between the two surfaces in contact.
Second Law: The frictional force (static as well as kinetic) always acts parallel to the surfaces in contact and its direction is opposite to the motion or attempted motion.

Third Law: The frictional force (static as well as kinetic) is independent of the area of contact of the two surfaces.

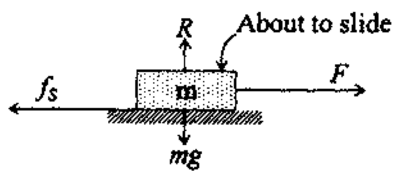
Fourth Law: The frictional force (static as well as kinetic) depends on the nature of the two surfaces in contact and their state of roughness.

Fifth Law: The frictional force is independent of the speed (applicable to kinetic friction only) of one surface relative to other surface.

Magnitude of limiting friction:
(1) According to the first law of limiting friction

$$f_S \propto R$$

Or $$f_S = \mu_S R$$



μ_S is called coefficient of static friction. Its value depends upon the nature and condition of the surfaces in contact.

$$\mu_S = \frac{f_S}{R} = \frac{\text{Limiting friction}}{\text{Normal reaction}}$$

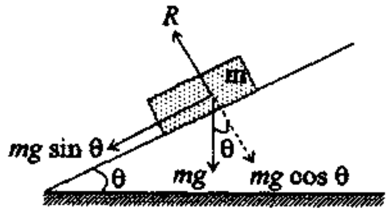
μ_S has no unit and no dimensional formula i.e., it is a pure number.

(2) According to the first law of limiting friction

$$f_S \propto R$$

Or $f_S = \mu_S R$

Or $f_S = \mu_S mg\cos\theta$



Magnitude of kinetic friction:

When the body is actually moving over the surface of another body, we replace f_S by f_K, the kinetic friction and μ_S by μ_K. Thus

$$f_K \propto R$$

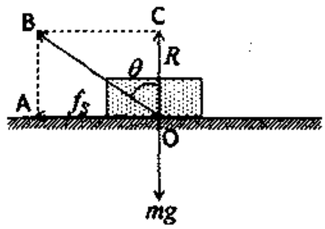
Or $f_K = \mu_K R$

μ_K is called coefficient of kinetic friction between two surfaces in contact. Its value depends upon the nature and condition of the surfaces in contact.

$$\mu_K = \frac{f_K}{R} = \frac{\text{Kinetic friction}}{\text{Normal reaction}}$$

μ_K has no unit and no dimensional formula i.e., it is a pure number.

Angle of friction: The angle of friction between two surfaces in contact is defined as the angle which the resultant of the force of limiting friction f_S and normal reaction R makes with the direction of the normal reaction R. It is represented by θ.



In ΔOBC,

$$\tan\theta = \frac{BC}{OC} = \frac{f_S}{R} = \mu_S$$

Angle of repose or angle of sliding: Angle of repose or angle of sliding is defined as the minimum angle of inclination of a plane with the horizontal, such that a body placed on the plane just begins to slide down. It is represented by α. Its value depends on the material and nature of the surfaces in contact.

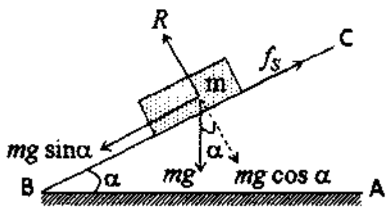
The various forces involved are:

(i) weight, *mg* of the body, acting vertically downwards

(ii) normal reaction *R* acting perpendicular to BC

(iii) force of friction f_S acting up the plane BC

Now *mg* can be resolved into two rectangular components, *mg* cosα opposite to *R* and *mg* sinα opposite to f_S. In equilibrium,



$$f_S = mg\sin\alpha$$

$$R = mg\cos\alpha$$

Or $\frac{f_S}{R} = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha$

Or $\mu_S = \tan\alpha$

Hence coefficient of limiting friction between any two surfaces in contact is equal to the tangent of the angle of repose between them.

Note:

Angle of friction is $\mu_S = \tan\theta$

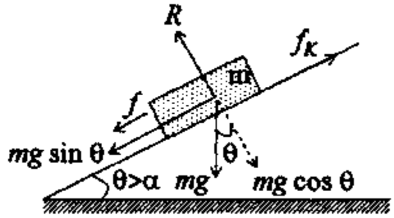
Angle of repose is $\mu_S = \tan\alpha$

$$\therefore \tan\theta = \tan\alpha \qquad \Rightarrow \theta = \alpha$$

i.e., angle of friction is equal to angle of repose.

Acceleration of a body down a rough inclined plane:

When a plane is inclined to the horizontal at an angle θ, which is more than the angle of repose, the body placed on the inclined plane slides down with an acceleration a.



The net force down the inclined plane is

$$f = mg\sin\theta - f_K$$

$$ma = mg\sin\theta - \mu_K R$$

$$ma = mg\sin\theta - \mu_K mg\cos\theta$$

$$a = \frac{mg\sin\theta - \mu_K mg\cos\theta}{m}$$

$$a = g\,(\sin\theta - \mu_K\cos\theta)$$

Note:

(1) As the body is sliding down, $mg\sin\theta > f_K$, so the net force f is in the direction of $mg\sin\theta$.

(2) It is clear that $a < g$ i.e., acceleration of a body down a rough inclined plane is always less than acceleration due to gravity (g).

(3) If the block slides at constant velocity down the inclined plane, then $a = 0$. Therfore

$$0 = g\,(\sin\theta - \mu_K\cos\theta)$$

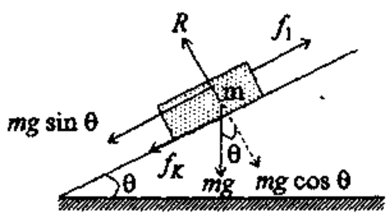
$$\sin\theta = \mu_K\cos\theta$$

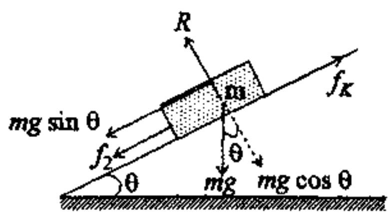
$$\frac{\sin\theta}{\cos\theta} = \mu_K$$

$$\tan\theta = \mu_K$$

(4) If $\theta < \alpha$, then the minimum force required to move the body up the inclined plane is

$$f_1 = mg\sin\theta + f_K = mg\sin\theta + \mu_K mg\cos\theta = mg\,(\sin\theta + \mu_K\cos\theta)$$

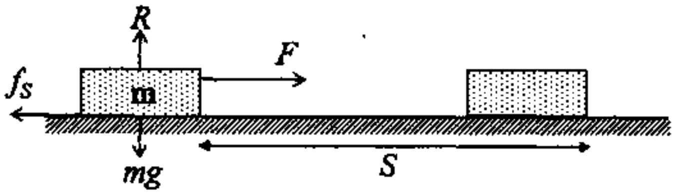




(5) The minimum force required to push the body down the inclined plane is

$$f_2 = f_K - mg\sin\theta = \mu_K mg\cos\theta - mg\sin\theta = mg\,(\mu_K\cos\theta - \sin\theta)$$

Work done in moving a body over a rough horizontal surface:



To move a body on a level track, we have to apply a force F which is atleast equal to the force of friction.

As work done = Force × Displacement

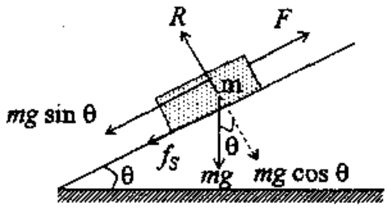
$$W = F \times S$$

$$W = f_S \times S$$

$$W = \mu_S R \times S$$

$$W = \mu_S mg \times S$$

Work done in moving a body up a rough inclined plane:



Again As work done = Force × Displacement

$$W = F \times S$$

$$W = mg(\sin\theta + \mu_S \cos\theta) \times S$$

Note: Friction is a non-conservative force i.e., work done against friction is path dependent. In the presence of friction, some energy is always lost in the form of heat etc. Thus mechanical energy is not conserved.

Advantages of friction:

(1) Walking will not be possible without friction. Our foot pressing the ground will only slip.
(2) No two bodies will stick to each other if there is no friction.
(3) Brakes of the vehicles will not work without friction.
(4) Nuts and bolts for holding the parts of machinery together will not work.
(5) Writing on black board or on paper will also not be possible without friction.
(6) Adhesives will lose their purpose.
(7) Cleaning with sand paper will not be possible without friction.

Disadvantages of friction:

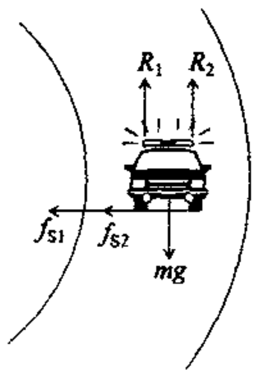
(1) Friction causes wear and tear of the parts of machinery in contact. Thus their life time reduces.
(2) Frictional forces result in the production of heat, which causes damage to the machinery.
(3) Since energy due to friction is converted into heat, it lowers the efficiency of every machinery.

Methods of reducing friction:

(1) By polishing
(2) By lubrication
(3) By proper selection of materials

(4) By streamlining
(5) By using ball bearings

Rounding a level curved surface: If the mass of the car is m and it is moving with a speed v on a circular tract of radius r, then the external forces acting on the car are
(1) Weight of the car mg
(2) Normal contact force R_1 an R_2
(3) Frictional force f_{S1} and f_{S2}.



Obviously,

$$R_1 + R_2 = mg$$
$$f_{S1} = \mu_S R_1$$
$$f_{S2} = \mu_S R_2$$
$$f_{S1} + f_{S2} = \mu_S R_1 + \mu_S R_2 = \mu_S(R_1+R_2) = \mu_S mg$$

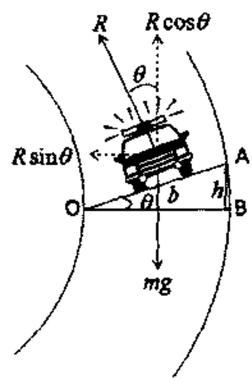
When the car is moving in a circular tract, for save turn we must have

$$m\frac{v^2}{r} \le (f_{S1} + f_{S2})$$
$$m\frac{v^2}{r} \le \mu_S mg$$
$$v^2 \le \mu_S\, rg$$
$$v \le \sqrt{\mu_S\, rg}$$

Hence, if a car takes a turn with a velocity greater than v it will skid outward.
Note: When a car moves at a steady speed around an unbanked curve, the centrifugal force keeping the car on the curve comes from the static friction between the road and the tyres. It is static rather than kinetic friction because the tyres are not slipping with respect to the radial direction.

Banking of roads: The outer edge of a curved road is raised more than the inner edge of the road to facilitate vehicles to turn without slipping. This is known as banking of road.
If v is the velocity of the vehicle over the banked road of radius r, then centrifugal force required is

$$F_C = m\frac{v^2}{r}$$



In equilibrium, $R\cos\theta$ balances the weight i.e.,

$$R\cos\theta = mg \text{ ------ } (1)$$

and $R\sin\theta$ provides the necessary centrifugal force i.e.

$$R\sin\theta = F_C$$
$$R\sin\theta = m\frac{v^2}{r} \text{ ------- } (2)$$

Dividing equation (2) by equation (1) we get

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{r\,mg}$$
$$\tan\theta = \frac{v^2}{r\,g} \text{ ------ } (3)$$

$$Or\ \ v = \sqrt{r\,g\tan\theta}$$

From the figure, we have

$$OB = \sqrt{OA^2 - AB^2}$$

$$OB = \sqrt{b^2 - h^2}$$

$$\tan\theta = \frac{AB}{OB} = \frac{h}{\sqrt{b^2 - h^2}}$$

As $h << b$, therefore h^2 is negligibly small compared to b^2. So

$$\tan\theta = \frac{h}{b} \text{-------}(4)$$

Equating equation(3) and (4) we have

$$\tan\theta = \frac{v^2}{r\,g} = \frac{h}{b}$$

Note:

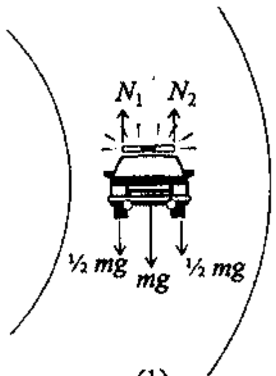
(1) When the vehicle is stationary:

In figure (1):

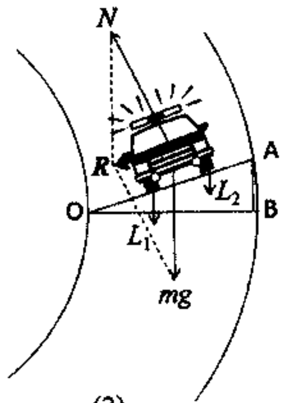
$$N_1 = N_2 = \frac{1}{2}mg$$

$$N_1 + N_2 = mg$$

The weight mg of the vehicle is perpendicular to the ground, and it is also equal and opposite to $N_1 + N_2$. So the vehicle is stable. Here the resultant force is zero.



(1)



(2)

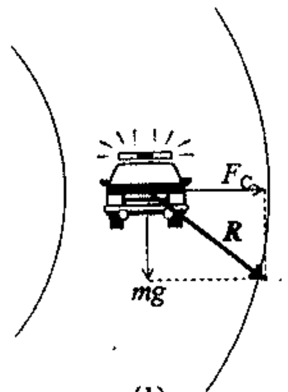
In figure (2): The forces or the pressures exerted by the two tyres are not the same, and

$$L_1 > L_2$$

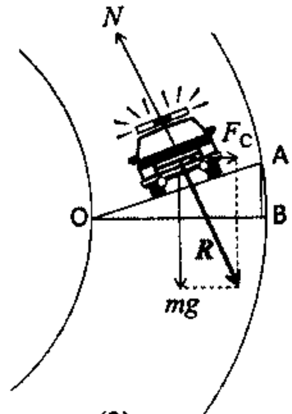
The Resultant force between N and mg acts sideway i.e., the vehicle is not stable when it is standing still in a bank road.

(2) When the vehicle is moving:

In figure (1): The Resultant force of the weight mg of the vehicle and the centrifugal force F_C is not perpendicular to the ground, so the vehicle is not stable while turning.



(1)



(2)

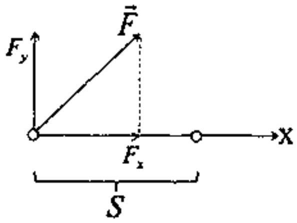
In figure (2): The Resultant force of the weight mg of the vehicle and the centrifugal force F_C is perpendicular to the ground, and is also equal and opposite to N. So the vehicle is stable while turning in a bank road.

<u>WORK ENERGY AND POWER</u>

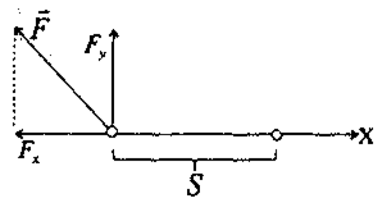
Work: Work is said to be done by a force acting on a body, if the body is displaced in any direction except in the direction perpendicular to the direction of the force.

Work done: Work done on an object by a force is defined as the product of the component of the force in the direction of motion and the displacement undergone by the object. Thus

Work done = Component of force $\vec{F}$ in the direction of motion × Object's displacement



(1)



(2)

In figure (1) $W = F_x\, S$

In figure (2) $W = -F_x\, S$

Or

Work done is defined as the dot product of the force vector $\vec{F}$ and the displacement vector $\vec{S}$ i.e.,

$$W = \vec{F} \cdot \vec{S}$$
$$W = F\, S \cos\theta$$

Where θ is the angle between the force $\vec{F}$ and the displacement $\vec{S}$.

Work done is a scalar quantity, its SI unit is joule (J) and the dimensional formula is $[ML^2T^{-2}]$

Definition of joule: One joule is defined as the work done by a force of one newton when its point of application moves by one metre along the line of action of the force.

Note:

(1) Here $F_x = F\cos\theta$

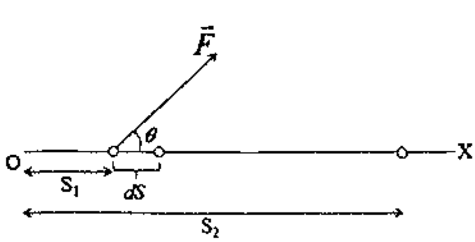
(1) If $\theta < 90^0$, $\cos\theta$ is positive so that work done is positive.

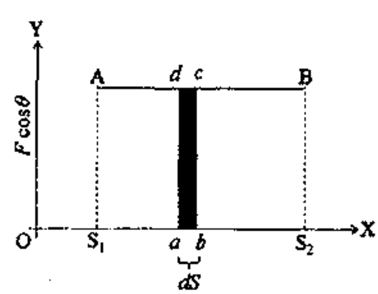
(2) If $\theta > 90^0$, $\cos\theta$ is negative so that work done is negative.

(3) If $\theta = 90^0$, $\cos\theta$ is zero so work done is zero.

(4) In cgs system, the unit of Work done is erg. 1 joule = 10^7 erg

Work done by a constant force: When the force F acting on a body has a constant magnitude and acts at a constant angle θ from the straight line path of the particle as shown in the figure then,





$dW = \vec{F} \cdot d\vec{S}$

Or $dW = FdS\cos\theta$ = Area of a small element abcd.

Or $dW = F\cos\theta\, dS$

The total work done when the body moves from S_1 to S_2 is

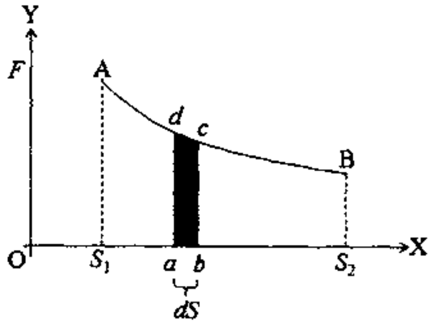
Or $W = \int_{S_1}^{S_2} F\cos\theta\, dS = F\cos\theta \int_{S_1}^{S_2} dS$

Or $W = F\cos\theta \left[S\right]_{S_1}^{S_2}$

Or $W = F\cos\theta \left[S_2 - S_1\right]$

The graphical representation of work done by a constant force is W = Area S_1ABS_2

Work done by a variable force: If the body is subjected to a varying force F and displaced along X axis as shown in the figure, then Work done is



$dW = \vec{F} \cdot d\vec{S}$

Or $dW = FdS\cos\theta$ = Area of a small element *abcd*.

The total work done when the body moves from S_1 to S_2 is

$$W = \int_{S_1}^{S_2} FdS\cos\theta = \text{Area } S_1ABS_2$$

Power: It is defined as the rate at which work is done.

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

Or $P = \dfrac{W}{t}$

Power is a scalar quantity. Its unit is *watt* (Js^{-1}) and dimensional formula is $[ML^2T^{-3}]$.

Definition of power: Power is said to be one watt, when one joule of work is said to be done in one second.

Note:

(1) The power P of an agent can also be expressed in terms of the force applied $\vec{F}$ and the velocity $\vec{v}$ of the body.

$$\because P = \frac{W}{t} = \frac{\vec{F} \cdot \vec{S}}{t} = \vec{F} \cdot \frac{\vec{S}}{t}$$

Or $P = \vec{F} \cdot \vec{v}$

If θ is the angle between $\vec{F}$ and $\vec{v}$, then

$$P = Fv\cos\theta$$

(2) $1kW = 10^3$ W
(3) $1MW = 10^6$ W
(4) $1\ W = 1Js^{-1} = 10^7$ erg s^{-1}
(5) 1 horse power (hp) = 746 W

Average power consumption in some common activities

	Activity	Power (watt)
1.	Heart beat	1.2
2.	Sleeping	75
3.	Slow walking	200
4.	Bicycling	500

Energy: The energy of a body is defined as the capacity or ability of the body to do work.

Note:
(1) When a body is capable of doing more work, it is said to possess more energy. The reverse is also true.
(2) Since energy is a stored work, so energy is also a scalar quantity.
(3) Energy is measured in the same units as work i.e., joule, erg etc.
(4) The dimensional formula of energy is $[ML^2T^{-2}]$ i.e., the same as for work.

Some practical units of energy

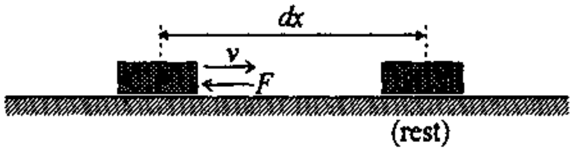
S.No	Unit	Symbol	Equivalence in (J)
1.	erg	*erg*	10^{-7}
2.	calorie	*cal*	4.2
3.	kilowatt-hour	*kW-h*	3.6×10^6
4.	electron volt	*eV*	1.6×10^{-19}

Mechanical energy: Mechanical energy is the energy associated with the motion and position of mechanical system. The mechanical energy of a body consists of kinetic energy and potential energy.
The energy possessed by an object because of motion is known as kinetic energy
The energy possessed by an object because of position or configuration (shape or size) is known as potential energy.

Note: Energy refers to the total amount of work a body can do, while power determines the rate of doing work. Thus, in power, time taken to complete the work is significant, but in energy, time is irrelevant.

Kinetic energy: The energy possessed by a body by virtue of its motion is called kinetic energy.

Expression for Kinetic energy: Consider a body of mass m moving with a velocity v in a straight line as shown in the figure. Suppose a constant force F resisting the motion of the body which produces a retardation a. Then



Force = mass × retardation

Or $F = -ma$

Let dx be the displacement of the body before it comes to rest. So

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v$$

$$\therefore F = -m\frac{dv}{dx} \times v = -mv \times \frac{dv}{dx}$$

Hence the work done in bringing the body to rest is given by,

$$W = \int F \cdot dx = -\int_v^0 mv \times \frac{dv}{dx} \cdot dx = -m\int_v^0 v\, dv$$

$$\text{Or} \quad W = -m\left[\frac{v^2}{2}\right]_v^0 = -\frac{1}{2}m\left[0 - v^2\right] = \frac{1}{2}mv^2$$

This work done is equal to kinetic energy of the body. Hence

$$KE = \frac{1}{2}mv^2$$

Note:

(1) The kinetic energy is a scalar quantity.

(2) The kinetic energy of a moving body depends on its speed.

(3) The expression $KE = \frac{1}{2}mv^2$ holds even when the force applied varies in magnitude or in direction or in both.

(4) The kinetic energy of the body is always positive. It can never be negative. (Because m and v^2 are both positive).

(5) The kinetic energy of the body depends upon the frame of reference. For example, the KE of a person of mass m sitting in a train moving with velocity v is $\frac{1}{2}mv^2$ with respect to a person in the frame of earth. The KE of the same person is zero with respect to a person in the frame of the train.

Relation between Kinetic energy and Linear momentum: Let m be the mass of the body and v be its velocity. Then the Linear momentum is

$$p = mv$$

$$\because KE = \frac{1}{2}mv^2$$

$$\text{Or} \quad KE = \frac{1}{2m}m^2v^2$$

$$\therefore KE = \frac{p^2}{2m}$$

Note:

(1) If p is constant, then $KE \propto \frac{1}{m}$

(2) If KE is constant, then $p \propto \sqrt{m}$

(3) If m is constant, then $p \propto \sqrt{KE}$

Work- Energy theorem: The work done by the net force on a body is equal to the change in its kinetic energy.
If the applied force increases the velocity of the body from v_0 to v, then work done W is

$$W = m\int_{v_0}^{v} v dv$$

Or $$W = m\left[\frac{v^2}{2}\right]_{v_0}^{v}$$

Or $$W = m\left[\frac{v^2}{2} - \frac{v_0^2}{2}\right]$$

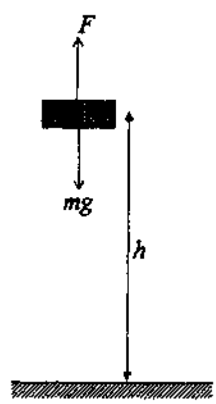
Or $$W = \frac{1}{2}m v^2 - \frac{1}{2}m v_0^2$$

i.e., Net work done = Change in Kinetic energy.

Note: This theorem can be applied to non-inertial frames also. In a non-inertial frames it can be written as
Work done by all forces including the Pseudo force = Change in Kinetic energy in non-inertial frame.

Potential energy: The energy possessed by a body by virtue of its position or configuration is called potential energy.

Expression for Potential energy: Let us consider a body of mass m, which is at rest at a height h above the ground as shown in the figure. The work done in raising the body from the ground to the height h is stored in the body as its potential energy.



Work done = Force × displacement

$$W = F \times h$$

Or $$W = mg \times h$$

This work done is stored as potential energy in the body. Hence

$$PE = mgh$$

Note:

(1) Although there is a single universal formula for the kinetic energy of a particle i.e., $\frac{1}{2}mv^2$ there is no single formula for potential energy (eg Elastic potential energy, Electric potential energy). The mathematical form of potential energy depends on the force or forces involved.
(2) The potential energy is defined only for conservative forces. It does not exist for non conservative forces.

(3) Potential energy depends upon frame of reference. It may be positive or negative.
(4) A body in motion may or may not have potential energy.

Conservative force: If the work done by a force in moving a body between two positions is independent of the path followed by the body, then such a force is called as a conservative force.
Examples: force due to gravity, spring force and elastic force.

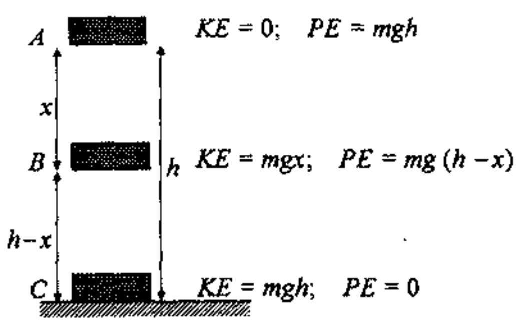
Note:
(1) The work done by the conservative forces depends only upon the initial and final position of the body.
(2) The work done by a conservative force around a closed path is zero i.e. $\oint \vec{F} \cdot d\vec{r} = 0$.

Non conservative force: Non-conservative force is the force, which can perform some resultant work along an arbitrary closed path of its point of application.
Examples: Frictional force, viscous force, etc.

Note: The work done by the non-conservative force depends upon the path of the displacement of the body i.e., $\oint \vec{F} \cdot d\vec{r} \neq 0$

Law of conservation of mechanical energy: If a body is under the action of conservative force or forces alone, the total mechanical energy of the body remains constant.

To show that the total mechanical energy ($KE + PE$) of the body at any point during its downward journey is constant. Let a body of mass m fall from a height h above the ground.



At point A: The body starts its downward motion with initial velocity v_0 equal to zero.

$$KE = 0; \quad PE = mgh$$

Total mechanical energy = $KE + PE = 0 + mgh = mgh$

At point B: Suppose at point B the body has velocity v_1

$$v_1^2 - v_0^2 = 2gx$$
$$v_1^2 - 0 = 2gx$$
$$v_1^2 = 2gx$$
$$KE = \frac{1}{2}mv_1^2 = \frac{1}{2}m(2gx) = mgx$$
$$PE = mg(h - x)$$

Total mechanical energy = $KE + PE = mgx + mg(h - x) = mgh$

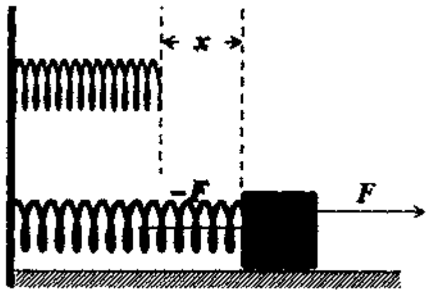
At point C: Suppose at point C just before the body touches the ground, its velocity is v_2

$$v_2^2 - v_0^2 = 2gh$$
$$v_2^2 - 0 = 2gh$$
$$v_2^2 = 2gh$$
$$KE = \frac{1}{2}mv_2^2 = \frac{1}{2}m(2gh) = mgh$$
$$PE = mg \times 0 = 0$$

Total mechanical energy = $KE + PE = mgh + 0 = mgh$

Potential energy of a spring: Potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

Expression for potential energy of a spring: For a small stretch or compression, spring obeys Hooke's law, i.e., for a spring



Restoring force ∝ stretch or compression

$$-F \propto x$$
Or $$-F = k\,x$$
Or $$F = -k\,x$$

Where k is a constant of the spring called spring constant.

Let the body be displaced further through a small distance dx, against the restoring force. So the small amount of work done is

$$dW = -Fdx = -(-k\,x)dx = k\,xdx$$

The total work done is

$$W = \int_{x=0}^{x=x} k\,xdx = k\int_{x=0}^{x=x} xdx = k\left[\frac{x^2}{2}\right]_{x=0}^{x=x} = k\left[\frac{x^2}{2} - \frac{0}{2}\right]$$

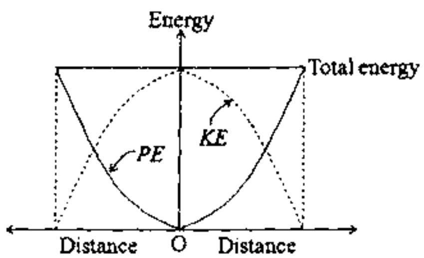
Or $$W = \frac{1}{2}k\,x^2$$

This work done is stored in the spring as potential energy.

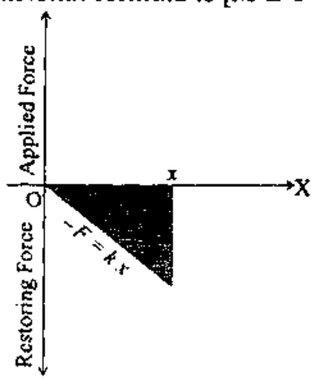
$$\therefore PE = \frac{1}{2}k\,x^2$$

Note:

(1) The SI unit of spring constant k is Nm^{-1}, the dimensional formula is $[M^1L^0T^{-2}]$.



(1)



(2)

(2) The variation of PE and KE with distance is shown in figure (1). Variation of restoring force with displacement is shown in figure (2).

(3) Spring force is conservative i.e., the work done (by the spring) to stretch a spring from one point to another is $W_1 = -F \cdot \Delta x$ and the work done (by the spring) to return back to the first point is $W_2 = F \cdot \Delta x$. The total work done is $W = W_1 + W_2 = -F \cdot \Delta x + F \cdot \Delta x = 0$

Collision: A collision is a short-time event and is said to have occurred if two or more particles physically collide against each other.

Types of collision:
(1) Elastic collision
(2) Inelastic collision

Note:
Perfectly inelastic collision: A collision in which the two objects stick together after the collision is called a perfectly inelastic collision.

(1) Elastic collision: A collision in which kinetic energy is conserved is called an elastic collision.

The basic characteristics of an elastic collision are
(i) The linear momentum is conserved
(ii) Total energy of the system is conserved
(iii) The kinetic energy is conserved
(iv) The forces involved during elastic collisions must be conservative forces.

(2) Inelastic collision: A collision in which kinetic energy is not conserved is called inelastic collision.

The basic characteristics of inelastic collision are
(i) The linear momentum is conserved
(ii) Total energy of the system is conserved
(iii) The kinetic energy is not conserved
(iv) Some or all of the forces involved during inelastic collision may be non conservative in nature.

Collision in one dimension (1D):
(A-1D) Elastic collision or perfectly elastic collision: Let m_1 and m_2 are the two masses moving in the same direction with velocity u_1 and u_2 respectively. After collision, they continue to move in the same direction with velocity v_1 and v_2 as shown in the figure.

Before collision ($u_1 > u_2$) During collision After collision ($v_1 < v_2$)

From the law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{-------} (1)$$

From the law of conservation of kinetic energy, we have

$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$$
$$m_1u_1^2+m_2u_2^2=m_1v_1^2+m_2v_2^2$$
$$m_1u_1^2-m_1v_1^2=m_2v_2^2-m_2u_2^2$$
$$m_1(u_1^2-v_1^2)=m_2(v_2^2-u_2^2) \text{ -------- } (2)$$

Dividing equation (2) by equation (1), we have

$$\frac{u_1^2-v_1^2}{u_1-v_1}=\frac{v_2^2-u_2^2}{v_2-u_2}$$
$$\frac{(u_1+v_1)(u_1-v_1)}{u_1-v_1}=\frac{(v_2+u_2)(v_2-u_2)}{v_2-u_2}$$
$$u_1+v_1=v_2+u_2$$
$$u_1-u_2=v_2-v_1$$

u_1-u_2 is the relative velocity of approach of the two masses m_1 and m_2.
v_2-v_1 is the relative velocity of separation of the two masses m_1 and m_2.

Thus, for perfectly elastic collision, the relative velocity of approach before collision is equal to the relative velocity of separation after collision.

Velocities after collision: After collision, the velocities of masses m_1 and m_2 are v_1 and v_2.

$$\because u_1-u_2=v_2-v_1$$
$$v_2=u_1-u_2+v_1 \text{ -------- } (1)$$

Again $\because m_1u_1+m_2u_2=m_1v_1+m_2v_2$ -------- (2)

Putting the value of v_2 in equation (2) we have

$$m_1u_1+m_2u_2=m_1v_1+m_2(u_1-u_2+v_1)$$
$$m_1u_1+m_2u_2=m_1v_1+m_2u_1-m_2u_2+m_2v_1$$
$$m_1u_1+m_2u_2=v_1(m_1+m_2)+m_2u_1-m_2u_2$$
$$m_1u_1+m_2u_2-m_2u_1+m_2u_2=v_1(m_1+m_2)$$
$$u_1(m_1-m_2)+2m_2u_2=v_1(m_1+m_2)$$
$$v_1=u_1\left(\frac{m_1-m_2}{m_1+m_2}\right)+u_2\left(\frac{2m_2}{m_1+m_2}\right) \text{ -------- } (3)$$

Putting the value of v_1 in equation (1) we have

$$v_2=u_1-u_2+u_1\left(\frac{m_1-m_2}{m_1+m_2}\right)+u_2\left(\frac{2m_2}{m_1+m_2}\right)$$
$$v_2=u_1\left(1+\frac{m_1-m_2}{m_1+m_2}\right)+u_2\left(\frac{2m_2}{m_1+m_2}-1\right)$$
$$v_2=u_1\left(\frac{m_1+m_2+m_1-m_2}{m_1+m_2}\right)+u_2\left(\frac{2m_2-(m_1+m_2)}{m_1+m_2}\right)$$
$$v_2=u_1\left(\frac{m_1+m_2+m_1-m_2}{m_1+m_2}\right)+u_2\left(\frac{2m_2-m_1-m_2}{m_1+m_2}\right)$$
$$v_2=u_1\left(\frac{2m_1}{m_1+m_2}\right)+u_2\left(\frac{m_2-m_1}{m_1+m_2}\right)$$

$$v_2 = u_2\left(\frac{m_2 - m_1}{m_1 + m_2}\right) + u_1\left(\frac{2m_1}{m_1 + m_2}\right) \text{-------} (4)$$

Different cases:

(I) When the target body i.e., m_2 is initially at rest: In this case $u_2 = 0$.

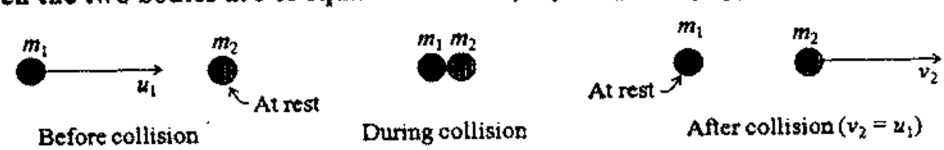
$$\because v_1 = u_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right) + u_2\left(\frac{2m_2}{m_1 + m_2}\right)$$

$$\therefore v_1 = u_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)$$

Also $$\because v_2 = u_2\left(\frac{m_2 - m_1}{m_1 + m_2}\right) + u_1\left(\frac{2m_1}{m_1 + m_2}\right)$$

$$\therefore v_2 = u_1\left(\frac{2m_1}{m_1 + m_2}\right)$$

(i) When the two bodies are of equal masses i.e., $m_1 = m_2 = m$ (say) and m_2 is at rest:



$$\because v_1 = u_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$v_1 = u_1\left(\frac{m - m}{m + m}\right)$$

$$v_1 = u_1\left(\frac{0}{2m}\right)$$

$$\therefore v_1 = 0$$

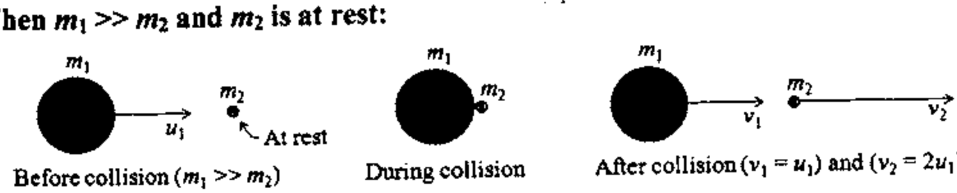
Also $$\because v_2 = u_1\left(\frac{2m_1}{m_1 + m_2}\right)$$

$$v_2 = u_1\left(\frac{2m}{m + m}\right)$$

$$v_2 = u_1\left(\frac{2m}{2m}\right)$$

$$\therefore v_2 = u_1$$

(ii) When $m_1 >> m_2$ and m_2 is at rest:



$$\because v_1 = u_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$v_1 = u_1\left(\frac{m_1}{m_1}\right)$$

$$\therefore v_1 = u_1$$

Also $$\because v_2 = u_1\left(\frac{2m_1}{m_1 + m_2}\right)$$

$$v_2 = u_1\left(\frac{2m_1}{m_1}\right)$$

$$\therefore v_2 = 2u_1$$

(iii) When $m_1 << m_2$ and m_2 is at rest:

$$\because v_1 = u_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$v_1 = u_1\left(\frac{-m_2}{m_2}\right)$$

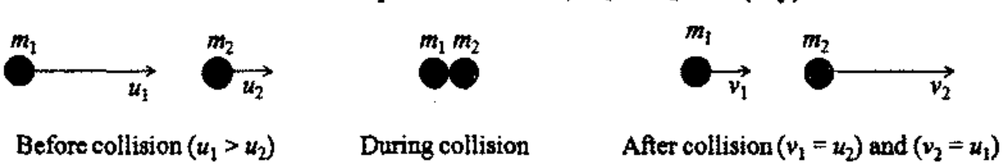
$$\therefore v_1 = -u_1$$

Also $$\because v_2 = u_1\left(\frac{2m_1}{m_1 + m_2}\right)$$

$$v_2 = u_1\left(\frac{2m_1}{m_2}\right)$$

$$\therefore v_2 \approx 0$$

(II) When the two bodies are of equal masses i.e., $m_1 = m_2 = m$ (say):



$$\because v_1 = u_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right) + u_2\left(\frac{2m_2}{m_1 + m_2}\right)$$

$$v_1 = u_1\left(\frac{m - m}{m + m}\right) + u_2\left(\frac{2m}{m + m}\right)$$

$$v_1 = u_1\left(\frac{0}{2m}\right) + u_2\left(\frac{2m}{2m}\right)$$

$$\therefore v_1 = u_2$$

Also $$v_2 = u_2\left(\frac{m_2 - m_1}{m_1 + m_2}\right) + u_1\left(\frac{2m_1}{m_1 + m_2}\right)$$

$$v_2 = u_2\left(\frac{m - m)}{m + m}\right) + u_1\left(\frac{2m}{m + m}\right)$$

$$v_2 = u_2\left(\frac{0}{2m}\right) + u_1\left(\frac{2m}{2m}\right)$$

$$\therefore v_2 = u_1$$

Note: This result is made use of, for slowing down neutrons in a nuclear reactor.

(B-1D) Inelastic collision: In an inelastic collision, the momentum is conserved but some of the kinetic energy is lost. Let m_1 and m_2 are the two masses moving in the same direction with velocity u_1 and u_2 respectively. After collision, they continue to move in the same direction with velocity v_1 and v_2 as shown in the figure.

Before collision ($u_1 > u_2$) During collision After collision ($v_1 < v_2$)

From the law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

(C-1D) Perfectly inelastic collision: Let m_1 and m_2 are the two masses moving in the same direction with velocity u_1 and u_2 respectively. After collision, they stick together and continue to move in the same direction with the same velocity v as shown in the figure.

Before collision ($u_1 > u_2$) During collision After collision

From the law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

Coefficient of restitution: The coefficient of restitution is defined as the ratio of the velocity of separation to the velocity of approach of the colliding particles. It is denoted by e.

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

Before collision After collision

Velocity of approach of the two masses m_1 and $m_2 = u_1 - u_2$
Velocity of separation of the two masses m_1 and $m_2 = v_2 - v_1$

So $$e = \frac{v_2 - v_1}{u_1 - u_2}$$

(1) For elastic collision, $u_1 - u_2 = v_2 - v_1$

$$\therefore e = \frac{v_2 - v_1}{u_1 - u_2} = 1$$

(2) For perfectly inelastic collision, $v_2 - v_1 = 0$

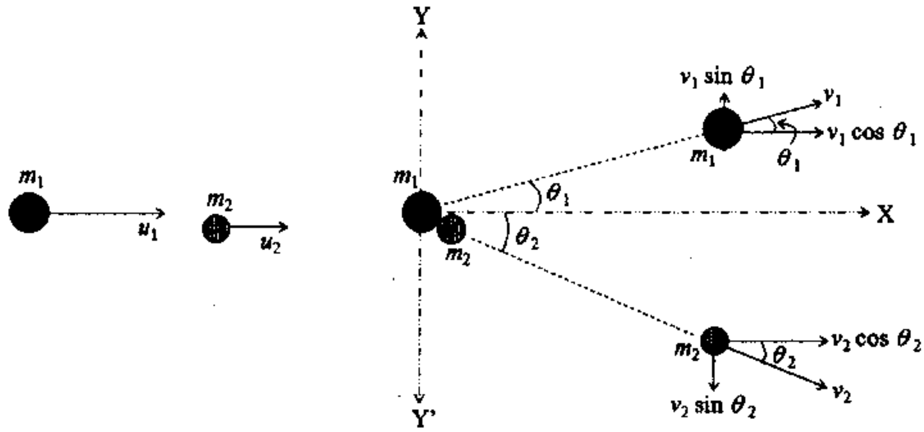
$$\therefore e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0}{u_1 - u_2} = 0$$

(3) For inelastic collision, $u_1 - u_2 > v_2 - v_1 > 0$

$$\therefore 0 < e < 1$$

Collision in two dimension (2D):

(A-2D) Elastic collision or perfectly elastic collision: Let m_1 and m_2 are the masses of two bodies moving initially along X axis with velocities u_1 and u_2 respectively. After collision, let v_1 and v_2 be the velocities of the two bodies as shown in the figure.



Before collision ($u_1 > u_2$) During collision After collision

From the law of conservation of momentum, we have

(1) Along X axis:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

(2) Along Y axis:

$$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2$$

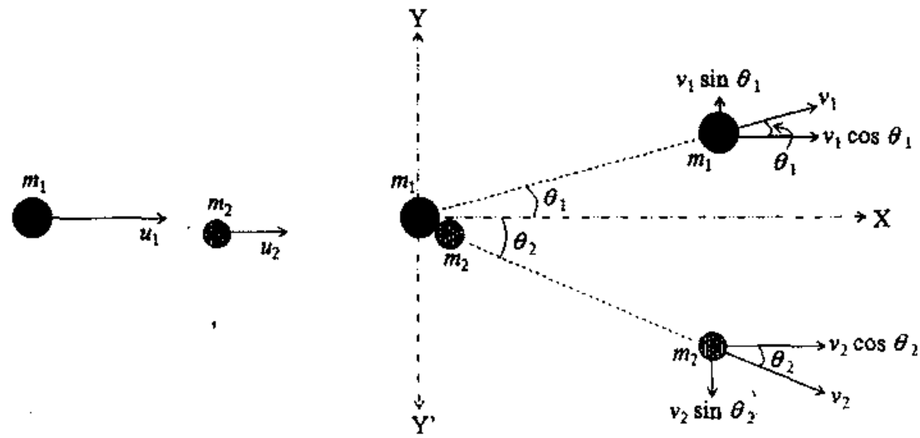
From the law of conservation of kinetic energy, we have

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1 u_1^2 + m_2 u_2^2 = m_1 v_1^2 + m_2 v_2^2$$

Note: In perfectly elastic collision, if $m_1 = m_2 = m$ and m_2 is at rest, then in this case $u_2 = 0$. After collision, $\theta_1 + \theta_2 = 90^0$.

(B-2D) Inelastic collision: Let m_1 and m_2 are the masses of two bodies moving initially along X axis with velocities u_1 and u_2 respectively. After collision, let v_1 and v_2 be the velocities of the two bodies as shown in the figure.



Before collision ($u_1 > u_2$) During collision After collision

From the law of conservation of momentum, we have

(1) Along X axis:

$$m_1u_1 + m_2u_2 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$$

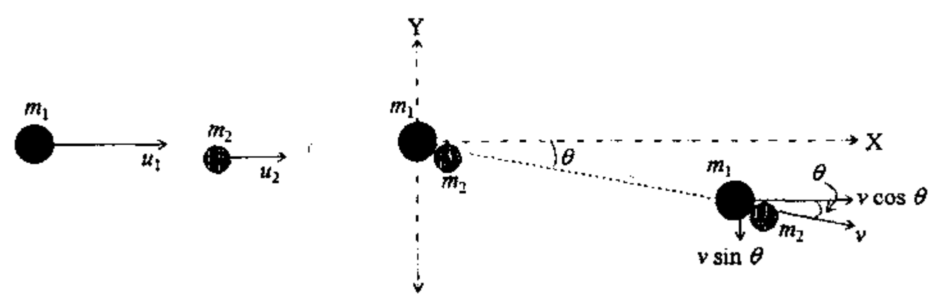
(2) Along Y axis:

$$0 = m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2$$

If we know the masses (m_1,m_2), initial velocities (u_1,u_2) and θ_1 and θ_2, we can find v_1 and v_2.

Note: For inelastic collision, if $m_1 = m_2 = m$ and m_2 is at rest, then after collision, $\theta_1 + \theta_2 < 90^0$.

(C-2D) Perfectly inelastic collision: Let m_1 and m_2 are the two masses moving initially along X axis with velocities u_1 and u_2 respectively. After collision, they stick together and continue to move with the same velocity v as shown in the figure.



Before collision ($u_1 > u_2$) During collision After collision

From the law of conservation of momentum, we have

(1) Along X axis:

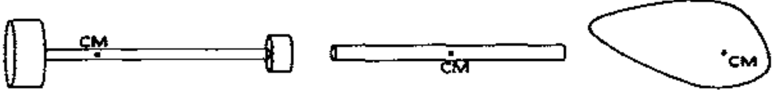
$$m_1u_1 + m_2u_2 = (m_1 + m_2)v\cos\theta$$

(2) Along Y axis:

$$0 = (m_1 + m_2)v\sin\theta$$

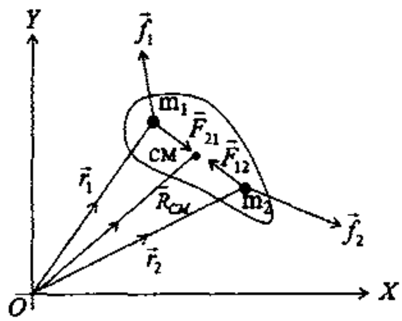
MOTION OF SYSTEM OF PARTICLES AND RIGID BODY

Centre of mass: A point in the system at which the whole mass of the body is supposed to be concentrated is called centre of mass of the body.



Note:
(1) The position of the centre of mass depends upon the shape , the size and the density of the body.
(2) The centre of mass of the body lies within or outside the body.
(3) In symmetrical bodies having uniform density, the centre of mass coincides with the geometrical centre of the body.
(4) The position of the centre of mass of a body changes in translatory motion but remains unchanged in rotatory motion.

Centre of mass of two particles system: Let us consider a system consisting of two particles of masses m_1 and m_2 as shown in the diagram.



The force that acts at m_1 is $\vec{F}_{21} + \vec{f}_1 = m_1 \frac{d^2\vec{r}_1}{dt^2} ----(1)$

And the force that acts at m_2 is

$$\vec{F}_{12} + \vec{f}_2 = m_2 \frac{d^2\vec{r}_2}{dt^2} ----(2)$$

Where $\vec{F}_{21}$ and $\vec{F}_{12}$ are the internal forces. They are equal and opposite. $\vec{f}_1$ and $\vec{f}_2$ are the external forces. $\vec{r}_1$ and $\vec{r}_2$ are the position vectors of the two particles.
Adding equation (1) and equation (2) gives

$$\vec{f}_1 + \vec{f}_2 = m_1 \frac{d^2\vec{r}_1}{dt^2} + m_2 \frac{d^2\vec{r}_2}{dt^2}$$

$$\vec{F}_{ext} = \frac{d^2}{dt^2}\left(m_1\vec{r}_1 + m_2\vec{r}_2\right) -----(3)$$

Multiply and divide the RHS of equation (3) by $(m_1 + m_2)$ we have

$$\vec{F}_{ext} = \left(m_1 + m_2\right)\frac{d^2}{dt^2}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}\right)$$

$$\vec{F}_{ext} = \left(m_1 + m_2\right)\frac{d^2}{dt^2}\left(\vec{R}_{CM}\right) -----(4)$$

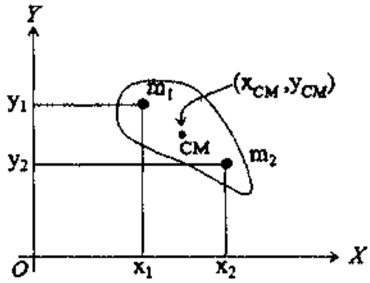
Where $\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$ is the position vector of the centre of mass.

Equation (4) can also be written as

$$\vec{F}_{ext} = \left(m_1 + m_2\right)\vec{a}_{CM} = M\,\vec{a}_{CM}$$

$\vec{a}_{CM} = \frac{\vec{F}_{ext}}{M}$ where M is the mass of the two particles and $\vec{a}_{CM}$ is the acceleration of the centre of mass.

Case - I

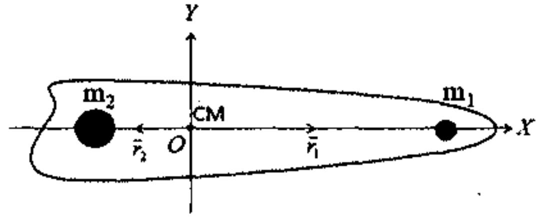


$\because \vec{R}_{CM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$

In component form we can write as

$x_{CM} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$ $\qquad y_{CM} = \dfrac{m_1y_1 + m_2y_2}{m_1 + m_2}$

Case - II



If $\vec{R}_{CM} = 0$ we have

$$0 = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

Or $m_1\vec{r}_1 + m_2\vec{r}_2 = 0$

Or $m_1\vec{r}_1 = -m_2\vec{r}_2$

Or $\vec{r}_1 = -\dfrac{m_2}{m_1}\vec{r}_2$

If $m_2 > m_1$, $\vec{r}_1 > \vec{r}_2$ thus the centre of mass (CM) lies closer to the heavier particle.

Case - III

If $m_1 = m_2 = m$

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{m\vec{r}_1 + m\vec{r}_2}{m + m} = \frac{m(\vec{r}_1 + \vec{r}_2)}{2m} = \frac{\vec{r}_1 + \vec{r}_2}{2}$$

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{mx_1 + mx_2}{m + m} = \frac{m(x_1 + x_2)}{2m} = \frac{x_1 + x_2}{2}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{my_1 + my_2}{m + m} = \frac{m(y_1 + y_2)}{2m} = \frac{y_1 + y_2}{2}$$

Note:

(1) In a two particle system, if the particles of masses m_1 and m_2 moving with velocities $\vec{v}_1$ and $\vec{v}_2$ respectively, then the velocity of the centre of mass is

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

(2) If the accelerations of the particles are $\vec{a}_1$ and $\vec{a}_2$ respectively, then the acceleration of the centre of mass is

$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$$

(3) Centre of mass of an isolated system has a constant velocity i.e., isolated system will remain at rest if it is initially at rest or will move with the same velocity if it is initially in motion.

Centre of mass of a body consisting of '*n*' particles: For *n* particles, the position vector of the centre of mass (CM) is

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...m_n\vec{r}_n}{m_1 + m_2 + ...m_n} = \frac{\sum_{i=1}^{i=n} m_i\vec{r}_i}{\sum_{i=1}^{i=n} m_i} = \frac{\sum_{i=1}^{i=n} m_i\vec{r}_i}{M}$$

Where M is the mass of all the *n* particles. In component form, we can write

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + ...m_nx_n}{m_1 + m_2 + ...m_n} = \frac{\sum_{i=1}^{i=n} m_ix_i}{\sum_{i=1}^{i=n} m_i} = \frac{\sum_{i=1}^{i=n} m_ix_i}{M}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + ...m_ny_n}{m_1 + m_2 + ...m_n} = \frac{\sum_{i=1}^{i=n} m_iy_i}{\sum_{i=1}^{i=n} m_i} = \frac{\sum_{i=1}^{i=n} m_iy_i}{M}$$

Momentum of a system of '*n*' particles: The external force for the system of *n* particles is

$$\vec{F}_{ext} = \frac{d^2}{dt^2}(m_1\vec{r}_1 + m_2\vec{r}_2 + ...m_n\vec{r}_n)$$

$$\vec{F}_{ext} = \frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2 + ...m_n\vec{v}_n)$$

$$\vec{F}_{ext} = \frac{d}{dt}(\vec{p}_1 + \vec{p}_2 + ...\vec{p}_n) = \frac{d}{dt}\sum_{i=1}^{i=n}\vec{p}$$

$$\vec{F}_{ext} = \frac{d}{dt}\vec{P}$$

$M\,\vec{a}_{CM} = \frac{d\vec{P}}{dt}$ where M is the total mass of all the *n* particles and $\vec{P}$ is the total momentum

Case - I

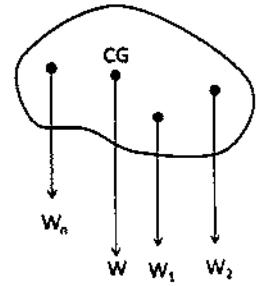
If $\vec{F}_{ext} = 0$ i.e., no external force acts on the system, then

$$\frac{d\vec{P}}{dt} = 0$$

$\therefore \vec{P}$ is constant. This is the conservation of linear momentum.

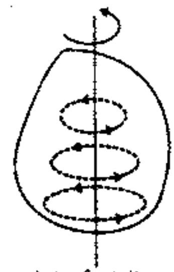
Example for the motion of centre of mass: The motion of the centre of mass of the body is not affected by the internal forces. If a hammer is tossed in the air, the motion of most part of the hammer is quite complex, but the motion of the centre of mass follow a simple parabolic path.

Centre of gravity: The centre of gravity of a body is the point at which the resultant of the weights of all the particles of the body acts, whatever may be the orientation or position of the body provided that its size and shape remain unaltered



Rigid body: A rigid body is defined as that body which does not undergo any change in shape or volume when external forces are applied on it.

Rotational motion: A rigid body is said to have pure rotational motion, if every particle of the body moves in a circle, whose centres lies on a straight line called the axis of rotation.



Equation of rotational motion:

(i) Prove that $\omega = \omega_0 + \alpha t$

Let a particle starts rotating with angular velocity ω_0 and angular acceleration α. At any instant t, let ω be the angular velocity of the particle.

$$\because \text{angular acceleration} = \frac{\text{change in angular velocity}}{\text{time taken}}$$

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$\omega - \omega_0 = \alpha t \qquad \Rightarrow \omega = \omega_0 + \alpha t$$

(ii) Prove that $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

Let a particle starts rotating with angular velocity ω_0 and angular acceleration α. At any instant t, let ω be the angular velocity of the particle and θ be the angular displacement produced by the particle.

The average angular velocity $<\omega> = \frac{\omega + \omega_0}{2}$

Total angular displacement = average angular velocity × time taken

$$\theta = \left(\frac{\omega + \omega_0}{2}\right) \times t = \left(\frac{\omega_0 + \alpha t + \omega_0}{2}\right) \times t = \left(\frac{2\omega_0 + \alpha t}{2}\right) \times t$$

$$\theta = \left(\omega_0 + \frac{\alpha t}{2}\right) \times t \qquad \Rightarrow \quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

(iii) Prove that $\omega^2 = \omega_0^2 + 2\alpha\theta$

Let a particle starts rotating with angular velocity ω_0 and angular acceleration α. At any instant t, let ω be the angular velocity of the particle and θ be the angular displacement produced by the particle.

$$\because \alpha = \frac{\omega - \omega_0}{t} \qquad \Rightarrow t = \frac{\omega - \omega_0}{\alpha}$$

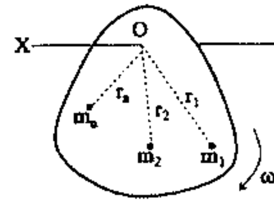
Also $\theta = <\omega> t$

$$\theta = \left(\frac{\omega + \omega_0}{2}\right)\left(\frac{\omega - \omega_0}{\alpha}\right) = \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta \qquad \Rightarrow \omega^2 = \omega_0^2 + 2\alpha\theta$$

Moment of inertia: The inability of the body to change its state of rest or uniform rotational motion by itself until and unless some external torque acts on it to change that state, is called the moment of inertia.

Rotational kinetic energy and the moment of inertia of a rigid body: Consider the particles of masses $m_1, m_2, ... m_n$ situated at distances $r_1, r_2, ... r_n$ respectively from the axis of rotation as shown in the diagram.



The kinetic energy of the 1st particle is

$$E_{K1} = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1(r_1\omega)^2 = \frac{1}{2}m_1 r_1^2\omega^2$$

The kinetic energy of the 2nd particle is

$$E_{K2} = \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_2(r_2\omega)^2 = \frac{1}{2}m_2 r_2^2\omega^2 \text{ and so on}$$

Adding the above equations, give the rotational kinetic energy of the rigid body

$$E_{K1} + E_{K2} + ... E_{Kn} = \frac{1}{2}\omega^2(m_1 r_1^2 + m_2 r_2^2 + ... m_n r_n^2)$$

$$E_R = \frac{1}{2}\omega^2\left(\sum_{i=1}^{i=n} m_i r_i^2\right) = \frac{1}{2}\omega^2 I$$

Where $I = \sum_{i=1}^{i=n} m_i r_i^2$ is the moment of inertia of a rigid body.

Thus the moment of inertia of a rigid body about the given axis of rotation is the sum of the products of the masses of its particles and the squares of their respective perpendicular distances from the axis of rotation.
Its unit is kg-m^2 and its dimensional formula is $[ML^2]$.

Note: The moment of inertia of a body depends upon the position of the axis of rotation, the orientation of the axis of rotation, the shape and size of the body and the distribution of mass of the body about the axis of rotation.

Radius of Gyration: The radius of gyration is equal to the root mean square distances of the particles from the axis of rotation of the body.
The radius of gyration can also be defined as the perpendicular distance between the axis of rotation and the point where the whole weight of the body is to be concentrated.

$$\because I = m_1 r_1^2 + m_2 r_2^2 + ... m_n r_n^2$$

If $m_1 = m_2 = ... = m_n = m$

$$\therefore I = m(r_1^2 + r_2^2 + ... r_n^2)$$

Divide and multiply the RHS of the above equation by n (the total number of particles) we have

$$I = mn\left(\frac{r_1^2 + r_2^2 + ... r_n^2}{n}\right) = MK^2$$

Where $M = mn$ is the mass of all the particles and

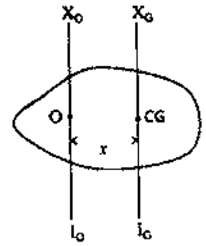
$K^2 = \frac{r_1^2 + r_2^2 + ... r_n^2}{n}$ $\Rightarrow K = \sqrt{\frac{r_1^2 + r_2^2 + ... r_n^2}{n}}$ is the radius of gyration

Theorems of moment of inertia:

(i) Parallel axes theorem

Statement:

The moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its centre of gravity and the product of the mass of the body and the square of the distance between the two axes.

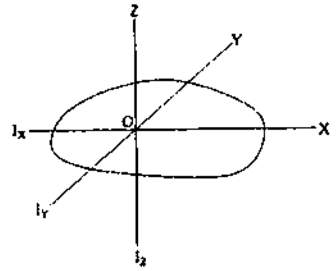


$$I_O = I_G + Mx^2$$

(ii) Perpendicular axes theorem

Statement:

The moment of inertia of a plane laminar body about an axis perpendicular to the plane is equal to the sum of the moments of inertia about two mutually perpendicular axes in the plane of the lamina such that the three mutually perpendicular axes have a common point of intersection.

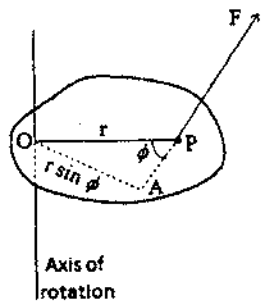


$$I_Z = I_X + I_Y$$

Note: Theorem of parallel axes is applicable for any type of rigid body whether it is a two or three dimensional, while the theorem of perpendicular axes is applicable for laminar type or two dimensional bodies only.

Torque or moment of a force: The turning effect of a force about a fixed point or axis is known as the moment of force or torque.

Let us consider a force F acting at the point P on the body as shown in the diagram. Then, the moment of the force F about the point O is



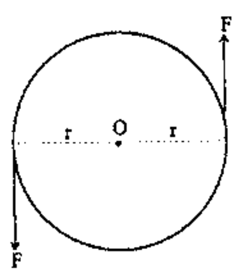
$$\tau = F \times OA = Fr\sin\phi$$

In vector form we can write

$$\vec{\tau} = \vec{r} \times \vec{F}$$

By convention, an anticlockwise moment is taken as positive and a clockwise moment as negative. Its SI unit is 'newton-metre' and its dimension is $[ML^2T^{-2}]$.

Couple and moment of a couple: Two equal and opposite forces whose lines of action do not coincide are said to constitute a couple in mechanics.

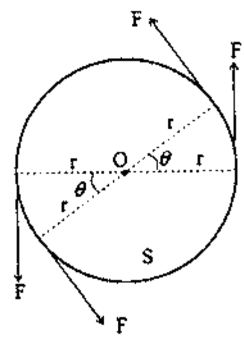


The moment of the couple is

$$\tau_c = F \times 2r = 2Fr$$

Work done by a couple: Suppose two equal and opposite forces F act tangentially to a wheel S, and rotate it through an angle θ.

Then the work done by each force = Force × distance



$$w = F \times r\theta$$

Total workdone $W = Fr\theta + Fr\theta = 2Fr\theta$

But $\tau_c = 2Fr$

$$\therefore W = \tau_c\theta$$

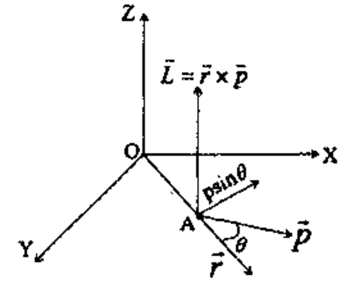
Angular momentum of a particle: The angular momentum of a particle is defined as the moment of linear momentum of the particle.

The angular momentum L of the particle about an axis passing through O perpendicular to XY plane is defined as the cross product of $\vec{r}$ and $\vec{p}$

i.e., $\vec{L} = \vec{r} \times \vec{p}$

Its magnitude is given by $L = rp\sin\theta$

The unit of angular momentum is kg m^2 s^{-1} and its dimensional formula is [M L^2 T^{-1}]



Angular momentum of a rigid body: Let us consider a system of n particles of masses m_1, m_2 ,... m_n situated at distances r_1, r_2,...r_n respectively from the axis of rotation. Let v_1,v_2,... v_n be the linear velocities of the particles respectively.

The linear momentum of the 1st particle is

$$p_1 = m_1 v_1 = m_1(r_1\omega)$$

So angular momentum of the 1st particle is

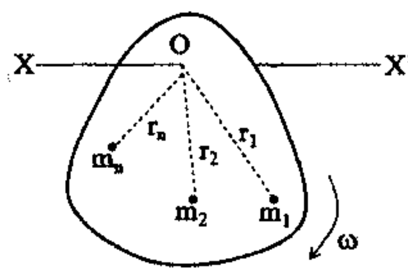
$$L_1 = r_1 p_1 = m_1 r_1^2 \omega$$

The linear momentum of the 2nd particle is

$$p_2 = m_2 v_2 = m_2(r_2\omega)$$

Also angular momentum of the 2nd particle is

$$L_2 = r_2 p_2 = m_2 r_2^2 \omega \text{ and so on}$$



The total angular momentum of the rigid body is

$$L = L_1 + L_2 + ...L_n$$

$$L = \omega(m_1 r_1^2 + m_2 r_2^2 + ...m_n r_n^2) = \omega\left(\sum_{i=1}^{i=n} m_i r_i^2\right)$$

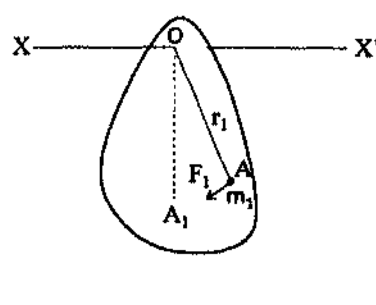
$$\therefore L = \omega I$$

where $I = \sum_{i=1}^{i=n} m_i r_i^2$ is the moment of inertia of the rotating rigid body.

Relation between torque and angular acceleration: Let us consider a rigid body rotating about a fixed axis XOX' with angular velocity ω. The force acting on a particle of mass m_1 situated at A, at a distance r_1, from the axis of rotation is

$$F_1 = m_1 a_1 = m_1 \frac{dv_1}{dt} = m_1 \frac{d(r_1\omega)}{dt} = m_1 r_1 \frac{d\omega}{dt}$$

$$F_1 = m_1 r_1 \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = m_1 r_1 \frac{d^2\theta}{dt^2}$$



The torque acting on m_1 about the axis of rotation is

$$\tau_1 = r_1 F_1 = m_1 r_1^2 \frac{d^2\theta}{dt^2} \text{ and so on.}$$

Thus the total torque on all the particles in the body is

$$\tau = \tau_1 + \tau_2 + ...\tau_n$$

$$\tau = (m_1 r_1^2 + m_2 r_2^2 + ...m_n r_n^2)\frac{d^2\theta}{dt^2}$$

$$\tau = \left(\sum_{i=1}^{i=n} m_i r_i^2\right)\frac{d^2\theta}{dt^2}$$

$$\tau = I\alpha$$

Relation between torque and angular momentum: The angular momentum of a rotating rigid body is

$$L = I\,\omega$$

Differentiating the above equation with respect to time, we have

$$\frac{dL}{dt} = I\frac{d\omega}{dt} = I\alpha = \tau$$

$$\therefore \tau = \frac{dL}{dt}$$

Conservation of angular momentum: The angular momentum of a rotating rigid body is

$$L = I\,\omega$$

The torque acting on a rigid body is

$$\tau = \frac{dL}{dt}$$

When no external torque acts on the body

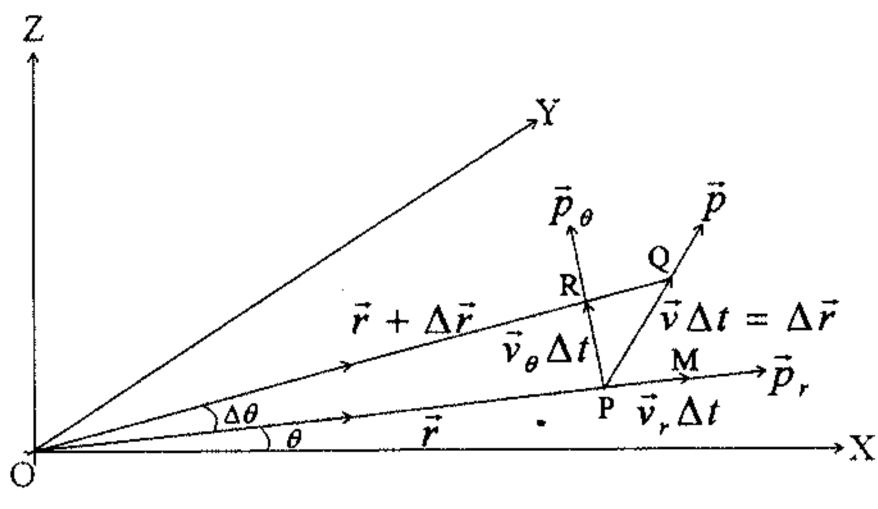
$$\tau = 0 \qquad \Rightarrow \frac{dL}{dt} = 0 \qquad \text{Or } L = \text{constant}$$

i.e., $I\omega = L =$ constant.

Total angular momentum of the body = constant

when no external torque acts on the body, the net angular momentum of a rotating rigid body remains constant. This is known as law of conservation of angular momentum.

Geometrical meaning of angular momentum:



Let a particle of mass m at P moves to Q in time Δt with velocity $\vec{v}$ and momentum $\vec{p}$. The position vector of P and Q be $\vec{r}$ and $\vec{r}+\Delta\vec{r}$ respectively. Therefore the displacement $PQ = \Delta\vec{r}$.

The components of $\vec{v}$ are $\vec{v}_\theta$ and $\vec{v}_r$ perpendicular to $\vec{r}$ and along $\vec{r}$, also the components of $\vec{p}$ are $\vec{p}_\theta$ and $\vec{p}_r$ perpendicular to $\vec{r}$ and along $\vec{r}$ respectively. Thus

$$PR = v_\theta \Delta t$$

The angular momentum $L = r\, p_\theta = r\, mv_\theta$

$$\text{Or} \quad r\, v_\theta = \frac{L}{m} \text{-------} (1)$$

In Δ OPQ, the area ΔA is given by

$$\Delta A = \frac{1}{2} OP \times PR$$

$$\text{Or} \quad \Delta A = \frac{1}{2} r \times v_\theta \Delta t$$

$$\text{Or} \quad \frac{\Delta A}{\Delta t} = \frac{1}{2} \cdot \frac{L}{m} \text{ by using equation (1)}$$

$$\text{Or} \quad L = 2m \cdot \frac{\Delta A}{\Delta t}$$

Where $\frac{\Delta A}{\Delta t}$ is called areal velocity and may be defined as the rate of sweeping out of area by radius vector from O.

$\therefore$ Angular momentum = $2m$ × Areal velocity.

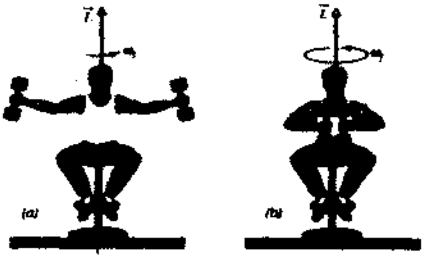
This is the geometrical significance of angular momentum for two dimensional motion.

$$\text{In vector form, } \vec{L} = 2m \times \frac{\Delta \vec{A}}{\Delta t}$$

Illustration of conservation of angular momentum: When no external torque acts on the body

$$\tau = 0 \qquad \Rightarrow \frac{dL}{dt} = 0 \qquad \text{Or } L = \text{constant}$$

i.e., $I\omega = L = mr^2\omega$ = constant.



Example for conservation of angular momentum

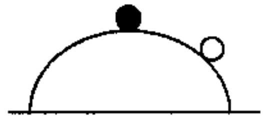
Mass (m) kg	Separation of the two masses $(r)^2$ m^2	Angular frequency (ω) rev/s	Angular momentum (L) kg m^2s^{-1} or Js
50	1^2	1	50
50	$(1/2)^2$	4	50
50	$(1/4)^2$	16	50
50	$(1/100)^2$	10,000	50

Equilibrium of bodies and types of equilibrium: A rigid body is said to be in equilibrium if the vector sum of the forces acting on the body is zero and the net torque acting on it is also zero. There are three types of equilibrium.

(i) Stable equilibrium: A body is in stable equilibrium if it returns to its equilibrium position after it has been displaced slightly. For stable equilibrium, the potential energy is minimum.

Stable equilibrium

(ii) Unstable equilibrium: A body is in unstable equilibrium if it does not return to its equilibrium position and does not remain in the displaced position after it has been displaced slightly. For unstable equilibrium, the potential energy is maximum.



Unstable equilibrium

(iii) Neutral equilibrium: A body is in neutral equilibrium if it stays in the displaced position after it has been displaced slightly. For neutral equilibrium, the potential energy is constant.

Neutral equilibrium

Moment of inertia of different bodies:

Body	Axis of Rotation	Moment of Inertia
Thin Uniform Rod	Axis passing through its centre of gravity and perpendicular to its length	$\frac{1}{12}Ml^2$ M = mass l = length
	Axis passing through the end and perpendicular to its length.	$\frac{1}{3}Ml^2$ M = mass l = length

Body	Axis of Rotation	Moment of Inertia
Thin Circular Ring	Axis passing through its centre and perpendicular to its plane.	MR^2 M = mass R = radius
	Axis passing through its diameter	$\frac{1}{2}MR^2$ M = mass R= radius
	Axis passing through a tangent	$\frac{3}{2}MR^2$ M = mass R= radius

Body	Axis of Rotation	Moment of Inertia
Circular Disc	Axis passing through its centre and perpendicular to its plane.	$\frac{1}{2}MR^2$ M = mass R= radius
	Axis passing through its diameter	$\frac{1}{4}MR^2$ M = mass R= radius
	Axis passing through a tangent	$\frac{5}{4}MR^2$ M = mass R= radius

Body	Axis of Rotation	Moment of Inertia
Solid Sphere	Axis passing through its diameter	$\frac{2}{5}MR^2$ M = mass R= radius
	Axis passing through a tangent	$\frac{7}{5}MR^2$ M = mass R= radius

Body	Axis of Rotation	Moment of Inertia
Solid Cylinder	Its own axis	$\frac{1}{2}MR^2$ M = mass R= radius
	Axis passing through its centre and perpendicular to its length	$M\left(\frac{R^2}{4}+\frac{l^2}{12}\right)$ M = mass R= radius l = length

GRAVITATION

Gravity: The gravitational pull of the earth is called gravity.

Acceleration due to gravity: The acceleration that results in an object due to earth's gravity is called acceleration due to gravity. It is denoted by g. At a given place, the value of g is the same for all bodies irrespective of their masses.
The value of g at sea-level is taken as the standard i.e., $g = 9.8\ m\ s^{-2}$. It always acts downward, towards the centre of the earth. It is a vector quantity.

Note:
(1) Acceleration due to gravity g is considered to be constant within a distance of $10 km$ above the surface of the earth.
(2) The value of g varies slightly from place to place on the surface of the earth.
(3) Since $W = mg$ or $g = W/m$. Therefore g can also be expressed as N/kg.
(4) The value of g on the moon is about one sixth of that on the earth and on the sun is about 27 times of that on the earth.

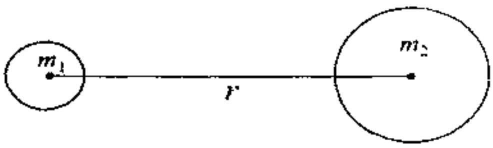
Vertical motion under gravity: For upward motion, g is negative and for downward motion g is positive.

Upward motion	Downward motion
$h = ut - \frac{1}{2}gt^2$	$h = ut + \frac{1}{2}gt^2$
$v = u - gt$	$v = u + gt$
$v^2 = u^2 - 2gh$	$v^2 = u^2 + 2gh$
$h_{nt} = u - \frac{g}{2}(2n-1)$	$h_{nt} = u + \frac{g}{2}(2n-1)$

Universal law of gravitation: The Law states that every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. The force acts along the line joining the centres of the two bodies.
Consider two bodies of masses m_1 and m_2 with their centres separated by a distance r. The gravitational force between them is

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{r^2}$$



$$\therefore F \propto \frac{m_1 m_2}{r^2}$$

Or $$F = G\frac{m_1 m_2}{r^2}$$

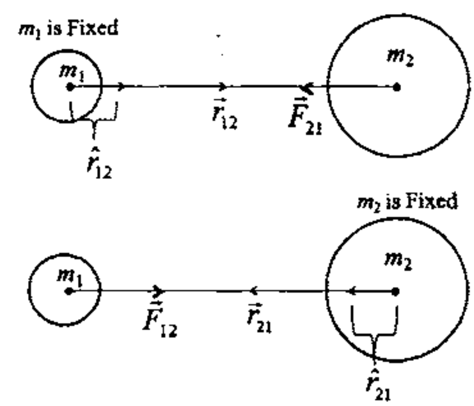
Where G is the universal gravitational constant.
The value of G is $6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$ and its dimensional formula is $[M^{-1} L^3 T^{-2}]$.

Note:
(1) The gravitational force F between the two bodies is always attracted.
(2) The gravitational force F between the two bodies is not altered by the presence of other

bodies.
(3) The value of G does not depend on the nature and size of the masses.
(4) The universal gravitational constant G is measured experimentally.
(5) The universal gravitational constant G is equal to the force of attraction between two bodies each of unit mass placed unit distance (centre-to-centre) apart i.e., $G = F$.
(6) This law fails if the distance between the objects is less than $10^{-9}\, m$ i.e., of the order of intermolecular distances.
(7) Newton's law of gravitation is valid for point masses. However it can be used for real objects whose centre of masses are at a distance r apart.
(8) Gravitational force is a central as well as conservative force.
(9) It is the weakest force in nature.
(10) It is about 10^{36} times smaller than the electrostatic force and about 10^{38} times smaller than the nuclear force.

Vector form of Newton's law of gravitation:
The force on mass m_2 due to mass m_1 is

$$\vec{F}_{21} = -G\frac{m_1 m_2}{(\vec{r}_{12})^2}\hat{r}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12}$$



The force on mass m_1 due to mass m_2 is

$$\vec{F}_{12} = -G\frac{m_1 m_2}{(\vec{r}_{21})^2}\hat{r}_{21} = -G\frac{m_1 m_2}{r^2}\hat{r}_{21}$$

$$\because \hat{r}_{12} = -\hat{r}_{21}$$

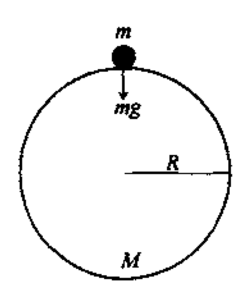
Thus $\vec{F}_{21} = -G\frac{m_1 m_2}{r^2}(-\hat{r}_{21})$

Or $\vec{F}_{21} = G\frac{m_1 m_2}{r^2}\hat{r}_{21}$

$\therefore \vec{F}_{12} = -\vec{F}_{21}$

Acceleration due to gravity at the surface of the earth: Consider a body of mass m on the surface of the Earth as shown in the Figure. Its distance from the centre of the Earth is R (radius of the Earth).
The gravitational force experienced by the body is

$F = G\frac{Mm}{R^2}$ where M is the mass of the Earth.



From Newton's second law of motion, Force $F = mg$.
Equating the above equations, we have

$$mg = G\frac{Mm}{R^2} \qquad \Rightarrow g = G\frac{M}{R^2}$$

This equation shows that g is independent of the mass of the body m. But, it varies with the distance from the centre of the Earth.

Mass of the earth:

$\because g = G\frac{M}{R^2}$

Or $M = \frac{gR^2}{G} = \frac{(9.8)(6.38\times10^6)^2}{6.67\times10^{-11}}\, kg$

$\therefore M = 5.98 \times 10^{24} kg$

Density of the earth:

$$\because \text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

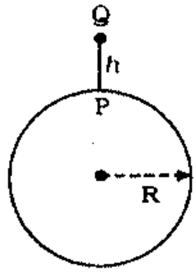
$$\text{Or} \quad \rho = \frac{\frac{gR^2}{G}}{\frac{4}{3}\pi R^3} = \frac{gR^2}{G} \times \frac{3}{4\pi R^3}$$

$$\text{Or} \ \rho = \frac{3g}{4\pi RG} = \frac{3(9.8)}{4(3.14)(6.37\times 10^6)(6.67\times 10^{-11})} kg\ m^{-3}$$

$$\therefore \rho = 5.5 \times 10^3 kg\ m^{-3}$$

Density of the Earth = 5.5 × Density of water

Variation of 'g' with altitude: Let P be a point on the surface of the Earth and Q be a point at an altitude h. Let the mass of the Earth be M and its radius be R.



The acceleration due to gravity at P is

$$g = \frac{GM}{R^2} - - - - - (1)$$

The acceleration due to gravity at Q is

$$g_h = \frac{GM}{(R+h)^2} - - - - - (2)$$

Dividing equation (2) by equation (1) we have

$$\frac{g_h}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM} = \frac{R^2}{(R+h)^2}$$

$$\frac{g_h}{g} = \frac{R^2}{\left\{R\left(1+\frac{h}{R}\right)\right\}^2} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g_h}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\text{Or} \quad g_h = g\frac{1}{\left(1+\frac{h}{R}\right)^2}$$

Case - I

If $h = R$

$$g_h = g\frac{1}{\left(1+\frac{R}{R}\right)^2} = g\frac{1}{(1+1)^2} = \frac{g}{4}$$

Case - II

If $h << R$

$$g_h = g\left(1+\frac{h}{R}\right)^{-2}$$

$$g_h = g\left(1-\frac{2h}{R}\right)$$

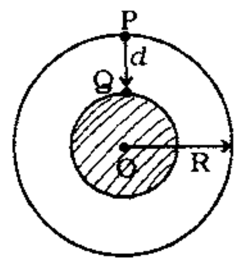
Variation of '*g*' with depth: Consider the Earth to be a homogeneous sphere with uniform density of radius *R* and mass *M*.

Let *P* be a point on the surface of the Earth and *Q* be a point at a depth *d* from the surface. The acceleration due to gravity at *P* is

$$g = \frac{GM}{R^2}$$

If ρ be the density, then, the mass of the Earth is

$$M = \frac{4}{3}\pi R^3 \rho$$



$$\therefore g = \frac{G\frac{4}{3}\pi R^3 \rho}{R^2} = G\frac{4}{3}\pi R\rho ------(1)$$

The acceleration due to gravity at *Q* is

$$g_d = \frac{GM_d}{(R-d)^2}$$

where M_d is the mass of the inner sphere of the Earth of radius $(R-d)$.

$$M_d = \frac{4}{3}\pi(R-d)^3\rho$$

$$\therefore g_d = \frac{G\frac{4}{3}\pi(R-d)^3\rho}{(R-d)^2} = G\frac{4}{3}\pi(R-d)\rho ------(2)$$

Dividing equation (2) by equation (1) we have

$$\frac{g_d}{g} = \frac{G\frac{4}{3}\pi(R-d)\rho}{G\frac{4}{3}\pi R\rho} = \frac{R-d}{R} = 1-\frac{d}{R}$$

$$\therefore g_d = g\left(1-\frac{d}{R}\right)$$

The value of acceleration due to gravity decreases with increase of depth.

Variation of '*g*' with '*r*': At the surface of the earth,

$$g = \frac{GM}{R^2} \qquad \Rightarrow \; GM = gR^2$$

Outside the earth:

At a distance *r* ($r > R$) from the centre of the earth, the acceleration due to gravity is

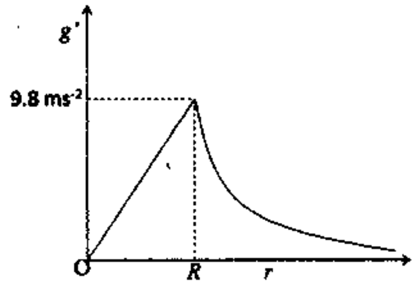
$$g' = \frac{GM}{r^2} \qquad \Rightarrow \; g' = g\frac{R^2}{r^2} \qquad \Rightarrow \; g' \propto \frac{1}{r^2}$$

Inside the earth:

At a distance *r* ($r < R$) from the centre of the earth, the acceleration due to gravity is

$$g' = \frac{GM'}{r^2}$$

Now $\frac{M'}{M} = \frac{\frac{4}{3}\pi r^3 \rho}{\frac{4}{3}\pi R^3 \rho} = \frac{r^3}{R^3}$ $\quad \Rightarrow M' = M \times \frac{r^3}{R^3}$



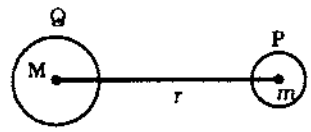
$$\therefore g' = \frac{G}{r^2}\left(M \times \frac{r^3}{R^3}\right) = GM\frac{r}{R^3} = gR^2\frac{r}{R^3} = g\frac{r}{R}$$

$\Rightarrow g' \propto r$

Gravitational field: The gravitational field is defined as the space around a mass in which it can exert gravitational force on other mass.

Gravitational field intensity: Gravitational field intensity or strength at a point is defined as the force experienced by a unit mass placed at that point. It is denoted by E. It is a vector quantity. Its unit is N kg^{-1}.

Consider a body of mass M placed at a point Q and another body of mass m placed at P at a distance r from Q.



The force experience by m due to M is

$$F = mE - - - - - (1)$$

The gravitational force of attraction between the masses m and M is

$$F = G\frac{Mm}{r^2} - - - - - (2)$$

Equating equation (1) and equation (2) gives

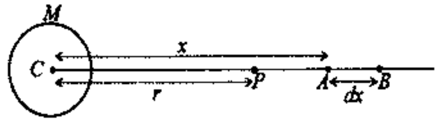
$$mE = G\frac{Mm}{r^2} \qquad \Rightarrow E = \frac{GM}{r^2}$$

Gravitational field intensity E is the measure of gravitational field.

Gravitational potential: Gravitational potential at a point is defined as the amount of work done in moving a unit mass from the point in the field to infinity against the gravitational field.

It is a scalar quantity, its unit is N m kg^{-1}.

Expression for gravitational potential at a point: Consider a body of mass M at the point C, and let P be a point at a distance r from C. To calculate the gravitational potential at P. We consider two points A and B in the field where A is at a distance x from C.



The gravitational field at A is

$$E = \frac{GM}{x^2}$$

Work done by the gravitational force in moving a unit mass from A to B is

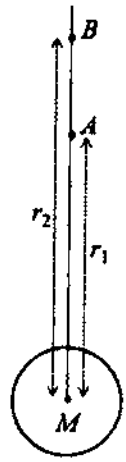
$$dw = -Fdx = -Edx = -\frac{GM}{x^2}dx \qquad \{\because m = 1\}$$

Work done by the gravitational force in moving a unit mass from P to infinity is

$$W = -GM\int_r^{\infty}\frac{dx}{x^2} = -GM\left[-\frac{1}{x}\right]_r^{\infty} = -GM\left[-\frac{1}{\infty} + \frac{1}{r}\right] = -\frac{GM}{r}$$

Or $V = -\dfrac{GM}{r}$

Gravitational potential difference: Gravitational potential difference between two points is defined as the amount of work done in moving a unit mass from one point to another point against the gravitational force of attraction.



The gravitational potential at a distance r_1 from the centre of the earth is

$$V_1 = -\frac{GM}{r_1}$$

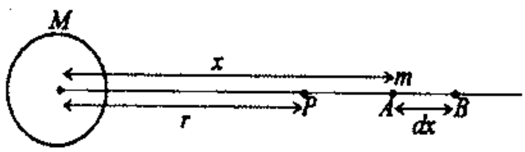
The gravitational potential at a distance r_2 from the centre of the earth is

$$V_2 = -\frac{GM}{r_2}$$

Potential difference is

$$\Delta V = V_2 - V_1 = GM\left[-\frac{1}{r_2} - \left(-\frac{1}{r_1}\right)\right] = GM\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Gravitational potential energy: Gravitational potential energy at a point is defined as the amount of work done in moving a body of mass *m* from a point in the field to infinity against the gravitational field.



Consider a body of mass *m* be placed at a point *A* at a distance *x* from the centre of the earth. Let *M* be the mass of the earth.

The gravitational force on *m* due to *M* is

$$F = \frac{GMm}{x^2}$$

The work done by the gravitational force in moving the mass *m* from *A* to *B* is

$$dw = -Fdx = -\frac{GMm}{x^2}dx$$

The work done by the gravitational force in moving the mass *m* from *P* to infinity is

$$W = -GMm\int_r^{\infty}\frac{dx}{x^2} = -GMm\left[-\frac{1}{x}\right]_r^{\infty}$$

$$W = -GMm\left[-\frac{1}{\infty} + \frac{1}{r}\right] = -\frac{GMm}{r}$$

Or $U = -\dfrac{GMm}{r}$

Gravitational potential energy is zero at infinity and decreases as the distance decreases.

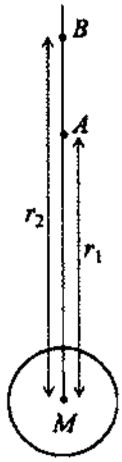
Change in gravitational potential energy: The gravitational potential energy of mass *m* at a distance r_1 from the centre of the earth is

$$U_1 = -\frac{GMm}{r_1}$$

The gravitational potential energy of mass *m* at a distance r_2 from the centre of the earth is

$$U_2 = -\frac{GMm}{r_2}$$

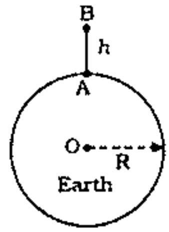
Change in gravitational potential energy is



$$\Delta GPE = U_2 - U_1 = GMm\left[-\frac{1}{r_2} - \left(-\frac{1}{r_1}\right)\right] = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= GM\left[\frac{1}{r_1} - \frac{1}{r_2}\right]m = (V_2 - V_1)m$$

Gravitational potential energy near the surface of the earth: Let the mass of the Earth be M and its radius be R. Consider a point A on the surface of the Earth and another point B at a height h above the surface of the Earth.



The gravitational potential energy of a body of mass m at A on the surface of the earth is

$$U_A = -\frac{GMm}{R}$$

The gravitational potential energy of a body of mass m at B, a height h from the surface of the earth is

$$U_B = -\frac{GMm}{R+h}$$

Change in gravitational potential energy is

$$\Delta GPE = U_B - U_A = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$\Delta GPE = GMm\left[\frac{R+h-R}{R(R+h)}\right] = GMm\frac{h}{R(R+h)}$$

If $h << R$ then

$$\Delta GPE = GMm\frac{h}{R^2}$$

$$\because g = \frac{GM}{R^2} \qquad \Rightarrow \quad GM = gR^2$$

$$\Delta GPE = gR^2 m\frac{h}{R^2}$$

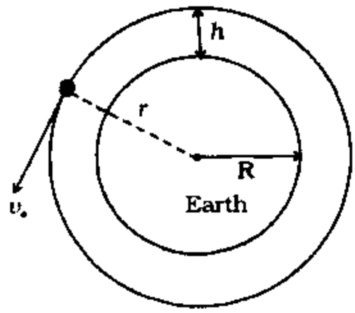
$$\therefore \Delta GPE = mgh$$

Inertial mass: Inertial mass of a body is a measure of the ability of a body to oppose the production of acceleration in it by an external force.

Gravitational mass: Gravitational mass is the mass of a body which determines the magnitude of gravitational pull between the body and the Earth.

Orbital velocity: The horizontal velocity that has to be imparted to a satellite at the determined height so that it makes a circular orbit around the planet is called orbital velocity.

Let us assume that a satellite of mass m moves around the Earth in a circular orbit of radius r with uniform speed v_o. Let the satellite be at a height h from the surface of the Earth. Hence, $r = R+h$, where R is the radius of the Earth. The centrifugal force required to keep the satellite in circular orbit is



$$F = \frac{mv_o^2}{r} = \frac{mv_o^2}{R+h}$$

The gravitational force between the Earth and the satellite is

$$F = \frac{GMm}{r^2} = \frac{GMm}{(R+h)^2}$$

For the stable orbital motion

$$\frac{mv_o^2}{R+h} = \frac{GMm}{(R+h)^2}$$

$$v_o^2 = \frac{GM}{R+h} \qquad \Rightarrow \quad v_o = \sqrt{\frac{GM}{R+h}}$$

$$\because GM = gR^2$$

$$\therefore v_o = \sqrt{\frac{gR^2}{R+h}}$$

The orbital velocity of a satellite is independent of the mass of the satellite and depends only upon its height h above the earth's surface.
If the satellite is at a height of few hundred kilometres, $(R+h)$ could be replaced by R.

$$\therefore v_o = \sqrt{\frac{gR^2}{R}} \qquad \Rightarrow \quad v_o = \sqrt{gR}$$

The orbital velocity for Earth is 8 $km\ s^{-1}$

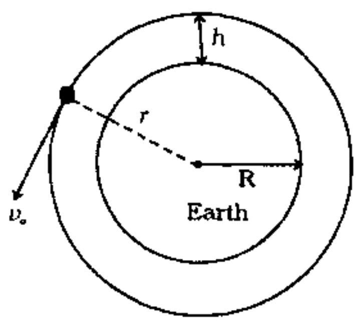
Note: If v is the speed of the satellite in its orbit and v_0 is the orbital velocity to move in the orbit, then
(i) If $v = v_0$, then the satellite revolves in a circular path around the earth.
(ii) If $v < v_0$, then the satellite will move on a parabolic path and finally falls back to earth.
(iii) If $v > v_0$, then the satellite will revolve around the earth in elliptical orbit.

Time period of a satellite: Time taken by the satellite to complete one revolution round the Earth is called time period.

$$\text{Time period} = \frac{\text{circumference of the orbit}}{\text{orbital velocity}}$$

$$T = \frac{2\pi r}{v_o} = \frac{2\pi(R+h)}{v_o}$$

$$T = 2\pi(R+h)\sqrt{\frac{R+h}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$



As $GM = gR^2$

$$\therefore T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

If the satellite orbits very close to the Earth, then $h << R$

$$\therefore T = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}$$

Note: Since $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

The mass of the earth M is volume × density i.e.,

$$M = \frac{4}{3}\pi R^3 \times \rho = \frac{4\pi R^3 \rho}{3}$$

$$\therefore T = 2\pi\sqrt{\frac{(R+h)^3}{G\times\frac{4\pi R^3\rho}{3}}} = \sqrt{\frac{3\times 4\pi^2 (R+h)^3}{G\times 4\pi R^3\rho}} = \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

If the satellite is very closed to the earth, then $R >> h$

$$\therefore T = \sqrt{\frac{3\pi R^3}{G\rho R^3}} = \sqrt{\frac{3\pi}{G\rho}}$$

Height of the satellite above the earth surface: The time period of the satellite is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

Squaring both sides gives

$$T^2 = \frac{4\pi^2(R+h)^3}{gR^2}$$

$$(R+h)^3 = \frac{gR^2T^2}{4\pi^2}$$

$$R+h = \left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3}$$

$$\therefore h = \left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3} - R$$

Geo-stationary satellites: Satellites which appear to remain in fixed positions at a specified height above the equator are called synchronous satellites or geo-stationary satellites.

The speed of the satellite in its orbit is

$$v = \frac{\text{circumference of the orbit}}{\text{time period}}$$

$$v = \frac{2\pi r}{T}$$

The centrifugal force is

$$F = \frac{mv^2}{r}$$

$$\therefore F = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4m\pi^2 r}{T^2}$$

The gravitational force on the satellite due to the Earth is

$$F = \frac{GMm}{r^2}$$

For the stable orbital motion

$$\frac{4m\pi^2 r}{T^2} = \frac{GMm}{r^2} \qquad \Rightarrow r^3 = \frac{GMT^2}{4\pi^2}$$

$$\because GM = gR^2$$

$$\therefore r^3 = \frac{gR^2T^2}{4\pi^2}$$

The orbital radius of the geo- stationary satellite is

$$r = \left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3}$$ This orbit is called parking orbit of the satellite.

Note: The distance of the satellite from the surface of the earth is 36000km, the radius of the orbit is 42400km, the time period is 24h, the orbital velocity is 3.1km/s and the angular velocity is $\frac{2\pi}{24} = \frac{\pi}{12} rad/h$

Polar satellite: The polar satellites revolve around the Earth in a north-south orbit passing over the poles as the Earth spins about its north-south axis.
The polar satellites positioned nearly 900 km above the earth, travels pole to pole in about 84 minutes with and orbital velocity of 8km/s. The polar orbit remains fixed in space as the earth rotates inside the orbit. As a result, most of the earth's surface crosses the satellite in a polar orbit.

Energy of an orbiting satellite: A satellite revolving in a circular orbit round the Earth possesses both potential energy and kinetic energy. If h is the height of the satellite above the Earth's surface and R is the radius of the Earth, then the radius of the orbit of satellite is $r = R+h$.

If m is the mass of the satellite, its potential energy is

$$E_p = -\frac{GMm}{r} = -\frac{GMm}{R+h}$$

The orbital velocity of the satellite is

$$v_o = \sqrt{\frac{GM}{R+h}}$$

Hence, its kinetic energy is

$$E_K = \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(\frac{GM}{R+h}\right) = \frac{GMm}{2(R+h)}$$

The total energy of the satellite is, $E = E_P + E_K$

$$E = -\frac{GMm}{R+h} + \frac{GMm}{2(R+h)}$$

$$\therefore E = -\frac{GMm}{2(R+h)}$$

The negative value of the total energy indicates that the satellite is bound to the Earth.

Escape speed from energy principle: The escape speed is the minimum speed with which a body must be projected in order that it may escape from the gravitational pull of the planet. Consider a body of mass m placed on the Earth's surface.

The gravitational potential energy is $E_P = -\frac{GMm}{R}$

If the body is projected up with a speed v_e, the kinetic energy is $E_K = \frac{1}{2}mv_e^2$

$\therefore$ the initial total energy of the body is $E_i = \frac{1}{2}mv_e^2 - \frac{GMm}{R}$

If the body reaches a height h above the Earth's surface,

The gravitational potential energy is $E_P = -\frac{GMm}{R+h}$

Let the speed of the body at the height h is v, then its kinetic energy is $E_K = \frac{1}{2}mv^2$

$\therefore$ The final total energy of the body is $E_f = \frac{1}{2}mv^2 - \frac{GMm}{R+h}$

$$\because E_i = E_f$$

$$\therefore \frac{1}{2}mv_e^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{R+h}$$

The body will escape from the Earth's gravity at a height where the gravitational field is zero. i.e., $h = \infty$. At the height $h = \infty$, the speed v of the body is zero.

$$\text{Thus} \quad \frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0$$

$$\text{Or} \quad v_e^2 = \frac{2GM}{R} \qquad \Rightarrow \quad v_e = \sqrt{\frac{2GM}{R}}$$

$$\text{Again} \quad \because GM = gR^2$$

$$v_e = \sqrt{\frac{2gR^2}{R}} \qquad \Rightarrow \quad v_e = \sqrt{2gR}$$

The escape speed for Earth is 11.2 $km\ s^{-1}$

Note: The relation between the escape velocity and the orbital velocity of the satellite is

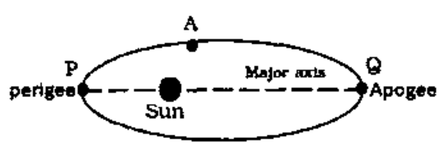
$$v_e = v_0\sqrt{2}$$

If the velocity of projection v is equal to the escape velocity v_e, then the satellite will escape away following a parabolic path. If the velocity of projection v is greater than the escape velocity v_e, then the satellite will escape away following a hyperbolic path.

Kepler's law of planetary motion:

(i) The law of orbits:

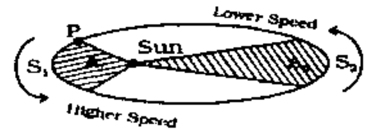
Each planet moves in an elliptical orbit with the Sun at one focus.



(ii) The law of areas:

The line joining the Sun and the planet (i.e., radius vector) sweeps out equal areas in equal interval of times.

i.e., $\frac{dA}{dt} = \text{constant}$

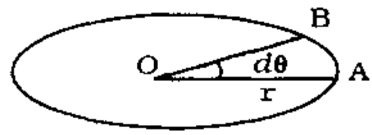


Proof for the law of areas:

Consider a planet moving from A to B. The radius vector OA sweeps a small angle $d\theta$ at the centre in a small interval of time dt. From the Figure, $AB = rd\theta$. The small area dA swept by the radius is

$$dA = \frac{1}{2} r \times (AB) \qquad \Rightarrow dA = \frac{1}{2} r \times rd\theta$$

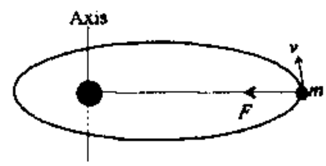
Dividing both sides by dt we have



$$\frac{dA}{dt} = \frac{1}{2} r^2 \times \frac{d\theta}{dt} \qquad \Rightarrow \frac{dA}{dt} = \frac{1}{2} r^2 \times \omega$$

$$\because L = mr^2\omega \qquad \Rightarrow r^2\omega = \frac{L}{m}$$

Hence $\frac{dA}{dt} = \frac{1}{2} \cdot \frac{L}{m}$



Since the line of action of gravitational force passes through the axis, the external torque is zero. Hence, the angular momentum is conserved.

$$\therefore \frac{dA}{dt} = \text{constant}$$

i.e., the area swept by the radius vector in unit time is the same.

(iii) The law of periods:

The square of the period of revolution of a planet around the Sun is directly proportional to the cube of the mean distance between the planet and the Sun.

i.e., $\frac{T^2}{r^3} = \text{constant}$

Proof for the law of periods:

Let us consider a planet of mass m moving with the velocity v around the Sun of mass M in a circular orbit of radius r.

The gravitational force of attraction of the Sun on the planet is $F = \frac{GMm}{r^2} - - - - - (1)$

The centripetal force is $F = \frac{mv^2}{r} - - - - - (2)$

Equating equation (1) and equation (2) gives

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \qquad \Rightarrow v^2 = \frac{GM}{r} - - - - - (3)$$

If T be the period of revolution of the planet around the Sun, then

$$v = \frac{2\pi r}{T} \qquad \Rightarrow v^2 = \frac{4\pi^2 r^2}{T^2} - - - - - (4)$$

Equating equation (3) and equation (4) we have $\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2} \qquad \Rightarrow \frac{T^2}{r^3} = \frac{4\pi^2}{GM}$

The quantity $\frac{4\pi^2}{GM}$ is a constant that depends only on the mass M of the central body about which the planet orbits.

$$\therefore \frac{T^2}{r^3} = \frac{4\pi^2}{GM} = k$$

Note: $k = \frac{4\pi^2}{GM} = \frac{4\pi^2}{G} \times \frac{1}{M}$

$$\therefore k \propto \frac{1}{M}$$

Derivation of Newton's law of gravitation from Kepler's third law: Let us consider a planet of mass m moving with the velocity v around the Sun of mass M in a circular orbit of radius r, and T is the time period.

$$\therefore v = \frac{2\pi r}{T}$$

$$\text{Or} \quad v^2 = \frac{4\pi^2 r^2}{T^2}$$

The centripetal force between the Sun and the planet is F and is given by

$$F = \frac{mv^2}{r} = \frac{m}{r} \times \frac{4\pi^2 r^2}{T^2}$$

According to Kepler's third law we have

$$T^2 = k\, r^3$$

$$\therefore F = \frac{m}{r} \times \frac{4\pi^2 r^2}{k\, r^3} = \frac{4\pi^2}{k} \times \frac{m}{r^2}$$

$$\because \frac{4\pi^2}{k} \propto M$$

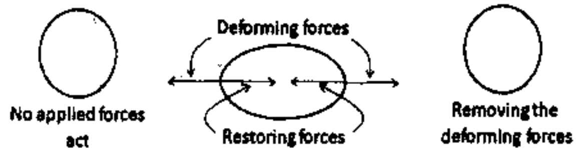
$$\text{Or} \quad \frac{4\pi^2}{k} = GM$$

$$\therefore F = G\frac{M\, m}{r^2}$$

Which is the Newton's law of gravitation.

PROPERTIES OF BULK MATTER

Elasticity: The property of a material to regain its original state when the deforming force is removed is called elasticity. Examples quartz fiber, phosphor bronze, etc.



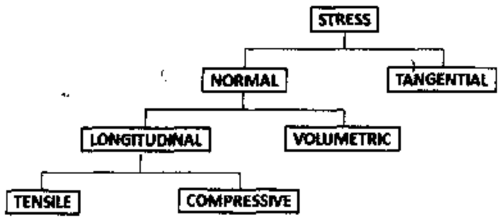
Plasticity: The inability the of a material to regain its original state when the deforming force is removed is called plasticity. Examples plastic, bakelite, etc.

Stress: The internal restoring force acting per unit area of the deformed body is called stress.

$$\text{Stress} = \frac{\text{restoring force}}{\text{area}} = \frac{F}{A}$$

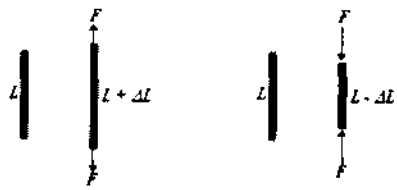
The dimensional formula is $[ML^{-1}T^{-2}]$

Types of stress:

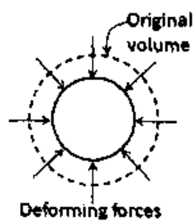


(1) Normal stress (2) Tangential or shearing stress

(1) Normal stress: When the restoring force or the deforming force acts perpendicular to the cross-sectional area of the body, the stress is called Normal stress. Normal stress are of two types. Longitudinal and volumetric. Longitudinal stress again has two types, Tensile stress and compressive stress.



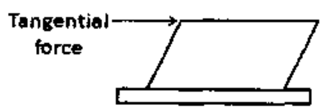
(a) Tensile stress: When there is an increase in the length of an elastic body in the direction of the deforming force, the stress set up is called Tensile stress.
(b) Compressive stress: When there is a decrease in the length of an elastic body due to the deforming force, the stress set up is called Compressive stress.



(c) Volumetric stress: When there is a change in the volume of an elastic body in the direction of the deforming force, the stress set up is called Volumetric or Hydraulic stress.

(2) Tangential or shearing stress: The restoring force per unit area developed in an elastic body due to the applied tangential force is known as Tangential or shearing stress.

$$\text{Tangential stress} = \frac{\text{tangential force}}{\text{area}} = \frac{F}{A}$$



Strain: Strain produced in a body is defined as the ratio of change in dimension of a body to the original dimension.

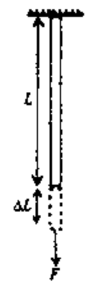
$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

Strain is the ratio of two similar quantities. Therefore it has no unit.

Types of strain:

(i) Longitudinal strain (ii) Shearing strain (iii) Volumetric strain

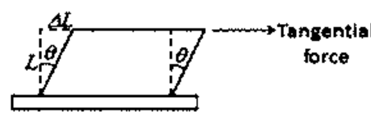
(i) Longitudinal strain: If the deforming force acting on an elastic body produces a change in length only, then the change in length per unit original length of the body is known as longitudinal strain.



$$\text{Longitudinal Strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

(ii) Shearing strain: Shearing strain can be defined as the ratio of the displacement of a surface under a tangential force to the perpendicular distance of the displaced surface from the fixed surface.

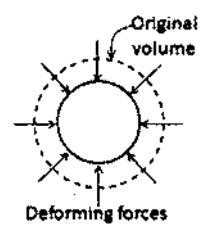
$$\text{Shearing strain} = \frac{\Delta L}{L} = \tan\theta \approx \theta$$



θ is measured in radian

(iii) Volumetric strain: If the deforming force acting on an elastic body produces a change in volume per unit original volume of the body, is known as Volumetric strain.

$$\text{Volumetric Strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$



Elastic limit: The limit beyond which permanent deformation occurs is called the elastic limit.

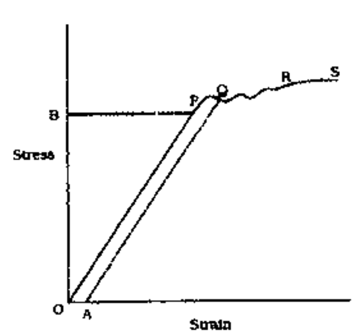
Hooke's law: According to Hooke's law, within the elastic limit, strain produced in a body is directly proportional to the stress that produces it.

i.e., stress α strain $\Rightarrow$ stress = k strain

$$\frac{\text{Stress}}{\text{Strain}} = k$$

Where k is known as modulus of elasticity .Its unit is N m^{-2} and its dimensional formula is $[ML^{-1}T^{-2}]$.

Study of stress-strain relationship: Let a wire be suspended from a rigid support. At the free end of the wire, a weight hanger is provided on which weights could be added to study the behaviour of the wire under different load conditions. The extension of the wire is suitably measured and a stress-strain graph is plotted as shown in the figure.



(i) In the figure the region OP is linear. Within a normal stress, strain is proportional to the applied stress. Here Hooke's law is obeyed. Upto P, when the load is removed the wire regains its original length along PO. The point P

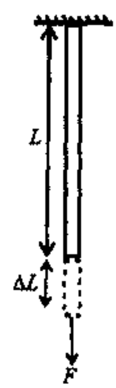
represents the elastic limit, PO represents the elastic range of the material and OB is the elastic strength.
(ii) Beyond P, the graph is not linear. In the region PQ, the material is partly elastic and partly plastic. From Q, if we start decreasing the load, the graph does not come to O via P, but traces a straight line QA. Thus a permanent strain OA is produced in the wire. This is called permanent set.
(iii) Beyond Q addition of even a very small load causes enormous strain. This point Q is called the yield point. The region QR is the plastic range.
(iv) Beyond R, the wire loses its shape and becomes thinner and thinner in diameter and ultimately breaks, say at S. Therefore S is the breaking point. The stress corresponding to S is called breaking stress.

Elastic after-effect or Elastic relaxation time: Elastic after-effect is the temporary delay in regaining the original configuration by an elastic object when all the deforming forces acting on the object are removed.

Elastic fatigue: The property of an elastic body due to which its behaviour becomes less elastic under the action of repeated alternating deforming forces is called elastic fatigue.

Elastic moduli: The ratio of the stress to the corresponding strain produced in a body within the elastic limit is called modulus of elasticity or coefficient of elasticity.
There are three different types of modulus of elasticity. They are Young's modulus, Bulk modulus and Rigidity modulus

Young's modulus: The ratio of longitudinal stress to the longitudinal strain within the elastic limits is called Young's modulus of elasticity. It is denoted by Y.
Consider a wire of length L and cross sectional area A stretched by a deforming force F acting along its length. Let ΔL be the extension produced.



$$\text{Longitudinal stress} = \frac{\text{deforming force}}{\text{area}} = \frac{F}{A}$$

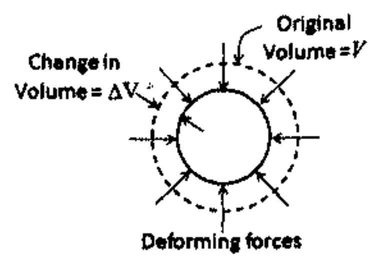
$$\text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

$$\text{Young's modulus} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} = \frac{F}{A} \times \frac{L}{\Delta L}$$

$$\therefore Y = \frac{FL}{A\Delta L}$$

Its SI unit is N/m^2 or pascal and its dimensional formula is $[ML^{-1}T^{-2}]$.

Bulk modulus: The ratio of normal stress to the volumetric strain produced in the body within the elastic limits is called Bulk modulus of elasticity. It is denoted by K.
Consider a sphere of volume V and surface area A is reduced by the deforming force F acting normally everywhere on the surface of the sphere. Let the decrease in volume be ΔV.



$$\text{Bulk stress} = \frac{\text{deforming force}}{\text{area}} = \frac{F}{A}$$

$$\text{Bulk strain} = \frac{\text{change in volume}}{\text{original volume}} = -\frac{\Delta V}{V}$$

$$\text{Bulk modulus} = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\frac{F}{A}}{\frac{\Delta V}{V}} = -\frac{F}{A} \times \frac{V}{\Delta V} = -\frac{FV}{A\Delta V}$$

$$\therefore K = -\frac{PV}{\Delta V} \qquad \left[\because P = \frac{F}{A}\right]$$

Its SI unit is N/m^2 or pascal and its dimensional formula is $[ML^{-1}T^{-2}]$.

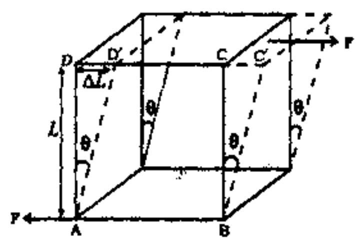
Compressibility: The compressibility of the material is a measure of how easy the material can be compressed. In other words, compressibility is just the reciprocal of Bulk modulus i.e.,

$$\text{Compressibility } k = \frac{1}{K}$$

Its SI unit is $N^{-1}m^2$ and cgs unit is $\text{dyne}^{-1}\text{cm}^2$.

Note: Steel is more elastic than rubber. Solid are more elastic and gasses are least elastic.

Rigidity modulus or shear modulus: The ratio of tangential stress to the tangential strain produced in the body within the elastic limits is known as Shear modulus or rigidity modulus. It is denoted by η. Let us apply a force F tangential to the top surface of a block whose bottom AB is fixed. Under the action of this tangential force, the body suffers a slight change in shape, its volume remaining unchanged. The side AD of the block is sheared through an angle θ to the position AD'. If the area of the top surface is A then



$$\text{Tangential stress or Shear stress} = \frac{\text{tangential force}}{\text{area}} = \frac{F}{A}$$

$$\text{Tangential strain or Shear strain} = \frac{\Delta L}{L} = \tan\theta \approx \theta$$

$$\text{Rigidity modulus} = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\frac{F}{A}}{\theta} = \frac{F}{A\theta}$$

$$\therefore \eta = \frac{F}{A\theta} = \frac{FL}{A\Delta L}$$

Its SI unit is N/m^2 or pascal and its dimensional formula is $[ML^{-1}T^{-2}]$.

Note: Young's modulus Y and modulus of rigidity η are possessed materials only.

Poisson's ratio: The ratio of the lateral strain to the longitudinal strain is called the Poisson's ratio. It is denoted by σ.
Let L and D are the original length and diameter of the wire respectively, ΔL is a small increase in the length and ΔD is a small decrease in the diameter.

$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

$$\text{Lateral strain} = -\frac{\Delta D}{D}$$

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\sigma = \frac{-\frac{\Delta D}{D}}{\frac{\Delta L}{L}} = -\frac{\Delta D}{D} \times \frac{L}{\Delta L} = -\frac{L\Delta D}{D\Delta L}$$

Theoretically the value of σ lies between -1 and +0.5 for all substances. However practically it lies between 0 and +0.5.

Elastic potential energy in a stretched wire: When a wire is stretched, the deforming force does some work against the internal restoring forces acting between the various particles of the wire. This work done is stored in the form of potential energy in the wire and is known as elastic potential energy.

Let L is the length of the wire and A is the area of cross-section. F is the deforming force, produces an increase ΔL in the length of the wire.

When the wire is stretched, the deforming force increases from 0 to F.

$$\therefore \text{the average force} = \frac{0+F}{2} = \frac{F}{2}$$

Work done = Average force × increase in length

$$W = \frac{F}{2} \times \Delta L$$

$$\text{Or } U = \frac{F}{2} \times \Delta L$$

Multiplying and dividing the RHS of the above equation by AL we have

$$U = \frac{F}{2} \times \Delta L \times \frac{AL}{AL}$$

$$U = \frac{1}{2}\frac{F}{A} \times \frac{\Delta L}{L} \times AL$$

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume of the wire}$$

The elastic potential energy per unit volume of the wire is

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

Fluids: A fluid is a substance that can flow when external force is applied on it. Liquid and gases are fluids.

Density: The ratio of mass of a substance to the volume occupied by the substance is known as density. It is denoted by ρ

Relative density: The ratio of the density of a substance to the density of water at 4^0 C is known as relative density

$$\text{Relative density} = \frac{\text{density of substance}}{\text{density of water at } 4^0 C}$$

$$= \frac{\text{weight of the substance in air}}{\text{lost of weight in water}}$$

Relative density is also known as specific gravity and has no unit and dimension.

Pressure: The force acting normally on a unit area of the surface is called pressure i.e.,

$$P = \frac{F}{A}$$

Thrust: The total force exerted normally on the surface is called Thrust. Since thrust is a force, its SI unit is newton (N)

Pressure due to a liquid column: Let h be the height of the liquid column in a cylinder of cross sectional area A. If ρ is the density of the liquid, then weight of the liquid column is given by

$$W = mg = (Ah\rho)g$$

$$\because \text{Pressure} = \frac{\text{Weight of liquid column}}{\text{Area of cross - section}}$$

$$\therefore P = \frac{(Ah\rho)g}{A} = \rho gh$$

Atmospheric pressure: The pressure exerted by the atmosphere on the surface of the earth is called atmospheric pressure. It is about $10^5\ N/m^2$.

Note: At sea level, atmospheric pressure is equal to 76 *cm* of mercury column. Then atmospheric pressure is $\rho gh = 13.6 \times 980 \times 76\ dyne/cm^2$.

Pascal's law: Pascal's law states that if the effect of gravity can be neglected then the pressure in a fluid in equilibrium is the same everywhere.

Effect of gravity in fluid pressure: When gravity is taken into account, Pascal's law is to be modified. Consider a cylindrical liquid column of height h, density ρ and area of cross-section of the circular face is A as shown in the figure. If the effect of gravity is neglected, then pressure at M will be equal to pressure at N. But, if force due to gravity is taken into consideration, then they are not equal. At equilibrium

(i) Force P_1A acting vertically down on the top surface.
(ii) Weight mg of the liquid column acting vertically downwards
(iii) Force P_2A acting vertically upwards.
Where P_1 and P_2 are the pressures at the top and bottom faces, and m is the mass of the cylindrical liquid column.

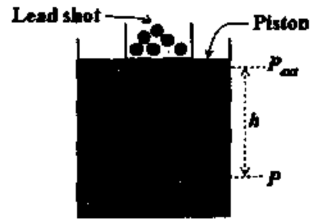
$$\therefore P_2A = P_1A + mg \qquad \Rightarrow P_2 = P_1 + \frac{mg}{A}$$

$$P_2 = P_1 + \frac{(A\rho h)g}{A} = P_1 + \rho gh \qquad \Rightarrow P_2 = P_1 + \rho gh$$

This equation proves that, if g is zero, the pressure P_1 is equal to the pressure P_2.

Pascal's principle: Any change in the pressure applied to a completely enclosed fluid is transmitted undiminished to all parts of the fluid and the enclosing walls.

Demonstrating Pascal's principle: Let a tall cylinder containing an incompressible liquid fitted with a piston on which the container of lead shots rest on it is shown in the figure. The atmosphere, container and the lead shots exert a pressure P_{ext} on the piston and thus on the liquid.



The pressure P at a point Q in the liquid is

$$P = P_{ext} + \rho gh$$

If we add little more lead shots to the container to increase P_{ext} by an amount ΔP_{ext}, the total pressure P will increase by ΔP i.e.,

$$P + \Delta P = P_{ext} + \Delta P_{ext} + \rho gh = \Delta P_{ext} + (P_{ext} + \rho gh)$$

$$P + \Delta P = \Delta P_{ext} + P$$

$$\therefore \Delta P = \Delta P_{ext}$$

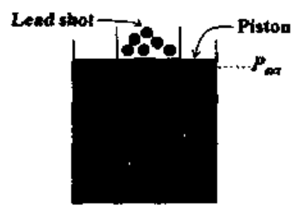
$\because \rho$, g and h remain unchanged.

This results in another statement of Pascal's law which can be stated as, change in pressure at any point in an enclosed fluid at rest is transmitted undiminished to all points in the fluid and act in all directions.

Example

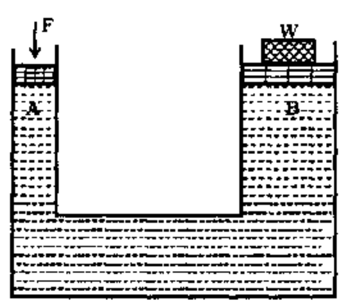
As shown in the figure,

In equilibrium,	Point S	Point Q	Point T
Total pressure P	40	50	60
External pressure P_{ext}		10	
$\rho g h$	30	40	50



If we add little more lead shots to the container to increase P_{ext} by an amount $\Delta P_{ext} = 5$, the change in pressure at S is 5, the change in pressure at Q is 5 and the change in pressure at T is also 5. In all the cases $\rho g h$ is constant in all the points. So the distribution of pressure is the same everywhere to all the points in the fluid.

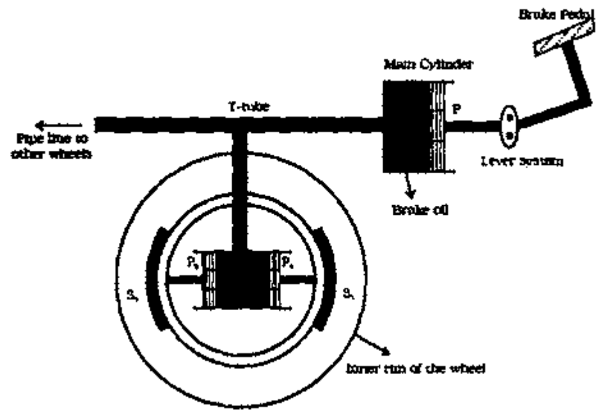
Hydraulic lift: An important application of Pascal's law is the hydraulic lift used to lift heavy objects. A schematic diagram of a hydraulic lift is shown in the figure. It consists of a liquid container which has pistons fitted into the small and large opening cylinders. If a_1 and a_2 are the areas of the pistons A and B respectively, F is the force applied on A and W is the load on B, then



$$\frac{F}{a_1} = \frac{W}{a_2} \qquad \Rightarrow W = \frac{a_2}{a_1} F$$

This is the load that can be lifted by applying a force F on A.

Hydraulic brakes: When a driver applies a little force by his foot on the brake paddle to stop the vehicle, the pressure so applied gets transmitted through the brake oil, to the piston of slave cylinders, which, in turn, pushes the break shoes against the break drum in all four wheels, simultaneously. The wheels stop rotating at the same time and the vehicle comes to stop instantaneously.



Gauge pressure: The difference between the total or absolute pressure at a point inside a liquid and the atmospheric pressure is known as gauge pressure at that particular point i.e.,

$$P_G = P - P_{atm}$$

$$\because P = P_{atm} + \rho g h$$

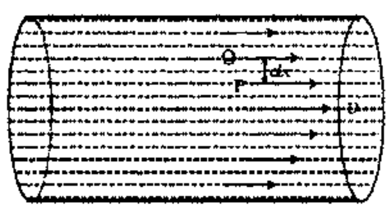
$$P_G = P_{atm} + \rho g h - P_{atm}$$

$$\therefore P_G = \rho g h$$

Viscosity: Viscosity is the property of the fluid by virtue of which it opposes the relative motion between its different layers. Both liquids and gases exhibit viscosity but liquids are much more viscous than gases.

Importance of viscosity:
(i) Blood circulation through arteries depends upon the viscosity of blood
(ii) Oil used as lubricant should have proper value of viscosity
(iii) The viscosity of blood depends upon the concentration of red blood corpuscles.

Co-efficient of viscosity: Consider a liquid flows steadily through a pipe as shown in the figure. The layers of the liquid which are in contact with the walls of the pipe have zero velocity. As we move towards the axis, the velocity of the liquid layer increases and the layer at the centre has the maximum velocity *v*. Consider any two layers P and Q separated by a distance *dx*. Let *dv* be the difference in velocity between the two layers. The viscous force *F* acting tangentially between the two layers of the liquid is proportional to

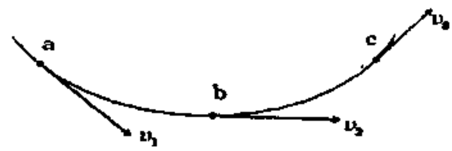


(i) area A of the layers in contact
(ii) velocity gradient $\frac{dv}{dx}$ perpendicular to the flow of liquid.

$$\therefore F \propto A\frac{dv}{dx} \qquad \Rightarrow F = \eta A\frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid. It is defined as the tangential viscous drag acting per unit area between two parallel liquid layers moving with unit velocity gradient. This is also known as Newton's law of viscous flow in fluids. The unit of η is $N\,s\,m^{-2}$. Its dimensional formula is $[ML^{-1}T^{-1}]$.

Streamline flow: The flow of a liquid is said to be steady, streamline or laminar if every particle of the liquid follows exactly the path of its preceding particle and has the same velocity of its preceding particle at every point.



Critical velocity: Critical velocity of a liquid can be defined as the maximum velocity upto which the flow is streamlined, and above it the flow becomes turbulent.

Turbulent flow: The motion of particles of a liquid becomes disorderly or irregular when the liquid moves with velocity greater than the critical velocity. Such a flow of liquid is known as turbulent flow.

Reynolds number: Reynolds number is a pure number which determines the nature of flow of a liquid through a pipe. It is denoted by N_R.

$$N_R = \frac{\rho v D}{\eta}$$

where v is the average velocity of the liquid, ρ is the density, η is the co-efficient of viscosity of the liquid and D is the diameter of the pipe. If N_R lies between 0 and 2000, the flow of a liquid is said to be streamline. If the value of N_R is above 3000, the flow is turbulent. If N_R lies between 2000 and 3000, the flow becomes unsteady.

Stokes' law: When a body falls through a highly viscous liquid, it experiences a viscous force which acts in the direction opposite to the direction of motion of the body.
According to Stokes' law, viscous force F acting on the sphere varies directly with
(1) the coefficient of viscosity η of the fluid,
(2) velocity v of the spherical body,
(3) radius r of the spherical body.

Mathematically $F = 6\pi \eta r v$

Proof

$$\text{Let } F = k\,\eta^a r^b v^c$$
$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a[L]^b[LT^{-1}]^c$$
$$[MLT^{-2}] = [M^a L^{-a+b+c} T^{-a-c}]$$

Equating the indices of M, L and T on both sides, we have

$a = 1,$

$-a - c = -2$ Or $-1 - c = -2$ Or $-c = -1$ Or $c = 1$

$-a + b + c = 1$ Or $-1 + b + 1 = 1$ Or $b = 1$

Or $F = k\,\eta\, r\, v$

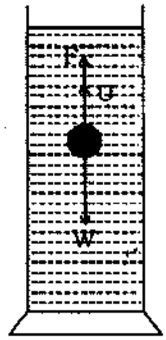
The value of k was found to be 6π in case of small sphere

$$\therefore F = 6\pi\ \eta\ r\ v$$

Importance of Stoke's law:

(i) This law is account for the formation of clouds.
(ii) This law is used in the determination of electronic charge by Millikan in his oil drop experiment.

Terminal velocity: The constant velocity, acquired by a freely falling body in a viscous medium, is known as terminal velocity.



If a sphere of density ρ is falling under gravity in a fluid of density σ, the forces acting on it are its weight W, the upthrust U and the viscous drag F as shown in the figure. The net downward force is $W - (U + F)$. As the velocity of the sphere increases, the viscous drag F is also increases. At a certain stage, the net force on the sphere becomes zero, and therefore it moves with constant velocity. This constant maximum velocity of the sphere is called terminal velocity (v_T).

$$\text{i.e., } W - (U + F) = 0$$
$$\text{Or } W = U + F$$
$$\text{Or } F = W - U$$

The viscous force $F = 6\pi\,\eta\,r\,v$

The weight of the sphere $W = \frac{4}{3}\pi\,r^3\rho\,g$

The upthrust or the buoyant force $U = \frac{4}{3}\pi\,r^3\sigma\,g$

$$6\pi\,\eta\,r\,v = \frac{4}{3}\pi\,r^3\rho\,g - \frac{4}{3}\pi\,r^3\sigma\,g$$

$$6\pi\,\eta\,r\,v = \frac{4}{3}\pi\,r^3(\rho-\sigma)g$$

$$v = \frac{\frac{4}{3}\pi\,r^3(\rho-\sigma)g}{6\pi\,\eta\,r} = \frac{4\pi\,r^3(\rho-\sigma)g}{3\times 6\pi\,\eta\,r} = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

$$\therefore v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

Note: The terminal velocity of a skydiver is about 54 ms^{-1}, for a feather it may be as small as 0.1 ms^{-1}, for a raindrop of 2 mm diameter, the terminal velocity is about 7 ms^{-1}.

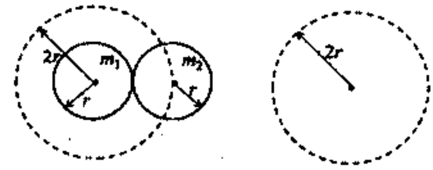
Cohesive force: Cohesive force is the force of attraction between the molecules of the same substance. This cohesive force is very strong in solids, weak in liquids and extremely weak in gases.

Adhesive force: Adhesive force is the force of attraction between the molecules of two different substances.

Water wets glass because the cohesive force between water molecules is less than the adhesive force between water and glass molecules. Whereas, mercury does not wet glass because the cohesive force between mercury molecules is greater than the adhesive force between mercury and glass molecules.

Molecular range: Molecular range is the maximum distance up to which a molecule can exert force of attraction on another molecule. It is of the order of $10^{-9}\,m$ for solids and liquids.

Sphere of influence: Sphere of influence is a sphere drawn around a particular molecule as centre and molecular range as radius. The central molecule exerts a force of attraction on all the molecules lying within the sphere of influence.

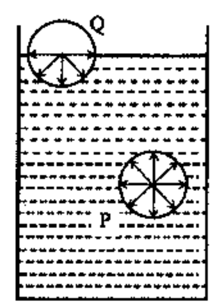


Surface tension of a liquid: Surface tension is the property of the free surface of a liquid at rest to behave like a stretched membrane in order to acquire minimum surface area.
Surface tension is defined as the force per unit length acting perpendicular on an imaginary line drawn on the liquid surface, tending to pull the surface apart along the line.

$$T = \frac{F}{l}$$

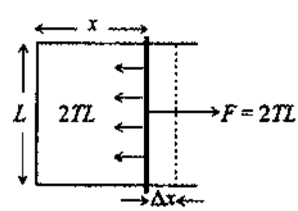
Its unit is $N\,m^{-1}$ and dimensional formula is $[MT^{-2}]$.

Molecular theory of surface tension: Consider two molecules P and Q as shown in the figure. A sphere of influence is drawn around them. The molecule P is attracted in all directions equally by neighbouring molecules. Therefore the net force acting on P is zero. The molecule Q is on the free surface of the liquid. It experiences a net downward force because the number of molecules in the lower half of the sphere is more and that on the upper half. Therefore all the molecules lying on the surface of a liquid experience only a net downward force.



If a molecule from the interior is to be brought to the surface of the liquid, work has to be done against this downward force. This work done on the molecule is stored as potential energy. Thus we can conclude that the surface of the liquid has potential energy. Greater the number of molecules on the surface, greater is the potential energy of the surface. To attain stable equilibrium, the surface of the liquid has to have minimum potential energy so the number of molecules in the surface is minimum. Since the thickness of the free surface is fixed, the surface of the liquid tends to assume minimum surface area by contracting and remains in a state of tension like a stretched elastic membrane.

Surface energy: Surface energy is defined as the ratio of the work done in increasing the surface area to the increase in surface area.



$$\text{Surface energy} = \frac{\text{Work done in increasing the surface area}}{\text{increase in surface area}}$$

$$\sigma = \frac{W}{\Delta S}$$

$$\sigma = \frac{F \times \Delta x}{\Delta S} = \frac{2(T \times L) \times \Delta x}{2(L \times \Delta x)}$$

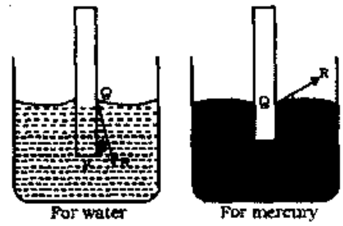
Where the number 2 indicates the two free surfaces of the film and T is the surface tension.

$$\therefore \sigma = T$$

The surface energy is numerically equal to surface tension.

Formation of drops and bubbles: A liquid drop or a soap bubble is spherical because the gravitational force acting on it is extremely small and the spherical shape of the liquid is mainly due to the surface tension.

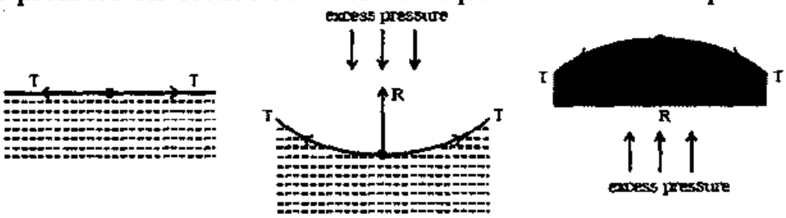
Angle of contact: The angle between the tangent to the liquid surface at the point of contact of the liquid with the solid and the solid surface inside the liquid is called angle of contact.



The angle of contact depends on the nature of liquid and solid in contact. For water and glass, θ lies between 8° and 18°. For pure water and clean glass, it is very small and hence it is taken as zero. The angle of contact of mercury with glass is 138°.

Pressure difference across a liquid surface: There is always an excess of pressure on the concave side of a curved liquid surface over the pressure on its convex side due to surface tension.

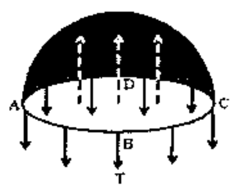
(i) If the free surface of a liquid is plane, then the surface tension acts horizontally. As a result, there is no pressure difference between the liquid side and the vapour side.



(ii) If the surface of the liquid is concave, then the resultant force R due to surface tension acts vertically upwards, and it will cease when the free surface becomes plane. To balance this, an excess of pressure acting downward on the concave side is necessary.

(iii) If the surface of the liquid is convex, the resultant force R due to surface tension acts downwards, and there must be an excess of pressure on the concave side acting upwards.

Excess pressure inside a liquid drop or excess pressure inside an air bubble inside a liquid: Let the drop be divided into two equal halves. Considering the equilibrium of the upper hemisphere of the drop, the upward force on the plane face ABCD due to excess pressure P is $P(\pi r^2)$.



If T is the surface tension of the liquid, the force due to surface tension acting downward along the circumference of the circle ABCD is $T(2\pi r)$.

At equilibrium, $P(\pi r^2) = T(2\pi r)$

$$\therefore P = \frac{2T}{r}$$

Excess pressure inside a soap bubble: A soap bubble has two liquid surfaces in contact with air, one inside the bubble and the other outside the btubble.

The force due to surface tension = $2 \times T(2\pi r)$

The force due to excess pressure = $P \pi r^2$

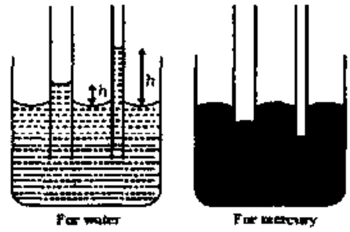
At equilibrium, $P \pi r^2 = 2 \times T(2\pi r)$

$$\therefore P = \frac{4T}{r}$$

Note: The excess of pressure inside a drop is inversely proportional to its radius i.e., $P \propto \frac{1}{r}$.

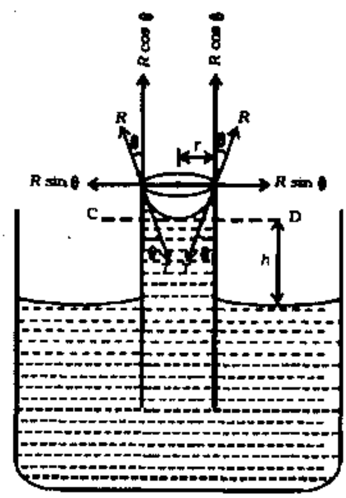
As $P \propto \frac{1}{r}$, the pressure needed to form a very small bubble is high. This explains why one needs to blow hard to start a balloon growing. Once the balloon has grown, less air pressure is needed to make it expand more.

Capillarity: The rise of a liquid in a capillary tube is known as capillarity. The height h indicates the capillary rise (for water) or capillary fall (for mercury).



Illustrations of capillarity:
(i) A blotting paper absorbs ink by capillary action. The pores in the blotting paper act as capillaries.
(ii) The oil in a lamp rises up the wick through the narrow spaces between the threads of the wick.
(iii) A sponge retains water due to capillary action.
(iv) Walls get damped in rainy season due to absorption of water by bricks.

Rise of liquid in a capillary tube: Let us consider a capillary tube of uniform bore dipped vertically in a beaker containing water. Due to surface tension, water rises to a height h in the capillary tube as shown in the figure. The surface tension T of the water acts inwards and the reaction of the tube R outwards. R is equal to T in magnitude but opposite in direction. This reaction R can be resolved into two rectangular components, $R\sin\theta$ and $R\cos\theta$.
The horizontal component $R\sin\theta$ acting all along the circumference of the tube cancel each other whereas the vertical component $R\cos\theta$ balances the weight of water column in the tube.



Total upward force = $R\cos\theta \times 2\pi r = 2\pi r \times T\cos\theta$ -----(1)

Total downward force $= \pi r^2 h\rho g + \left(\pi r^2 r - \frac{1}{2}\frac{4}{3}\pi r^3\right)\rho g$

$$= \pi r^2 h\rho g + \pi r^3\left(1 - \frac{2}{3}\right)\rho g$$

$$= \pi r^2 h\rho g + \frac{1}{3}\pi r^3 \rho g \text{ ------(2)}$$

Equating (1) and (2) gives

$$\pi r^2 h\rho g + \frac{1}{3}\pi r^3 \rho g = 2\pi r \times T\cos\theta$$

$$\pi r^2 h\rho g = 2\pi r \times T\cos\theta - \frac{1}{3}\pi r^3 \rho g$$

$$h = \frac{2\pi r \times T\cos\theta}{\pi r^2 \rho g} - \frac{\frac{1}{3}\pi r^3 \rho g}{\pi r^2 \rho g}$$

$$\therefore h = \frac{2T\cos\theta}{r\rho g} - \frac{1}{3}r$$

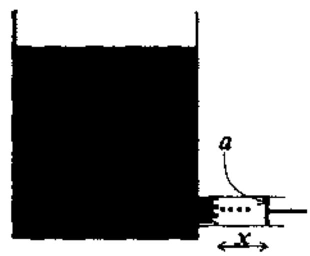
Factors affecting surface tension: Impurities present in a liquid appreciably affect surface tension. A highly soluble substance like salt increases the surface tension whereas sparingly soluble substances like soap decreases the surface tension.

Critical temperature of the liquid: The surface tension decreases with the rise in temperature. The temperature at which the surface tension of a liquid becomes zero is called critical temperature.

Applications of surface tension:
(i) During stormy weather, oil is poured into the sea around the ship. As the surface tension of oil is less than that of water, it spreads on water surface. Due to the decrease in surface tension, the velocity of the waves decreases. This reduces the wrath of the waves on the ship.
(ii) Lubricating oils spread easily to all parts because of their low surface tension.
(iii) Dirty clothes cannot be washed with water unless some detergent is added to water. When detergent is added to water, one end of the hairpin shaped molecules of the detergent get attracted to water and the other end, to molecules of the dirt. Thus the dirt is suspended surrounded by detergent molecules and this can be easily removed. This detergent action is due to the reduction of surface tension of water when soap or detergent is added to water.
(iv) Cotton dresses are preferred in summer because cotton dresses have fine pores which act as capillaries for the sweat.

Total energy of a liquid: A liquid in motion possesses pressure energy, kinetic energy and potential energy.

(i) Pressure Energy: It is the energy possessed by a liquid by virtue of its pressure.
Consider a liquid of density ρ contained in a wide tank having a side tube near the bottom of the tank as shown in the figure. A frictionless piston of cross sectional area 'a' is fitted to the side tube. If x is the distance through which the piston is pushed inwards, then work done or the pressure energy in pushing the liquid of mass $ax\rho$ is



$$\text{Work done} = \text{Force} \times \text{displacement} = (\text{Pressure} \times \text{area}) \times \text{displacement}$$

Or pressure energy $= (P \times a) \times x = Pax$

Pressure energy per unit mass of the liquid $= \dfrac{Pax}{ax\rho} = \dfrac{P}{\rho}$

(ii) Kinetic Energy: It is the energy possessed by a liquid by virtue of its motion.
If m is the mass of the liquid moving with a velocity v,
the kinetic energy of the liquid is $\frac{1}{2}mv^2$

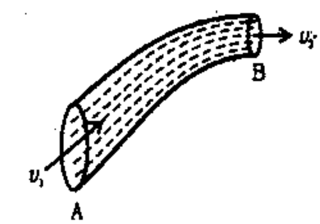
Kinetic energy per unit mass $= \dfrac{\frac{1}{2}mv^2}{m} = \dfrac{v^2}{2}$

(iii) Potential Energy: It is the energy possessed by a liquid by virtue of its height above the ground level.
If m is the mass of the liquid at a height h from the ground level,
the potential energy of the liquid is mgh

Potential energy per unit mass $= \dfrac{mgh}{m} = gh$

$\therefore$ Total energy of a liquid is $\dfrac{P}{\rho} + \dfrac{v^2}{2} + gh$

Equation of continuity: Consider a non-viscous liquid in streamline flow through a tube AB of varying cross section as shown in the figure. Let a_1 and a_2 be the area of cross section, v_1 and v_2 be the velocity of flow of the liquid at A and B respectively.



If ρ is the density of the liquid,
then mass of liquid entering per second at A = $a_1 v_1 \rho$.
Similarly, mass of liquid leaving per second at B = $a_2 v_2 \rho$.
If there is no loss of liquid in the tube and the flow is steady, then
mass of liquid entering per second at A = mass of liquid leaving per second at B

i.e., $a_1 v_1 \rho = a_2 v_2 \rho$ or $a_1 v_1 = a_2 v_2$

i.e., av = constant

This is called as the equation of continuity. From this equation $v \propto \frac{1}{a}$

i.e., the larger the area of cross section the smaller will be the velocity of flow of liquid and vice-versa.

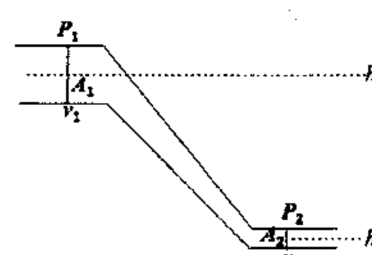
Bernoulli's principle: Bernoulli's Principle states that where the velocity of a fluid is high, the pressure is low and where the velocity of the fluid is low, pressure is high.

Bernoulli's theorem: According to Bernoulli's theorem, for the streamline flow of a non-viscous and incompressible liquid, the sum of the pressure energy, kinetic energy and potential energy per unit mass is a constant. i.e.,

$$\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant}$$

This equation is known as Bernoulli's equation.

Bernoulli's equation: If a volume ΔV of a liquid has a kinetic energy $\frac{1}{2}(\rho \Delta V)v_2^2$ (ρ density, v_2 velocity) at a point where the tube cross-section is A_2, and the kinetic energy $\frac{1}{2}(\rho \Delta V)v_1^2$ (v_1 velocity) at another point where the cross-section is A_1, then the difference $\Delta W_{KE} = \frac{1}{2}(\rho \Delta V)v_2^2 - \frac{1}{2}(\rho \Delta V)v_1^2$ must originate from the pressure difference and the difference of the potential energies.



Change in pressure energy is $\Delta W_p = P_1 \Delta V - P_2 \Delta V$ and the change in potential energy is $\Delta W_{PE} = (\Delta V \rho)gh_1 - (\Delta V \rho)gh_1$ (h_1, h_2 are the corresponding heights).

$$\frac{1}{2}(\rho \Delta V)v_2^2 - \frac{1}{2}(\rho \Delta V)v_1^2 = (\Delta V \rho)gh_1 - (\Delta V \rho)gh_2 + P_1 \Delta V - P_2 \Delta V$$

$$\frac{v_2^2}{2} - \frac{v_1^2}{2} = gh_1 - gh_2 + \frac{P_1}{\rho} - \frac{P_2}{\rho}$$

$$\therefore \frac{v_2^2}{2} + gh_2 + \frac{P_2}{\rho} = \frac{v_1^2}{2} + gh_1 + \frac{P_1}{\rho}$$

$$or \ \frac{v^2}{2} + gh + \frac{P}{\rho} = \text{constant}$$

In this equation, all the three terms carry the dimensions of energy per unit mass. If the above equation is divided by g, we get

$$\frac{v^2}{2g} + h + \frac{P}{\rho g} = \text{constant}$$

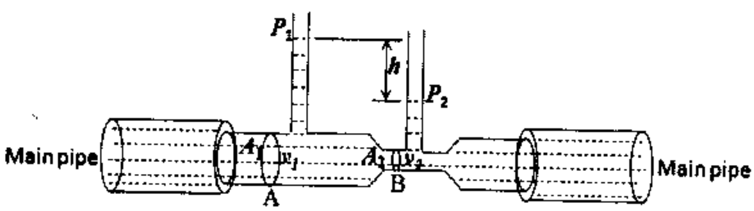
In this equation, all the three terms carry the dimensions of length. For this reason, $\frac{v^2}{2g}$ is called velocity head, h is the elevation (gravitational) head and $\frac{P}{\rho g}$ is called the pressure head. If $h = 0$, then

$$\frac{1}{2}\rho v^2 + P = \text{constant}$$

P is called the static pressure and $\frac{1}{2}\rho v^2$ is called the dynamic pressure.

Applications of Bernoulli's theorem:

(i) Flow meter or Venturimeter: It is a device used to measure the rate of flow of liquids through pipes. The device is inserted in the flow pipe, as shown in the figure.



It consists of a manometer, whose two limbs are connected to a tube having two different cross-sectional areas say A_1 and A_2 at A and B, respectively. Suppose the main pipe is horizontal above the ground. Then applying Bernoulli's theorem for the steady flow of liquid through the venturimeter at A and B, we can write

Total Energy at A = Total Energy at B

$$\frac{v_1^2}{2} + \frac{P_1}{\rho} = \frac{v_2^2}{2} + \frac{P_2}{\rho}$$

$$\frac{P_1}{\rho} - \frac{P_2}{\rho} = \frac{v_2^2}{2} - \frac{v_1^2}{2} \qquad \Rightarrow P_1 - P_2 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) = \frac{\rho v_1^2}{2}\left[\left(\frac{v_2}{v_1}\right)^2 - 1\right]$$

It shows that if $P_1 > P_2$, then $v_2 > v_1$. This is called *Venturi's Principle.*

For steady flow through the ventrurimeter,

volume of liquid entering per second at A = volume of liquid leaving per second at B.

$$A_1 v_1 = A_2 v_2$$

$$\frac{v_2}{v_1} = \frac{A_1}{A_2}$$

$$\therefore P_1 - P_2 = \frac{\rho v_1^2}{2}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] \qquad \text{Or} \quad v_1^2 = \frac{2(P_1 - P_2)}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}$$

$$or \quad v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}$$

If h denotes level difference between the two limbs of the venturimeter, then

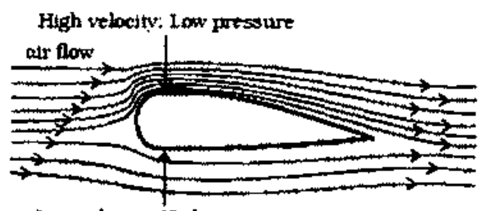
$$P_1 - P_2 = \rho gh$$

$$\text{and} \quad v_1 = \sqrt{\frac{2\rho gh}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}} = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

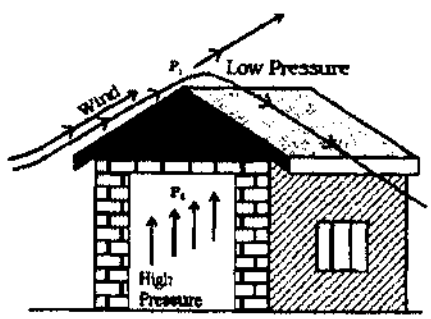
From this we note that $v_1 \propto \sqrt{h}$ since all other parameters are constant for a given venturimeter. Thus $v_1 = K\sqrt{h}$ where K is a constant.

The volume of liquid flowing per second is given by $V = A_1 v_1 = A_1 \times K\sqrt{h}$

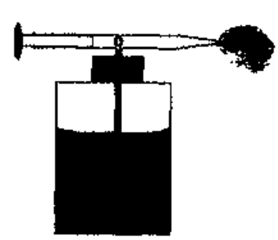
(ii) Lift of an Aircraft wing: A section of an aircraft wing and the flow lines are shown in the figure. The orientation of the wing relative to the flow direction causes the flow lines to crowd together above the wing. This corresponds to increased velocity in this region and hence the pressure is reduced. But below the wing, the pressure is nearly equal to the atmospheric pressure. As a result of this, the upward force on the underside of the wing is greater than the downward force on the topside. Thus there is a net upward force or lift.



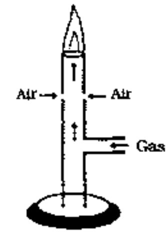
(iii) Blowing of roofs: During a storm, the roofs of huts or tinned roofs are blown off without any damage to other parts of the hut. The blowing wind creates a low pressure P_1 on top of the roof. The pressure P_2 under the roof is however greater than P_1. Due to this pressure difference, the roof is lifted and blown off with the wind.



(iv) Atomiser or sprayer: When the piston is moved in, it blows the air out of the narrow hole 'O' with large velocity creating a region of low pressure in its neighbourhood. The liquid (e.g. insecticide) is sucked through the narrow tube attached to the vessel end having its opening just below 'O'. The liquid on reaching the end gets sprayed by out blown air from the piston.



(v) Bunsen burner: In a Bunsen burner, the gas comes out of the nozzle with high velocity. Due to this the pressure in the stem of the burner decreases. So, air from the atmosphere rushes into the burner.



(vi) Motion of two parallel boats: When two boats separated by a small distance row parallel to each other along the same direction, the velocity of water between the boats becomes very large compared to that on the outer sides. Because of this, the pressure in between the two boats gets reduced. The high pressure on the outer side pushes the boats inwards. As a result of this, the boats come closer and may even collide.

Heat: Heat is a form of energy called thermal energy.
The SI unit of heat is joule (J).
When mechanical energy (work) is converted into heat, the ratio of work done (W) to the amount of heat produced (Q) is always a constant, represented by J, i.e.,

$$\frac{W}{Q} = J \qquad \text{Or} \quad W = JQ$$

Where J is called Joule's mechanical equivalent of heat.

Note: J is not a physical quantity, but a conversion factor whose value is 4.186 joule/calorie

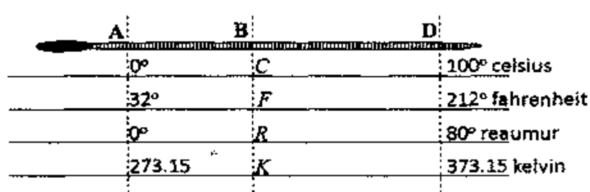
Calorie: The amount of heat required to raise the temperature of $1g$ of water by 1^0C i.e., from 14.5^0C to 15.5^0C is known as one calorie.

$$1 \text{ kcal} = 10^3 \text{ cal.}$$
$$\text{Also } 1 \text{ cal} = 4.18 \text{ J}$$

Temperature: Temperature of a body is the degree of hotness or coldness of the body.
Heat flows from a body at high temperature to a body at low temperature when they are in contact with each other.
The SI unit of temperature is kelvin (K), where as degree celsius (^oC) is a commonly used unit of temperature.

Different scales of temperature: Three scales of measurement of temperature are used.
(i) Centigrade or Celsius scale
(ii) Fahrenheit scale
(iii) Reaumur scale.

Absolute temperature scale and its relation with celsius, fahrenheit and reaumur scales:
The Absolute scale of temperature begins at -273.15^oC and is called $0K$ (kelvin). The unit of temperature in this scale is identical with the centigrade scale.



According to Celsius scale $\dfrac{AB}{AD} = \dfrac{C-0}{100}$

According to Fahrenheit scale $\dfrac{AB}{AD} = \dfrac{F-32}{180}$

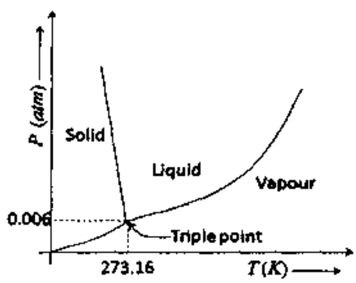
According to Reaumur scale $\dfrac{AB}{AD}=\dfrac{R-0}{80}$

According to Kelvin scale $\dfrac{AB}{AD}=\dfrac{K-273.15}{100}$

$$\therefore \frac{AB}{AD}=\frac{C}{100}=\frac{F-32}{180}=\frac{R}{80}=\frac{K-273.15}{100}$$

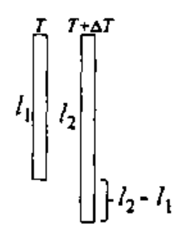
$$Or \quad \frac{C}{5}=\frac{F-32}{9}=\frac{R}{4}=\frac{K-273.15}{5}$$

Triple point of water: Triple point of water is the temperature at which the three phases of water (ice, water and water vapour) are all equally stable and co-exist in equilibrium. This temperature is taken to be 273.16K and the corresponding pressure is 4.58 *mm* of mercury or 0.006 *atm*.



Thermal expansion in solids: Thermal expansion of a solid is defined as the increase in dimension of the solid due to increase in its temperature.
Increase in length is called linear expansion
Increase in area is called area expansion
Increase in volume is called volume expansion

Coefficient of linear expansion (α): It is defined as the increase in length per unit length per degree rise in temperature.
Let l_1 = original length of the solid bar at temperature T
l_2 = length of the bar at temperature $T+\Delta T$, then

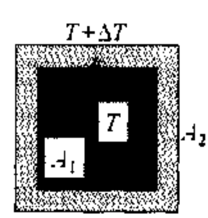


$$\alpha=\frac{l_2-l_1}{l_1\,\Delta T} \qquad \text{Or } l_2-l_1=\alpha(l_1\,\Delta T)$$

$$\therefore l_2=l_1\,(1+\alpha\,\Delta T)$$

The unit of α is (1/^oC) or (1/K). The value of α depends upon the nature of the material of the solid.

Coefficient of area (superficial) expansion (β): It is defined as the increase in area per unit area per degree rise in temperature.
Let A_1 = original surface area at temperature T
A_2 = surface area at temperature $T+\Delta T$, then



$$\beta=\frac{A_2-A_1}{A_1\,\Delta T} \qquad \text{Or } A_2-A_1=\beta(A_1\,\Delta T)$$

$$\therefore A_2=A_1\,(1+\beta\,\Delta T)$$

The unit of β is (1/^oC) or (1/K). The value of β depends upon the nature of the material.

Relation between β and α: Consider a square of area l_1^2 at temperature T, and at temperature $T+\Delta T$ its area is l_2^2.

$$\because \beta=\frac{A_2-A_1}{A_1\,\Delta T}=\frac{l_2^2-l_1^2}{l_1^2\Delta T}=\frac{(l_2+l_1)(l_2-l_1)}{l_1^2\Delta T}$$

$$\beta = \frac{(l_2 + l_1)}{l_1}\alpha \qquad \because \alpha = \frac{l_2 - l_1}{l_1 \Delta T}$$

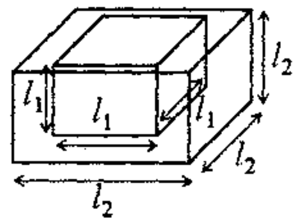
If we consider that $l_2 \approx l_1$, we have

$$\beta = 2\alpha$$

Coefficient of volume (cubical) expansion (γ): It is defined as the increase in volume per unit volume per degree rise in temperature.

Let V_1 = original surface area at temperature T

V_2 = surface area at temperature $T+\Delta T$, then



$$\gamma = \frac{V_2 - V_1}{V_1\, \Delta T} \qquad \text{Or } V_2 - V_1 = \gamma(V_1\, \Delta T)$$

$$\therefore V_2 = V_1\,(1 + \gamma\, \Delta T)$$

The unit of γ is ($1/^oC$) or ($1/K$). The value of β depends upon the nature of the material.

Relation between γ and α: Consider a cube of volume l_1^3 at temperature T, and at temperature $T + \Delta T$ its volume is l_2^3.

$$\because \gamma = \frac{V_2 - V_1}{V_1\, \Delta T} = \frac{l_2^3 - l_1^3}{l_1^3 \Delta T} = \frac{(l_2 - l_1)^3 + 3l_2 l_1 (l_2 - l_1)}{l_1^3 \Delta T}$$

$$Or\ \ \gamma = (l_2 - l_1)\left[\frac{(l_2 - l_1)^2 + 3l_2 l_1}{l_1^3 \Delta T}\right] = \frac{(l_2 - l_1)}{l_1 \Delta T}\left[\frac{(l_2 - l_1)^2 + 3l_2 l_1}{l_1^2}\right]$$

$$\gamma = \alpha \frac{(l_2 - l_1)^2 + 3l_2 l_1}{l_1^2} \qquad \because \alpha = \frac{l_2 - l_1}{l_1 \Delta T}$$

If we consider that $l_2 \approx l_1$, we have

$$\gamma = 3\alpha$$

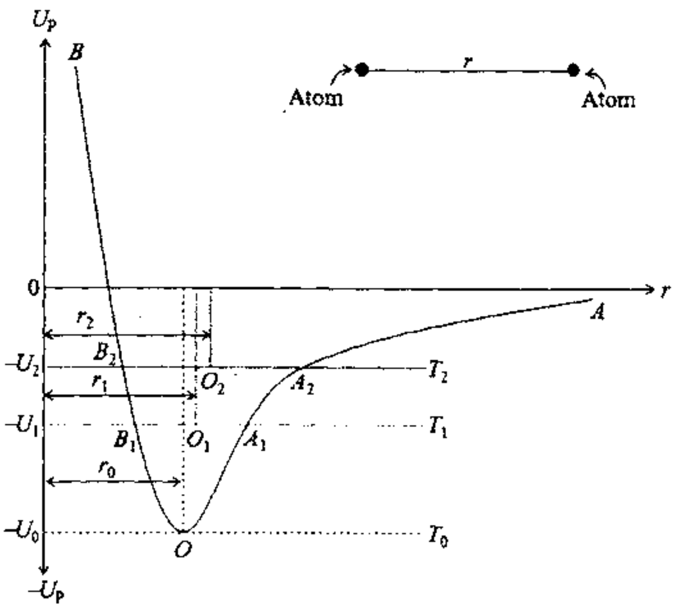
Relation between α, β and γ:

$$\because \beta = 2\alpha \qquad \Rightarrow \alpha = \frac{\beta}{2}$$

Also $\gamma = 3\alpha \qquad \Rightarrow \alpha = \frac{\gamma}{3}$

$$\therefore \alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

Molecular explanation of thermal expansion: Two atoms are separated by a distance r. If $r = \infty$, the force of attraction between them is zero, and the potential energy is also zero. In the diagram, $U_P = 0$. As r decreases, the force between them starts increases i.e., attractive and become maximum and then with



further decreases of r, the force starts decreases i.e., repulsive, becomes zero and then positive as shown in the figure.

(i) At temperature $T_0 = 0K$, the atoms have only potential energy $-U_0$. In this case, $r = r_0$ and the atoms are at fixed lattice point. They are unable to vibrate and no other value of r is allowed.

(ii) At temperature T_1, the atoms gain vibrational energy $-U_1$, and as a result, the atoms vibrate between points A_1 and B_1 i.e., about O_1. In this case the average distance between the atoms is r_1, where $r_1 > r_0$. As a result of this the solid expands.

(iii) At temperature T_2, the energy of the atoms further increases by an amount $-U_2$. Consequently, the atoms vibrate between the points A_2 and B_2 i.e., about O_2. In this case the average distance between the atoms is r_2, where $r_2 > r_1$. As a result of this the solid has expanded further. So with rise in temperature, solids expand.

Practical applications of thermal expansion of solids:

(i) When rails are laid down on the ground, space is left between the end of two rails.
(ii) The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of the wheel.
(iii) The transmission cables are not tightly fixed to the poles.
(iv) A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottle.

Expansion of solids: In liquids only expansion in volume takes place on heating.

(i) Apparent expansion of liquid: When expansion of the container containing liquid, on heating is not taken into account, then observed expansion is called apparent expansion of liquids. Coefficient of apparent expansion of a liquid

$$\gamma_a = \frac{\text{apparent increase in volume}}{\text{original volume} \times \text{rise in temperature}}$$

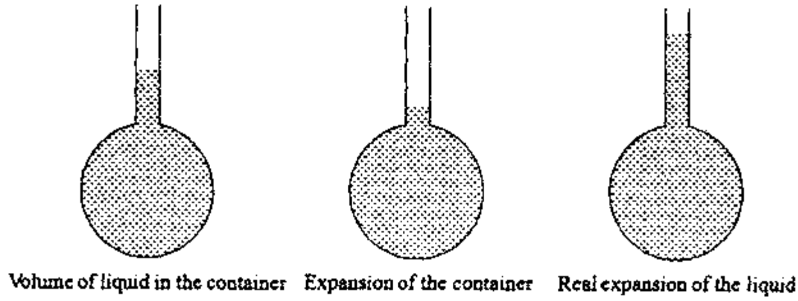
(ii) Real expansion of liquid: When expansion of the container containing liquid, on heating is also taken into account, then observed expansion is called real expansion of liquids. Coefficient of real expansion of a liquid

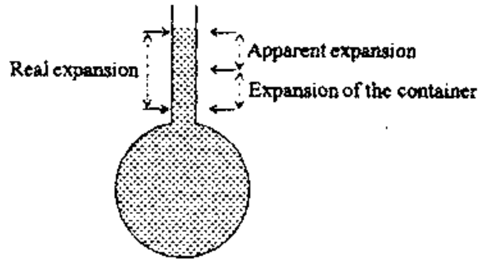
$$\gamma_r = \frac{\text{real increase in volume}}{\text{original volume} \times \text{rise in temperature}}$$

Both γ_a and γ_r are measured in ${}^0C^{-1}$.

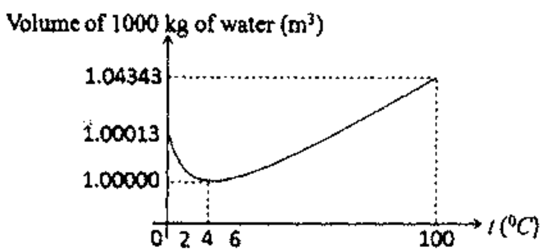
Note: It can be shown that $\gamma_r = \gamma_a + \gamma_g$

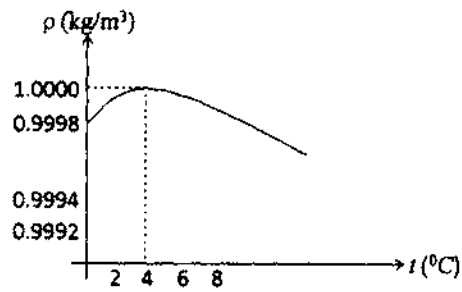
where γ_a and γ_r are coefficient of apparent and real expansion of a liquids and γ_g is coefficient of cubical expansion of the container.





Anamalous expansion of water: When temperature of water is increased from 0^0C, then its volume decreases upto 4^0C, becomes minimum at 4^0C and then increases. This behaviour of water around 4^0C is called anamalous expansion of water.





Note: Water is an exception as its density is greatest at 4^0C and less for both higher and lower temperatures.

Expansion of gases: There are two types of coefficient of expansion in gases.

(i) Volume coefficient (γ_V): At constant pressure, the change in volume per unit volume per degree celsius is called volume coefficient.

$$\gamma_V = \frac{V_2 - V_1}{V_0(t_2 - t_1)}$$

where V_0, V_1 and V_2 are volumes of the gas at 0^0C, $t_1{}^0C$ and $t_2{}^0C$.

(ii) Pressure coefficient (γ_P): At constant volume, the change in pressure per unit pressure per degree celsius is called pressure coefficient.

$$\gamma_P = \frac{P_2 - P_1}{P_0(t_2 - t_1)}$$

where P_0, P_1 and P_2 are volumes of the gas at 0^0C, $t_1{}^0C$ and $t_2{}^0C$.

Methods of heat transmission: There are three method of heat transmission.

(i) Conduction : Heat is transmitted through the solids by the process of conduction only. When one end of the solid is heated, the atoms or molecules of the solid at the hotter end becomes more strongly agitated and start vibrating with greater amplitude. The disturbance is transferred to the neighbouring molecules.

Applications : (i) The houses of Eskimos are made up of double walled blocks of ice. Air enclosed in between the double walls prevents transmission of heat from the house to the coldest surroundings. (ii) Birds often swell their feathers in winter to enclose air between their body and the feathers. Air prevents the loss of heat from the body of the bird to the cold surroundings. (iii) Ice is packed in gunny bags or sawdust because, air trapped in the saw dust prevents the transfer of heat from the surroundings to the ice. Hence ice does not melt.

(ii) Convection : It is a phenomenon of transfer of heat in a fluid with the actual movement of the particles of the fluid.
When a fluid is heated, the hot part expands and becomes less dense. It rises and upper colder part replaces it. This again gets heated, rises up replaced by the colder part of the fluid. This process goes on. This mode of heat transfer is different from conduction where energy transfer takes place without the actual movement of the molecules.

Application : It plays an important role in ventilation and in heating and cooling system of the houses.

(iii) Radiation : It is the phenomenon of transfer of heat without any material medium. Such a process of heat transfer in which no material medium takes part is known as radiation.

Thermal radiation: The energy emitted by a body in the form of radiation on account of its temperature is called thermal radiation. It depends on, (i) temperature of the body (ii) nature of the radiating body

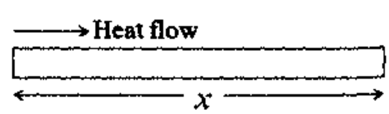
Note: The wavelength of thermal radiation ranges from $8 \times 10^{-7}\ m$ to $4 \times 10^{-4}\ m$. They belong to infra-red region of the electromagnetic spectrum.

Properties of thermal radiations

(i) Thermal radiations can travel through vacuum.
(ii) They travel along straight lines with the speed of light.
(iii) They can be reflected and refracted. They exhibit the phenomenon of interference and diffraction.
(iv) They do not heat the intervening medium through which they pass.
(v) They obey inverse square law.

Steady state: The state of a conducting rod in which no part of the rod absorbs heat is called the steady state.

Temperature gradient: The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient.



$$\text{Temperature gradient} = \frac{\text{Change in temperature}}{\text{Perpendicular distance}} = -\frac{\Delta\theta}{\Delta x}$$

Its SI unit is ^0C per meter and dimension is $[L^{-1}K]$.
The negative sign indicates that heat flows in the direction of decreasing temperature.

Coefficient of thermal conductivity: The amount of heat flow in the conducting rod is

$$Q \propto \frac{A(\Delta\theta)t}{x}$$

$$\text{Or} \quad Q = \frac{kA(\Delta\theta)t}{x}$$

where k is the coefficient of thermal conductivity, A is the area of cross-section, x is the length of the rod, $\Delta\theta$ is the temperature difference between the ends of the rod and t is the time.
The SI unit of k is $J\,s^{-1}\,m^{-1}\,K^{-1}$. Since $J\,s^{-1}$ is watt (W), the unit of k can also be expressed as $W\,m^{-1}\,K^{-1}$. Its dimension is $[MLT^{-3}]$

In calculus, the above equation can be written as $\frac{dQ}{dt} = -kA\frac{d\theta}{dx}$

Hence the coefficient of thermal conductivity of the material may be defined as the rate of flow of heat per unit area per unit temperature gradient when the heat flow is perpendicular to the isothermal surfaces.

Thermal (heat) capacity: Heat capacity of any body is equal to the amount of heat energy required to increase its temperature to 1^0C. It is denoted by C, and its SI unit is joule/kelvin (J/K) or ($J/^0C$)

Specific heat: The amount of heat required to raise the temperature of unit mass of the substance through 1^0C is called its specific heat. It is denoted by s.

Its SI unit is joule/kelvin-kg ($J\,kg^{-1}\,K^{-1}$) or ($J\,kg^{-1}\,{}^0C^{-1}$), and its dimension is $[L^2T^{-2}K^{-1}]$.

Note:

(i) The heat capacity per unit mass is called specific heat s.

$$s = \frac{C}{m} \qquad \Rightarrow C = ms$$

(ii) The specific heat of water is 4200 $J\,kg^{-1}\,K^{-1}$ which is high compared with most other substances.

(iii) Gases have two types of specific heat. The specific heat capacity at constant volume (C_V) and the specific heat capacity at constant pressure (C_P).
The specific heat capacity at constant pressure (C_P) is greater than the specific heat capacity at constant volume (C_V) i.e., $C_P > C_V$.

(iv) For molar specific heats $C_P - C_V = R$
where R is called gas constant, and this relation is called Mayer's formula.

(v) The ratio of two principal specific heats of a gas is represented by γ.

$$\gamma = \frac{C_P}{C_V}$$

The value of γ depends on atomicity of the gas.

(vi) Amount of heat energy required to change the temperature of any substance is given by

$$Q = ms\Delta\theta$$

where m is the mass of the substance, s is the specific heat of the substance and $\Delta\theta$ is the change in temperature.

Molar specific heat: The amount of heat required to raise the temperature of one mole of a substance through a unit degree is called the molar specific heat of the substance. It is denoted by C'.

Note: 1 mol = 6.02×10^{23} elementary units

Relation between the molar specific heat (C') and the ordinary specific heat (s): Amount of heat energy Q required to raise the temperature of μ mole of a substance through a temperature change $\Delta\theta$ is

$$Q = \mu C' \Delta\theta$$

$$\because \mu = \frac{m}{M}$$

where m is the mass of the substance, and M is the mass of 1 mol (molar mass) of that substance.

$$\therefore Q = \frac{m}{M} C' \Delta\theta$$

Also $\quad Q = ms\Delta\theta$

Equating the above two equations we have

$$\frac{m}{M} C' \Delta\theta = ms\Delta\theta$$

$$\frac{C'}{M} = s \qquad \Rightarrow C' = Ms$$

Hence the molar specific heat equals the specific heat times the mass M of 1 mol (molar mass).

Note:

$C' = mN_A s$

where m is the molecular mass, N_A is the Avogadro number and $M = mN_A$

Specific heat of water: The specific heat of water varies by only about 1 percent from 0^0C to 100^0C at a pressure of 1 *atm*. We can usually neglect this variation and take the specific heat of water to be 4.184 *kJ/kg.K*.

Thermal equilibrium: When there is no transfer of heat between two bodies in contact, then the bodies are said to be in thermal equilibrium.

Water equivalent: It is the quantity of water whose thermal capacity is same as the heat capacity of the body. It is denoted by w.

$$w = ms = \text{Heat capacity of the body}$$

In order to raise the temperature of m grams of the body through $\Delta\theta$, the amount of heat required is

$$Q = ms\Delta\theta$$

where s is the specific heat of the body. Now if the same amount of heat is supplied to w grams of water which has specific heat $s = 1\ cal/gm.^0C$, such that the temperature of the water rises by $\Delta\theta$, then

$$Q = ws\Delta\theta = w\Delta\theta \qquad [\because s = 1\,cal/gm.^0C]$$

Equating the above two equations , we have

$$w\Delta\theta = ms\Delta\theta \qquad \Rightarrow w = ms$$

Latent heat: The heat energy absorbed or released at constant temperature per unit mass for change of state is called latent heat i.e.,

$$L = \frac{Q}{m}$$

$$\text{Or} \quad Q = mL$$

where m is the mass of the substance and L is the latent heat.
The SI unit of latent heat is cal/gm or J/kg and its dimension is $[L^2\ T^{-2}]$.

For water at its normal boiling point or condensation temperature (100^0C), the latent heat of vaporisation is

L = 540 cal/gm
L = 40.8 kJ/mol
L = 2260 kJ/kg

For water at its normal freezing temperature or melting point (0^0C), the latent heat of fusion is

L = 80 cal/gm
L = 60 kJ/mol
L = 336 kJ/kg

Note:
(i) It is more painful to get burn by steam rather than by boiling water at 100^0C. At 100^0C, steam gets converted to water then it gives out 536 cal of heat. So it is clear that steam at 100^0C has more heat than water at 100^0C.

(ii) There is more shivering effect of ice cream on teeth as compared to that of water (obtained from ice). This is because when ice cream melts down, it absorbs large amount of heat from teeth.

Principle of calorimetry: When a hot body is mixed with a cold body, then heat lost by hot body is equal to heat gained by cold body.

Heat lost = Heat gain.

Absorptive power: Absorptive power of a body for a given wavelength and temperature is defined as the ratio of the radiant energy absorbed per unit area per unit time to the total energy incident on it per unit area per unit time. It is denoted by a_λ.

Emissive power: Emissive power of a body at a given temperature is the amount of energy emitted per unit time per unit area of the surface for a given wavelength. It is denoted by e_λ. Its unit is $W\,m^{-2}$.

Perfectly black body: A perfect black body is the one which absorbs completely the heat radiations of all wave-lengths which fall on it and emits heat radiations of all wavelengths when heated. Since a perfect black body neither reflects nor transmits any radiation, the absorptive power of a perfectly black body is unity.

Kirchoff's law: According to this law, the ratio of emissive power to the absorptive power corresponding to a particular wavelength and at a given temperature is always a constant for all bodies. This constant is equal to the emissive power of a perfectly black body at the same temperature and the same wavelength. Mathematically

$$\frac{e_\lambda}{a_\lambda} = \text{constant} = E_\lambda$$

Note: If a body absorbs radiation of certain wavelength strongly then it will also strongly emit the radiation of same wavelength. In other words, good absorbers of heat are good emitters also.

Stefan's law: Stefan's law states that the total amount of heat energy radiated per second per unit area of a perfectly black body is directly proportional to the fourth power of its absolute temperature i.e.,

$$E \propto T^4$$

$$\text{or} \quad E = \sigma T^4$$

where σ is called the Stefan's constant. Its value is $5.67 \times 10^{-8}\ W m^{-2} K^{-4}$. It is also called Stefan - Boltzmann law.

Newton's law of cooling: Newton's law of cooling states that the rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings. i.e.,

$$\frac{dT}{dt} = E \propto (T - T_0)$$

where T and T_0 are the temperatures of the body and the surroundings.

The law holds good only for a small difference of temperature. Loss of heat by radiation depends on the nature of the surface and the area of the exposed surface.

Derivation of Newton's law of cooling from Stefan's law: Stefan's law is applicable for all temperatures of a hot body. But Newton's law is applicable when the difference of temperature between the hot body and the surrounding is small. Consider a hot body at a temperature T is placed in a surrounding of uniform temperature T_0. According to Stefan's law,

$$E = e\sigma(T^4 - T_0^4)$$

where e is the emissivity of the surface of the hot body.

$$E = e\sigma[(T^2)^2 - (T_0^2)^2] = (T^2 + T_0^2)(T^2 - T_0^2)$$

$$\text{Or} \quad E = e\sigma(T^2 + T_0^2)(T + T_0)(T - T_0)$$

$$\text{Or} \quad E = e\sigma(T - T_0)(T + T_0)[(T + T_0)^2 - 2TT_0]$$

For a small difference of temperature, i.e., $T \approx T_0$ we have

$$E = e\sigma(T - T_0)(T + T)[(T + T)^2 - 2TT]$$

Or $E = e\sigma(T - T_0)(2T)[4T^2 - 2T^2] = e\sigma(T - T_0)(2T)(2T^2)$

Or $E = 4e\sigma T^3(T - T_0)$

Taking $4e\sigma T^3 = k$, then

$$E = k(T - T_0)$$

Or $E \propto (T - T_0)$

This equation represents Newton's law of cooling and is true when the difference of temperature is small.

THERMODYNAMICS

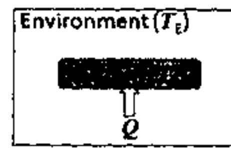
Heat: Heat is a form of energy called thermal energy. Rubbing our hands against each other produces heat. Joule's paddle wheel experiment led to the production of heat by friction.

$$1 \text{ kcal} = 10^3 \text{ cal.}$$

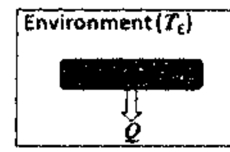
$$\text{Also } 1 \text{ cal} = 4.18 \text{ J}$$

Calorie: The amount of heat required to raise the temperature of $1g$ of water by 1^0C i.e., from 14.5^0C to 15.5^0C is known as one calorie.

Positive and negative heat: Heat is positive when energy is transferred to the system from the environment i.e., heat is absorbed by the system. Heat is negative when energy is transferred to the environment from the system i.e., heat is lost by the system.



$T_S < T_E$
$Q > 0$, **Heat is positive**



$T_S > T_E$
$Q < 0$, **Heat is negative**

Temperature: Temperature of a body is the property which determines whether or not it is in thermal equilibrium with other bodies.
Energy exchange between a glass of cold water and its surroundings continues until thermal equilibrium was reached. All bodies in thermal equilibrium have a common property, called temperature, whose value is same for all of them.

Or

Temperature of a body is the degree of hotness or coldness of the body.
Heat flows from a body at high temperature to a body at low temperature when they are in contact with each other.

Thermodynamical System: An assembly of an extremely large number of particles whose state can be expressed in terms of pressure, volume and temperature, is called thermodynamic system.
Thermodynamic system is classified into the following three systems.
(i) Open system: System that exchange both energy and matter with the surrounding is called open system.
(ii) Closed system: System that exchange only energy (not matter) with the surrounding is called closed system.
(iii) Isolated system: System that exchange neither energy nor matter with the surrounding is called isolated system.

Thermodynamic Parameters or Coordinates or Variables: The state of thermodynamic system can be described by specifying pressure, volume, temperature, internal energy, number of moles, etc. These are called thermodynamic parameters or coordinates or variables.

Equation of state: The general relationship between pressure, volume and temperature for the given mass of the system (eg gas) is called equation of state. Thus for n moles of an ideal gas, the equation of state is

$$PV = nRT$$

where R is called gas constant

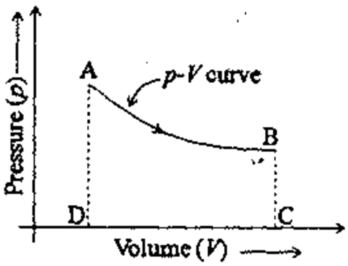
Note: At very low pressures, the forces of intermolecular attraction are negligible. Therefore, at low pressures only, a real gas obeys the equation $PV = RT$

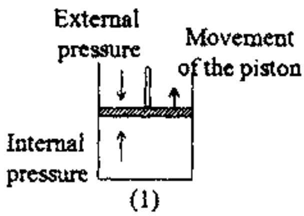
Work done by the thermodynamic system: Work done by the thermodynamic system given by

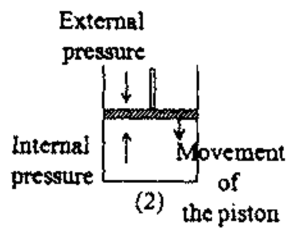
$$W = p \times \Delta V$$

where p = pressure and ΔV = change in volume.
Work done by the thermodynamic system is equal to the area enclosed between the p-V curve and the volume axis.

Work done in process A–B = area ABCDA







Note: Work done by the thermodynamic system depends not only upon the initial and final states of the system but also depend upon the path followed in the process.
In figure (1), W.D by the thermodynamic system (internal pressure) is positive, and W.D by external pressure is negative.
In figure (2), W.D by the thermodynamic system (internal pressure) is negative, and W.D by external pressure is positive.

Internal energy: The total energy possessed by any system due to molecular motion and molecular configuration, is called its internal energy.

Note: Internal energy of an ideal gas depends on temperature only i.e., $U = f(T)$. Whereas for a real gas the internal energy is a function of any two of the thermodynamic variables pressure, volume and temperature i.e., $U = f(P,T)$, $U = f(V,T)$ and $U = f(P,V)$.

Zeroth law of thermodynamics: Zeroth law of thermodynamics states that, if two systems which are individually in thermal equilibrium with the third system, they are also in thermal equilibrium with each other.

If two systems A and B are separately in thermal equilibrium with a third system C, then the three systems are in thermal equilibrium with each other.

Note: This Zeroth law was stated by Flower much later than both first and second laws of thermodynamics. This law helps us to define temperature in a more rigorous manner.

First law of thermodynamics: The first law of thermodynamics states that the amount of heat energy supplied to a system is equal to the sum of the change in internal energy of the system and the work done by the system. i.e.,

$$\Delta Q = \Delta U + \Delta W$$

Note: This law is in accordance with the law of conservation of energy.

Thermodynamic processes:

(1) Isothermal process: A process taking place in a thermodynamic system at constant temperature is called an isothermal process. Isothermal process are very slow processes. These process follows Boyle's law, according to which

$$pV = \text{constant.}$$

Example: Melting process is an isothermal change, because temperature of a substance remains constant during melting.

Note: In isothermal process, change in internal energy is zero $(\Delta U = 0)$ i.e.,

$$\Delta Q = \Delta U + \Delta W$$
$$\Delta Q = 0 + \Delta W$$
$$\therefore \Delta Q = \Delta W$$

(2) Adiabatic process: A process taking place in a thermodynamic system for which there is no exchange of heat between the system and its surroundings. Adiabatic processes are very fast processes. These processes follows Poisson's law, according to which

$$pV^{\gamma} = TV^{\gamma-1} = \frac{T^{\gamma}}{p^{\gamma-1}} = \text{constant.}$$

Examples:

(i) Sudden compression or expansion of a gas in a container with perfectly non-conducting wall.

(ii) Sudden bursting of the tube of a bicycle tyre.

Note: In adiabatic process, no exchange of heat takes place, i.e., $\Delta Q = 0$ i.e.,

$$\Delta Q = \Delta U + \Delta W$$
$$0 = \Delta U + \Delta W$$
$$\therefore \Delta U = -\Delta W$$

In adiabatic process, if gas expands, its internal energy and hence its temperature decreases and *vice-versa.*

Isobaric process: A process taking place in a thermodynamic system at constant pressure is called an isobaric process.

Isochoric process: A process taking place in a thermodynamic system at constant volume is called an isochoric process.

Cyclic process: When a thermodynamic system returns to its initial state after passing through several states, then the process is called a cyclic process.

Specific heat capacity of a gas: Specific heat capacity of a gas may have any value between $-\infty$ and $+\infty$ depending upon the way in which heat energy is given.

Let m be the mass of a gas and C its specific heat capacity. Then

$$\Delta Q = m \times C \times \Delta T$$

where ΔQ is the amount of heat absorbed and ΔT is the corresponding rise in temperature i.e.,

$$C = \frac{\Delta Q}{m \times \Delta T}$$

Case (i)

If the gas is insulated from its surroundings and is suddenly compressed, it will be heated up, and there is rise in temperature, even though no heat is supplied from outside i.e.,

$$\Delta Q = 0$$

$$C = \frac{\Delta Q}{m \times \Delta T} = \frac{0}{m \times \Delta T} = 0$$

Case (ii)

If the gas is allowed to expand slowly, in order to keep the temperature constant ($\Delta T = 0$), an amount of heat ΔQ is supplied from outside, then

$$C = \frac{\Delta Q}{m \times \Delta T} = \frac{\Delta Q}{m \times 0} = +\infty$$

$\because \Delta Q$ is positive as heat is supplied from outside.

Case (iii)

If the gas is compressed gradually and the heat generated ΔQ is conducted away so that temperature remains constant, then

$$C = \frac{\Delta Q}{m \times \Delta T} = \frac{-\Delta Q}{m \times 0} = -\infty$$

$\because \Delta Q$ is negative as heat is supplied by the system

Hence, in order to find the value of specific heat capacity of a gas, either the pressure or the volume of the gas should be kept constant. Consequently a gas has two specific heat capacities (i) Specific heat capacity at constant volume (ii) Specific heat capacity at constant pressure.

Specific heat capacity of a gas at constant volume: Specific heat capacity of a gas at constant volume C_V is defined as the quantity of heat required to raise the temperature of unit mass of a gas through 1^0C, keeping its volume constant i.e.,

$$C_V = \left(\frac{\Delta Q}{\Delta T}\right)_V$$

Specific heat capacity of a gas at constant pressure: Specific heat capacity of a gas at constant pressure C_p is defined as the quantity of heat required to raise the temperature of unit mass of a gas through 1^0C, keeping its pressure constant i.e.,

$$C_P = \left(\frac{\Delta Q}{\Delta T}\right)_P$$

C_P is greater than C_V: When a gas is heated at constant volume, no work is done, and hence whole of the heat supplied is used to raise its temperature through 1^0C. However more heat is required to raise its temperature through 1^0C at constant pressure. Hence $C_P > C_V$.

Relation between C_P and C_V (Meyer's relation): Let us consider one mole of an ideal gas enclosed in a cylinder provided with a frictionless piston of area A. Let P, V and T be the pressure, volume and absolute temperature of gas respectively.

A quantity of heat dQ is supplied to the gas at constant volume and this heat energy is used to increase the internal energy dU of the gas. Let the increase in temperature of the gas is dT, but the gas does not do any work ($dW = 0$).

$$\therefore dQ = dU = 1 \times C_V \times dT = C_V \times dT$$

Now let dQ' be the quantity of heat supplied to the same gas at constant pressure to increase its temperature to dT.

$$\therefore dQ' = dU + dW = dU + PdV \text{ ------- } (1)$$

Since the expansion takes place at constant pressure,

$$\therefore dQ' = 1 \times C_P \times dT = C_P \times dT$$

Equation (1) becomes

$$C_P \times dT = C_V \times dT + PdV \text{ ------- } (2)$$

The equation of state of one mole of an ideal gas is

$PV = RT$ where R is called gas constant.

Differentiating both sides gives

$$PdV = RdT \text{ ------- } (3)$$

Substituting equation (3) in equation (2) gives

$$C_P \times dT = C_V \times dT + RdT$$

$$\text{Or} \quad C_P = C_V + R$$

$$\therefore C_P - C_V = R$$

This equation is known as Meyer's relation

Reversible process: A process that can be reversed without the loss of energy form the system is called reversible process.

Condition for reversible process

(i) The process must be infinitely slow.

(ii) The system should remain in thermal equilibrium i.e., system and surrounding should remain at the same temperature.

Examples

(a) Let a gas be compressed isothermally so that heat generated is conducted away to the surrounding. When it is allowed to expand in the same small equal steps, the temperature falls but the system takes up the heat from the surrounding and maintains its temperature.

(b) Electrolysis can be regarded as reversible process, provided there is no internal resistance.

Irreversible process: A process that cannot reverse both the system and the surrounding to their original conditions is called Irreversible process.

Examples: diffusion of gases and liquids, passage of electric current through a wire, and heat energy lost due to friction. As an irreversible process is generally a very rapid one, temperature adjustments are not possible. Most of the chemical reactions are irreversible.

Second law of thermodynamics: Different scientists have stated this law in different ways to bring out its salient features.

(i) Kelvin's statement: It is impossible to obtain a continuous supply of work from a body by cooling it to a temperature below the coldest of its surroundings.

(ii) Clausius statement: It is impossible to transfer heat from a lower temperature body to a higher temperature body without the use of an external agency.

(iii) Kelvin - Planck's statement: It is impossible to construct a heat engine that will convert heat completely into work.

Entropy: Entropy is a measure of the disorder of a system.

Note:
(1) Entropy is a physical quantity that remains constant during a reversible adiabatic change.
Change in entropy is given by $dS = \frac{\delta Q}{T}$
where δQ is the heat supplied to the system and T is the absolute temperature. Entropy of a system never decreases, i.e., $dS \geq 0$.

(2) In any natural process, the entropy increases i.e., some energy becomes unavailable to do useful work.

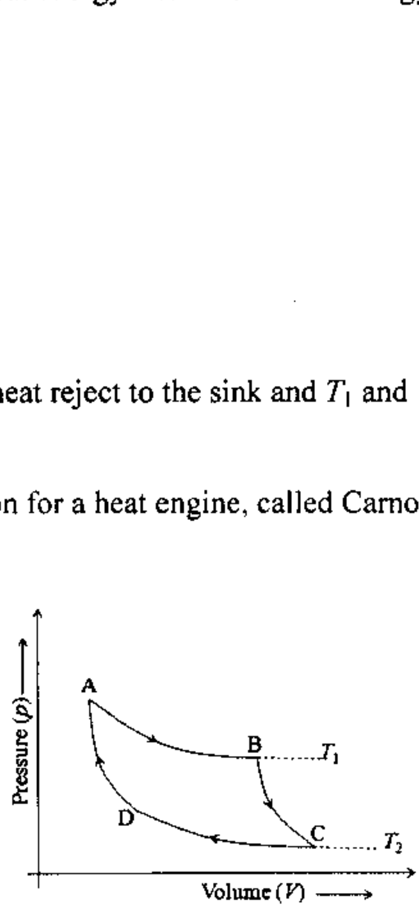
Heat engine: A heat engine is a device which converts heat energy into mechanical energy.
Heat engine consists of three parts
(i) Source of heat at higher temperature
(ii) Working substance
(iii) Sink of heat at lower temperature.
Thermal efficiency of a heat engine is given by

$$\eta = \frac{\text{work done/cycle}}{\text{total amount of heat absorbed/cycle}}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

where Q_1 is the heat absorbed from the source, Q_2 is the heat reject to the sink and T_1 and T_2 are temperatures of source and sink.

Carnot's cycle: Carnot devised an ideal cycle of operation for a heat engine, called Carnot's cycle.
A Carnot's cycle contains the following four processes
(i) Isothermal expansion (AB)
(ii) Adiabatic expansion (BC)
(iii) Isothermal compression (CD)
(iv) Adiabatic compression (DA)
The net work done per cycle by the engine is numerically equal to the area of the loop representing the Carnot's cycle.



The efficiency of the cycle is $\eta = 1 - \frac{T_2}{T_1}$

Note: Efficiency of Carnot engine is maximum for given temperatures T_1 and T_2.

Refrigerator or Heat pump: A Refrigerator or heat pump is a device used for cooling things. It absorbs heat from the sink at lower temperature and reject a large amount of heat to the source at higher temperature.
Coefficient of performance of refrigerator is given by

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

where Q_2 is the heat absorbed from the sink, Q_1 is the heat rejected to the source and T_1 and T_2 are temperatures of source and sink.

Relation between efficiency (η) and coefficient of performance (β)

$$\beta = \frac{1-\eta}{\eta}$$

BEHAVIOUR OF PERFECT GAS AND KINETIC THEORY

Boyle's law: At a constant temperature the volume of a given mass of a gas is inversely proportional to its pressure i.e.,

$$V \propto \frac{1}{P}$$

Or $V = k\frac{1}{P}$ where k is a constant of proportionality.

Or $PV =$ constant

For a given gas $P_1V_1 = P_2V_2$

Charles' law: At a constant pressure the volume of a given mass of a gas is directly proportional to its absolute temperature i.e.,

$$V \propto T$$

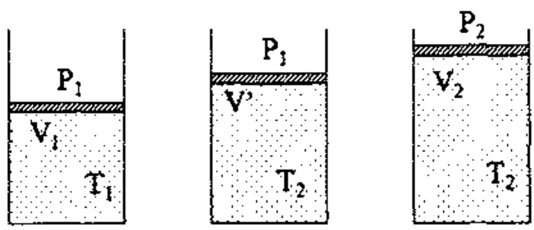
Or $V = kT$ where k is a constant of proportionality.

Or $\frac{V}{T} =$ constant

For a given gas $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Equation of state of a perfect gas or an ideal gas:

In figure (*a*)
Initial volume of the gas in a container = V_1
Initial temperature of the gas = T_1
Initial pressure of the gas = P_1
In figure (*b*)
Final volume of the gas in a container = V'
Final temperature of the gas = T_2 (here $T_2 > T_1$)
Final pressure of the gas = P_1 (constant)



(*a*) (*b*) (*c*)

From Charles' law $V \propto T$ at constant Pressure

$$\Rightarrow \frac{V}{T} = k \text{ (constant)}$$

$$\therefore \frac{V'}{T_2} = \frac{V_1}{T_1}$$

$$\text{Or } \frac{V'}{V_1} = \frac{T_2}{T_1} \text{ ----------(1)}$$

Again in figure (*b*)
Initial volume of the gas in a container = V'
Initial temperature of the gas = T_2
Initial pressure of the gas = P_1
In figure (*c*)
Final volume of the gas in a container = V_2
Final temperature of the gas = T_2 (constant)
Final pressure of the gas = P_2 (here $P_2 < P_1$)

From Boyles' law $V \propto \frac{1}{P}$ at constant Temperature

$$\Rightarrow VP = k \text{ (constant)}$$

$$\therefore V_2P_2 = V'P_1$$

$$\text{Or } \frac{V_2}{V'} = \frac{P_1}{P_2} \text{ ----------(2)}$$

Multiplying equation (1) and equation (2) we have

$$\frac{V'}{V_1} \times \frac{V_2}{V'} = \frac{T_2}{T_1} \times \frac{P_1}{P_2}$$

$$\frac{V_2}{V_1} = \frac{T_2P_1}{T_1P_2}$$

$$\text{Or } \frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

$$\therefore \frac{PV}{T} = \text{constant}$$

$$\text{Or } \frac{PV}{T} = R \text{ (Universal gas constant)}$$

$$\text{Or } PV = RT$$

For n mole of the gas,

$$PV = nRT$$

This is the equation of state. The value of $R = 8.31 J\,mol^{-1}\,K^{-1}$

Unit of the Universal gas constant R:

$$\because PV = nRT$$

$$\text{Or } R = \frac{PV}{nT}$$

$$\text{Or } R = \frac{Nm^{-2} \times m^3}{mol \times K} = \frac{Nm}{mol \times K}$$

$\therefore$ The unit of R is $J\,mol^{-1}K^{-1}$

Numerical value of the Universal gas constant R:

For 1 mole of a gas at STP,

$V = 22.4$ litre $= 22.4 \times 10^{-3}\,m^3$

$P = 760\,mm$ of mercury $= \rho gh = (13600) \times (9.8) \times (760 \times 10^{-3}) = 1.013 \times 10^5\,Nm^{-2}$

$T = 273.15\,K$

$$\therefore R = \frac{PV}{T} = \frac{(1.013\times10^5)\times(22.4\times10^{-3})}{273.15} = 8.31\,J\,mol^{-1}K^{-1}$$

Postulates of Kinetic theory of gases:

(1) A gas consists of a very large number of molecules. Each one is a perfectly identical elastic sphere.

(2) The molecules of a gas are in a state of continuous and random motion. They move in all directions with all possible velocities.

(3) The size of each molecule is very small as compared to the distance between them. Hence, the volume occupied by the molecule is negligible in comparison to the volume of the gas.
(4) There is no force of attraction or repulsion between the molecules and the walls of the container.
(5) The collisions of the molecules among themselves and with the walls of the container are perfectly elastic. Therefore, momentum and kinetic energy of the molecules are conserved during collisions.
(6) A molecule moves along a straight line between two successive collisions and the average distance travelled between two successive collisions is called the mean free path of the molecules.
(7) The collisions are almost instantaneous i.e., the time of collision of two molecules is negligible as compared to the time interval between two successive collisions.

Avogadro number N_A:
Avogadro number is defined as the number of molecules present in one mole of a substance. It is constant for all the substances. Its value is 6.023×10^{23}.

Pressure exerted by an ideal gas:
Consider a cubic container of side l containing N molecules of perfect gas moving with velocities $C_1, C_2, C_3 \dots C_N$. A molecule moving with a velocity C_1, will have velocities u_1, v_1 and w_1 as components along the X, Y and Z axes respectively. Similarly u_2, v_2 and w_2 are the velocity components of the second molecule and so on.

$$\therefore C_1^2 = u_1^2 + v_1^2 + w_1^2$$

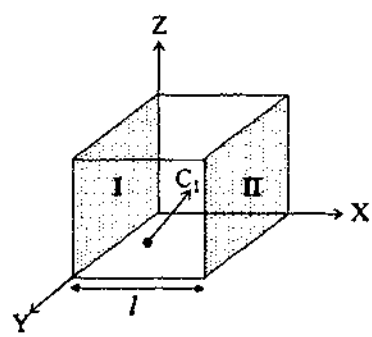
$$C_2^2 = u_2^2 + v_2^2 + w_2^2 \text{ and so on.}$$

The momentum of the first molecule moving along the positive X-axis, before striking the face II (shown in the figure) is mv_1 and reflected back with the momentum $-mv_1$.
Change in the momentum of the molecule = Final momentum – Initial momentum

$$= -mu_1 - mu_1 = -2mu_1$$

During each successive collision on face I the molecule must travel a distance $2l$ i.e., from face I to face II and back to face I.
Time taken between two successive collisions is

$$= \frac{2l}{v_1} \text{ second}$$



$$\therefore \text{Rate of change of momentum} = \frac{\text{change in momentum}}{\text{time taken}}$$

$$= \frac{-2mu_1}{\frac{2l}{u_1}} = -2mu_1 \times \frac{u_1}{2l}$$

$$\text{i.e., Force exerted on the molecule} = -\frac{mu_1^2}{l}$$

According to Newton's third law of motion,

$$\text{Force exerted by the molecule} = -\left(-\frac{mu_1^2}{l}\right) = \frac{mu_1^2}{l}$$

Force exerted by all the n molecules is

$$F_x = \frac{mu_1^2}{l} + \frac{mu_2^2}{l} + \frac{mu_3^2}{l} + \dots \frac{mu_N^2}{l}$$

Pressure exerted by the molecules is

$$P_x = \frac{F_x}{A} = \frac{1}{l^2}\left(\frac{mu_1^2}{l} + \frac{mu_2^2}{l} + \frac{mu_3^2}{l} + \dots \frac{mu_N^2}{l}\right)$$

$$= \frac{m}{l^3}\left(u_1^2 + u_2^2 + u_3^2 + \dots u_N^2\right)$$

Similarly, pressure exerted by the molecules along Y and Z axes are

$$P_y = \frac{m}{l^3}\left(v_1^2 + v_2^2 + v_3^2 + \dots v_N^2\right)$$

$$P_z = \frac{m}{l^3}\left(w_1^2 + w_2^2 + w_3^2 + \dots w_N^2\right)$$

Since the gas exerts the same pressure on all the walls of the container, then

$$P_x = P_y = P_z = P$$

$$\therefore P_x + P_y + P_z = P + P + P = 3P$$

$$\text{Or } P = \frac{P_x + P_y + P_z}{3}$$

$$P = \frac{1}{3}\cdot\frac{m}{l^3}\left[\left(u_1^2 + u_2^2 + u_3^2 + \dots u_N^2\right) + \left(v_1^2 + v_2^2 + v_3^2 + \dots v_N^2\right) + \left(w_1^2 + w_2^2 + w_3^2 + \dots w_N^2\right)\right]$$

$$P = \frac{1}{3}\cdot\frac{m}{l^3}\left[\left(u_1^2 + v_1^2 + w_1^2\right) + \left(u_2^2 + v_2^2 + w_2^2\right) + \left(u_3^2 + v_3^2 + w_3^2\right) + \dots\left(u_N^2 + v_N^2 + w_N^2\right)\right]$$

$$P = \frac{1}{3}\cdot\frac{m}{l^3}\left[C_1^2 + C_2^2 + C_3^2 + \dots C_N^2\right]$$

$$P = \frac{1}{3}\cdot\frac{m\,N}{l^3}\left(\frac{C_1^2 + C_2^2 + C_3^2 + \dots C_N^2}{N}\right)$$

$$P = \frac{1}{3}\cdot\frac{m\,N}{V}C^2 \text{ ----------(1)}$$

Where $C = \sqrt{\frac{C_1^2 + C_2^2 + C_3^2 + \dots C_N^2}{N}}$ is called the root mean square (RMS) velocity, which is defined as the square root of the mean value of the squares of velocities of individual molecules.

Equation (1) is the expression for the pressure exerted by the gas. Equation (1) can also be written as

(1) $P = \frac{1}{3}\cdot\frac{M}{V}C^2$ where m N = M is the mass of the gas

(2) $P = \frac{1}{3}\cdot\rho C^2$ where $\frac{M}{V} = \rho$ is the density of the gas

(3) $P = \frac{1}{3}\cdot\frac{M}{V}C^2$ $\Rightarrow PV = \frac{1}{3}MC^2$

(4) $PV = \frac{2}{3}\left(\frac{1}{2}\cdot MC^2\right) = \frac{2}{3}\overline{K.E}$ where $\overline{K.E}$ is the average kinetic energy of the gas.

Average kinetic energy of one mole of the gas:

Let us consider one mole of gas of mass M and volume V.

$$PV = \frac{1}{3} \cdot MC^2$$

From gas equation, $PV = RT$

$$\therefore RT = \frac{1}{3}MC^2$$

$$\text{Or } \frac{3}{2}RT = \frac{3}{2}\left(\frac{1}{3}MC^2\right)$$

$$\text{Or } \frac{3}{2}RT = \frac{1}{2}MC^2$$

i.e., Average kinetic energy of one mole of the gas is equal to $\frac{3}{2}RT$

Note:

(1) The root mean square velocity of the gas is given by

$$C = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{\rho V}} = \sqrt{\frac{3P}{\rho}}$$

(2) For a given gas $C \propto \sqrt{T}$

(3) For different gases $C \propto \frac{1}{\sqrt{M}}$

Average kinetic energy per molecule of the gas:

Since one mole of the gas contains N_A number of atoms where N_A is the Avogadro number.

$$\frac{3}{2}RT = \frac{1}{2}\mathrm{m}N_A C^2$$

$$\frac{3}{2}\frac{R}{N_A}T = \frac{1}{2}\mathrm{m}C^2$$

$$\frac{3}{2}k_B T = \frac{1}{2}\mathrm{m}C^2$$ where $k_B = \frac{R}{N_A}$ is the Boltzmann constant.

Its value is $1.38 \times 10^{-23}\, J\,K^{-1}$

$\therefore$ Average kinetic energy per molecule of the gas is equal to $\frac{3}{2}k_B T$

Another form of Perfect gas equation:

Let N is the number of molecules in the gas and 'n' is the number of moles

1 mole of a gas = N_A molecules (N_A = Avogadro number)

$\therefore n$ moles of the gas = nN_A molecules.

Again let N = nN_A

$$\therefore n = \frac{\mathrm{N}}{N_A}$$

$$\because PV = nRT$$

$$\therefore PV = \left(\frac{\mathrm{N}}{N_A}\right)RT = \mathrm{N}\left(\frac{R}{N_A}\right)T = \mathrm{N}\,k_B T$$

The above equation gives the perfect gas equation in terms of the number of molecules present in the gas.

Molecular interpretation of temperature:

∵ Average kinetic energy per molecule of the gas is equal to $\frac{3}{2}k_BT$

$$\text{Or } \frac{1}{2}\text{mC}^2 = \frac{3}{2}k_BT$$

$$\text{Or } \frac{1}{2}\text{mC}^2 \propto T$$

Thus the Kinetic energy of a gas molecules is directly proportional to the absolute temperature of the gas.

Note: At absolute zero i.e., $0K$ or -273.15^0 C, the kinetic energy of a gas molecule is zero. No gas molecule can have kinetic energy less than zero so no temperature can be lower than -273.15^0 C.

Real gases: Real gases deviate slightly from an ideal gas because in a real gas, the intermolecular forces are not zero and therefore, a definite amount of work has to be done in changing the distance between the molecules. Also real gas molecules occupy a finite volume.

Degrees of freedom:

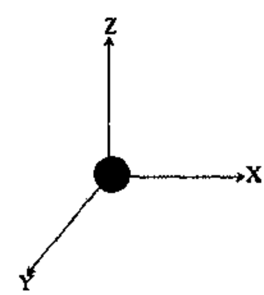
The number of degrees of freedom of a dynamical system is defined as the total number of co-ordinates or independent variables required to describe the position and configuration of the system.

Or

The degrees of freedom mean the number of independent ways the molecules can possess energy.

Monoatomic molecule:

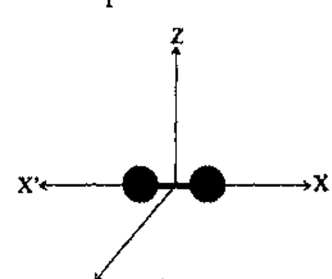
A monoatomic molecule consists of only a single atom of point mass, it has three degrees of freedom of translatory motion along the three co-ordinate axes as shown in the figure.



Examples: molecules of rare gases like helium, argon, etc.

Diatomic molecule:

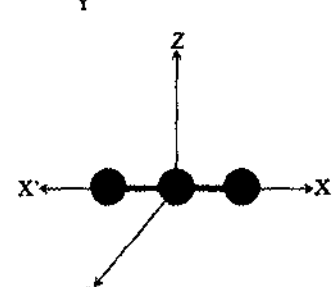
A diatomic molecule has two degrees of freedom of rotational motion, and three degrees of freedom of translational motion along the three axes. So, a diatomic molecule has five degrees of freedom.



Examples: molecules of O_2, N_2, CO, Cl_2, etc.

Triatomic molecule (Linear type):

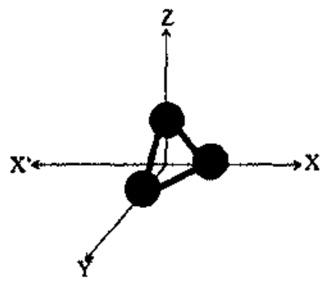
In the case of triatomic molecule of linear type, the centre of mass lies at the central atom. It therefore, behaves like a diatomic molecule with three degrees of freedom of translation and two degrees of freedom of rotation, totally it has five degrees of freedom.



Examples: molecules of CO_2, CS_2, etc.

Triatomic molecule (Non-linear type):

A triatomic non-linear molecule may rotate, about the three mutually perpendicular axes. Therefore, it possesses three degrees of freedom of rotation in addition to three degrees of freedom of translation along the three co-ordinate axes. Hence it has six degrees of freedom.



Examples: molecules of H_2O, SO_2, etc.

Note: The degrees of freedom of the system is given by:

$$f = 3N - k$$

Where f = degrees of freedom

N = number of particles in the system

k = independent relations among the particles

(i) For monoatomic molecule, $N = 1$ and $k = 0$

$$\therefore f = 3N - k = 3(1) - 0 = 3$$

(ii) For diatomic molecule, $N = 2$ and $k = 1$

$$\therefore f = 3N - k = 3(2) - 1 = 5$$

(iii) For triatomic molecule of linear type, $N = 3$ and $k = 2$

$$\therefore f = 3N - k = 3(3) - 2 = 7$$

(iv) For triatomic molecule of non-linear type, $N = 3$ and $k = 3$

$$\therefore f = 3N - k = 3(3) - 3 = 6$$

Note: At very high temperatures (above $5000K$), a gas molecule possesses vibratory motion also in addition to translatory and rotatory motion.

Law of equipartition of energy:

Law of equipartition of energy states that for a dynamical system in thermal equilibrium the total energy of the system is shared equally by all the degrees of freedom (translational as well as rotational). The energy associated with each degree of freedom per molecule is $\frac{1}{2}k_BT$, where k_B is the Boltzmann's constant.

Note: For a monoatomic molecule, the average kinetic energy is $\frac{3}{2}k_BT$. This energy is shared equally by all the degrees of freedom, and each degree of freedom has an energy $\frac{1}{2}k_BT$.

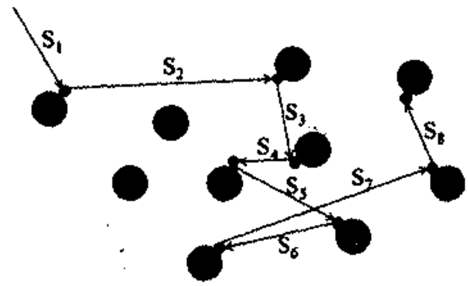
$$\therefore \frac{3}{2}k_BT = \frac{1}{2}k_BT + \frac{1}{2}k_BT + \frac{1}{2}k_BT$$

Thus a monoatomic molecule has three degrees of freedom.

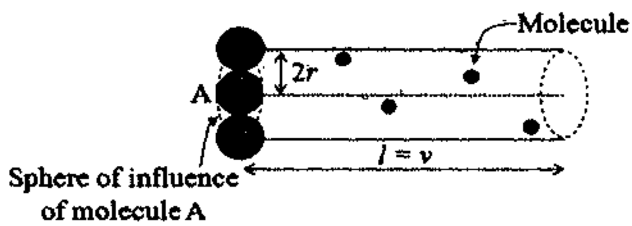
Concept of mean free path:

The mean free path λ is the average distance covered by a molecule between two successive collisions. If the total distance covered after *N* collisions is *S*, then the mean free path λ is given by

$$\lambda = \frac{S}{N}$$



In the figure, the mean free path is $\lambda = \frac{S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8}{8}$



Let the average velocity of the molecule A be *v* and the number of molecules per unit volume be *n*. Then in 1 second a molecule A will collide with all the molecules whose centres lies in the cylinder of radius 2*r* and length *l* as shown in the figure.

Length of the cylinder $l = vt = v \times 1 = v$

Volume of the cylinder $V = \pi\,(2r)^2 \times v = 4\pi r^2 v$

Number of molecules in the cylinder $= 4\pi r^2 v \times n$

Number of collisions made by the molecule A in 1 second $= 4\pi r^2 v \times n$

$\therefore$ One collision will take place in every $\frac{1}{4\pi r^2 v\,n}$ second

So the time interval between two successive collisions $= \frac{1}{4\pi r^2 v\,n}$ second

The distance travelled between the two successive collisions $= v \times \frac{1}{4\pi r^2 v\,n} = \frac{1}{4\pi r^2\,n}$

$\therefore$ mean free path $\lambda = \frac{1}{4\pi r^2\,n}$

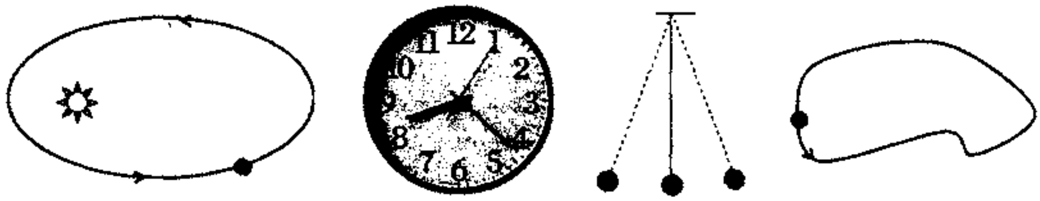
This is Clausius' expression for the mean free path.

Note: All the *n* molecules are considered to be at rest in Clausius' expression. But if the motion of the *n* molecules are taken into consideration, the corrected formula for the mean free path is $\lambda = \frac{1}{4\sqrt{2}\,\pi r^2\,n}$.

This is the Maxwell's expression for the mean free path.

OSCILLATIONS AND WAVES

Periodic motion: Any motion that repeats itself after regular interval of time is called periodic motion. Examples are the motion of planets around the Sun, motion of the hands of clock etc.



Oscillatory motion: If a body moves back and forth repeatedly about the mean position, it is said to possess oscillatory motion. Examples are the motion of a pendulum bob, oscillation of mass suspended from a spring.

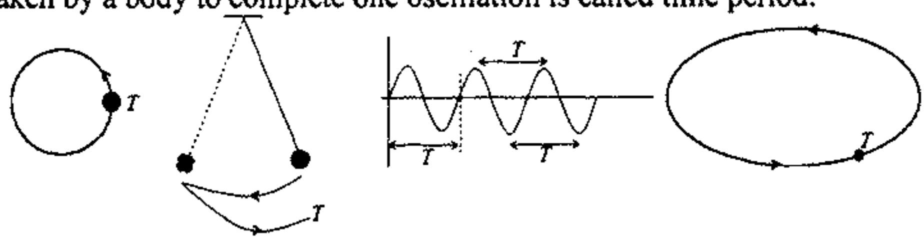
Note: All oscillatory motions are periodic but all periodic motion are not oscillatory. Motion of the Earth round the Sun is periodic but not oscillatory.

Vibratory motion: When the variable parameter is displacement, an oscillation is called vibration. Example is the vibration of a stretched string etc.

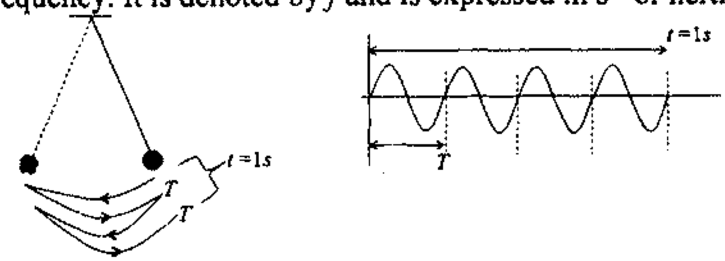
Time period: The smallest interval of time after which the motion is repeated is called Time period of the motion and is denoted by *T*. Its unit is second *s*.

Or

The time taken by a body to complete one oscillation is called time period.



Frequency: The number of complete oscillation or periodic motion executed by a body per second is called frequency. It is denoted by *f* and is expressed in s^{-1} or hertz or Hz.



(a) (b)

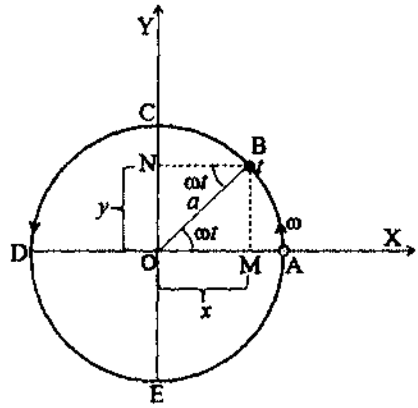
The relation between time period T and frequency ν is

$$f = \frac{1}{T}$$

In figure (*a*), the time period $T = \frac{1}{2}s$ $\Rightarrow f = \frac{1}{T} = \frac{1}{\frac{1}{2}} = 1 \times \frac{2}{1} = 2\ Hz$

In figure (b), the time period $T = \frac{1}{4} s$ $\quad \Rightarrow f = \frac{1}{T} = \frac{1}{\frac{1}{4}} = 1 \times \frac{4}{1} = 4\ Hz$

Displacement as a function of time: Displacement of a body executing a periodic motion can be expressed by a sine or cosine function of time, or by their linear combination.



Let a particle starts from the point A on the X axis, and move along the circumference of a circle of radius a to the point B with uniform angular velocity ω. Let t be the time taken by the particle to reach the point B, so the angular displacement is ωt. BM and BN are dropped perpendicular to the X and Y axis respectively. The position of the particle at B along the X and Y axis is represented by the point M and N, and the displacement of the particle at B along the X and Y axis is OM = y an ON = x.

As the particle moves from A to C, the point M moves from A to O, while the point N moves from O to C. For a complete motion of the particle from point A, to the point A again along the circumference, the point M moves from A to D and back to A, while the point N moves from O to C, C to E and E to O. So we see that the motion of the point M and N are oscillatory motion while the motion of the particle is periodic motion. Along the X axis, the displacement starts from the extreme position A, but along the Y axis , the displacement starts from the mean position O. In the figure, the displacement $y = a \sin \omega t$, and the displacement $x = a \cos \omega t$. Their linear combination is

$$\xi = y + x = a \sin \omega t + a \cos \omega t$$

$$\text{or } \xi = a(\sin \omega t + \cos \omega t)$$

Periodic function: The functions which are used to represent periodic motions are known as periodic functions.

In a periodic motion, displacement can be represented by a simple periodic function as

$$f(t) = A \sin \omega t = A \sin \frac{2\pi}{T} t$$

$$g(t) = B \cos \omega t = B \cos \frac{2\pi}{T} t$$

Since the time period of the periodic function is $T = \frac{2\pi}{\omega}$

If the variable t is changed to $t + T$, we have

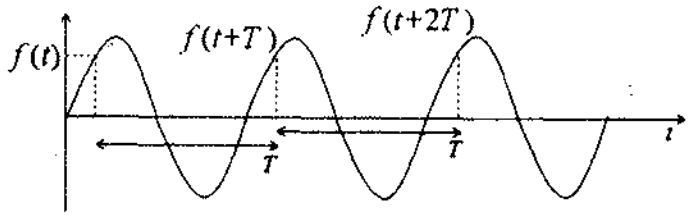
$$f(t+T) = A \sin \frac{2\pi}{T}(t+T) = A \sin\left(\frac{2\pi}{T} t + 2\pi\right) = A \sin \frac{2\pi}{T} t$$

$$\Rightarrow f(t+T) = f(t)$$

$$g(t+T) = B \cos \frac{2\pi}{T}(t+T) = B \cos\left(\frac{2\pi}{T} t + 2\pi\right) = B \cos \frac{2\pi}{T} t$$

$$\Rightarrow g(t+T) = g(t)$$

The periodic function can also be expressed as a linear combination of sine and cosine function as



$$\xi(t) = A\sin\omega t + B\cos\omega t$$

Now taking $A = D\cos\phi$ and $B = D\sin\phi$ then

$$\xi(t) = D\cos\phi\sin\omega t + D\sin\phi\cos\omega t$$
$$\xi(t) = D(\cos\phi\sin\omega t + \sin\phi\cos\omega t)$$
$$\xi(t) = D\sin(\omega t + \phi)$$

To find D and ϕ, we use the relation

$$A = D\cos\phi \qquad \text{or } A^2 = D^2\cos^2\phi \text{------}(1)$$
$$B = D\sin\phi \qquad \text{or } B^2 = D^2\sin^2\phi \text{------}(2)$$

Adding (1) and (2) we have

$$D^2\cos^2\phi + D^2\sin^2\phi = A^2 + B^2$$
$$D^2(\cos^2\phi + \sin^2\phi) = A^2 + B^2$$
$$D^2 = A^2 + B^2 \qquad \Rightarrow D = \sqrt{A^2 + B^2}$$

Again $\frac{B}{A} = \frac{D\sin\phi}{D\cos\phi} = \tan\phi$

$$\tan\phi = \frac{B}{A} \qquad \Rightarrow \phi = \tan^{-1}\left(\frac{B}{A}\right)$$

Simple harmonic motion (SHM): A special type of periodic motion in which a particle moves to and fro repeatedly about a mean position under the influence of a restoring force is known as simple harmonic motion.

This restoring force is always directed toward the mean position and its magnitude at any instant is directly proportional to the displacement of the particle from the mean position at that instant.

The characteristic of a simple harmonic motion are:-

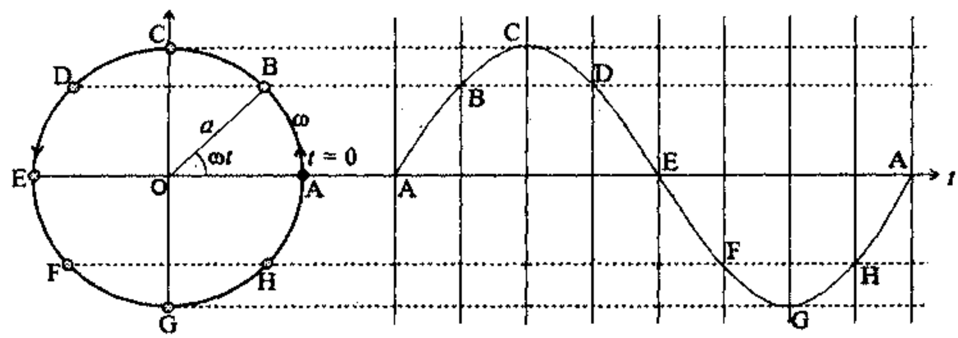
(i) It is periodic motion

(ii) It is to-and fro-motion

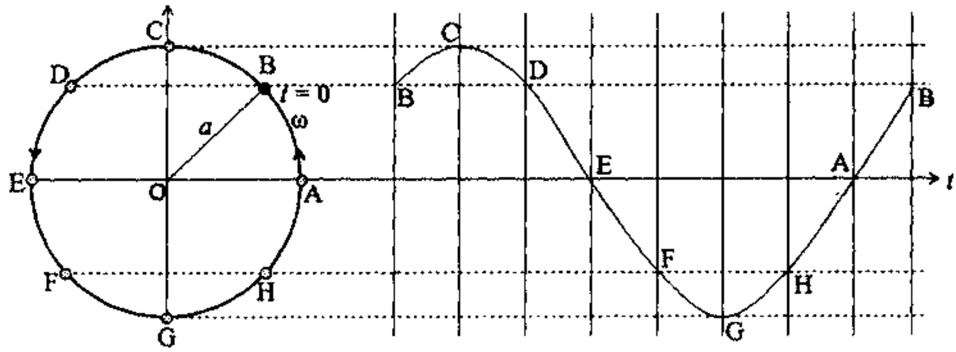
(iii) It is a linear motion

(iv) The value of the acceleration at any moment is proportional to the displacement from the mean position of rest.

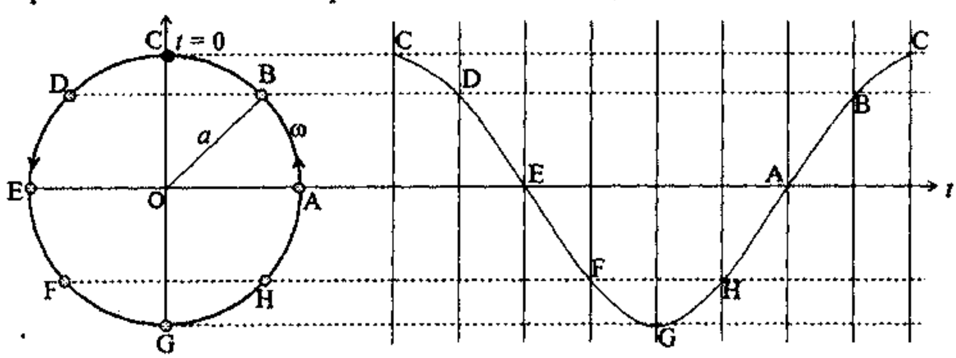
Analysis of SHM in terms of uniform circular motion: (i) Let the particle starts from the point A at time $t = 0$, and moves to points BCDEFGA in counter-clockwise direction along the circumference of a circle of radius a as shown in the figure. Let the motion be periodic, and moves with uniform angular velocity ω. Then from the figure we see that



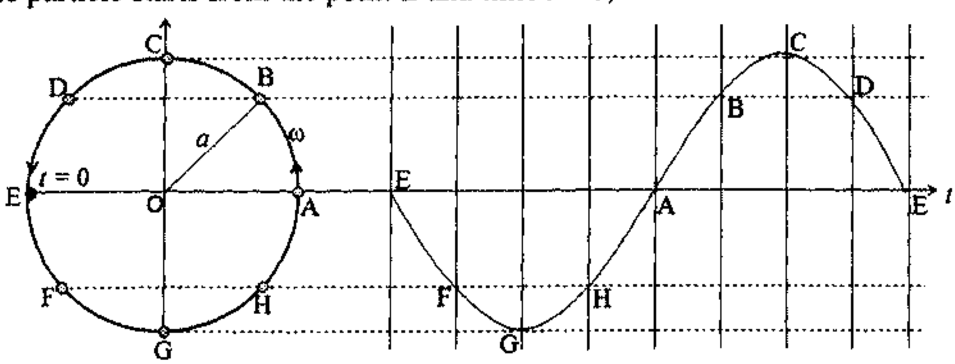
(ii) Let the particle starts from the point B and time $t = 0$,



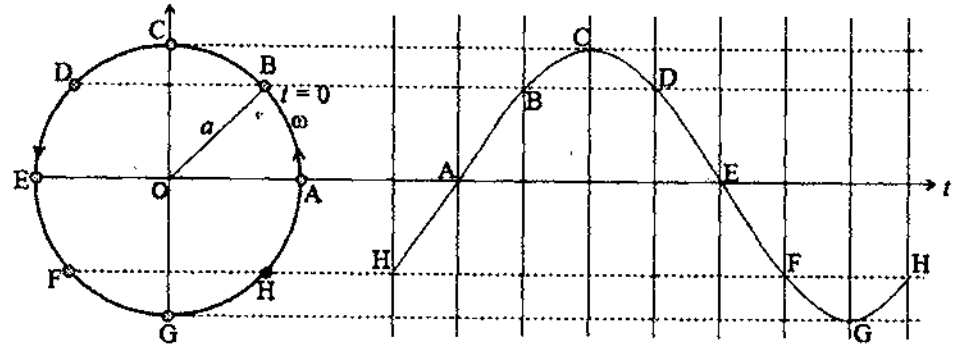
(iii) Let the particle starts from the point C and time $t = 0$,



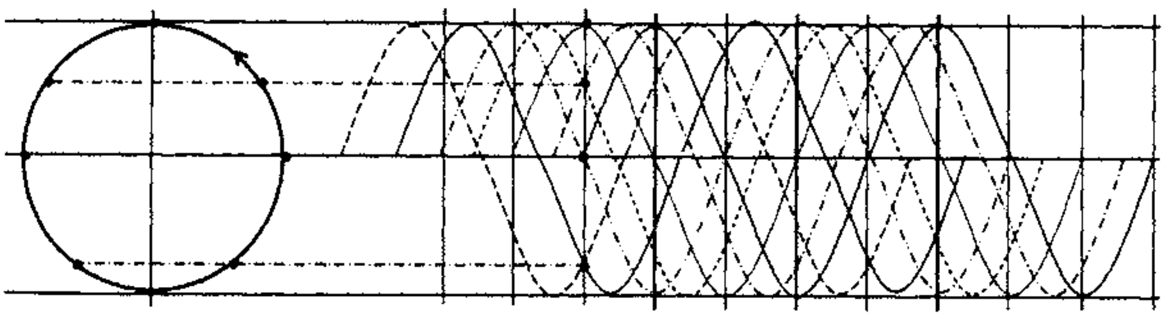
(iv) Let the particle starts from the point E and time $t = 0$,



(v) Let the particle starts from the point H and time $t = 0$,

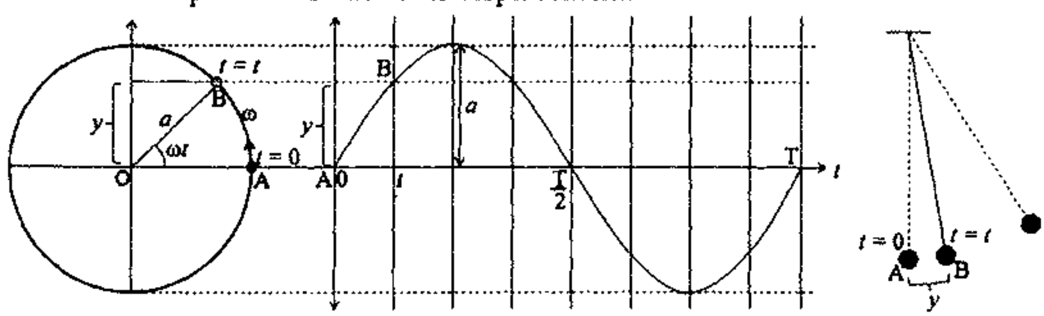


In the above examples, we see that particle starting from point C, leads the particle that starts from B and A. Also particle starting from point B, leads the particle that starts from point A. Particle starting from A and E are out of phase, and particle starting from H lags behind the particle that starts from A.



Some relations relating to vibration:

(i) Displacement in SHM: The distance travelled by the vibrating particle at any instant of time t from its mean position is known as displacement.



The displacement of the vibrating particle is $y = a \sin \omega t$

(ii) Amplitude: The amplitude of the vibrating particle is defined as its maximum displacement from the mean position. In the displacement equation $y = a \sin \omega t$, the amplitude is 'a'.

(iii) Time period: The time taken by a body to complete one oscillation is called time period.

$$\because y = a \sin \omega t = a \sin(\omega t + 2\pi)$$

Also $y = a \sin \omega t = a \sin \omega(t + T)$

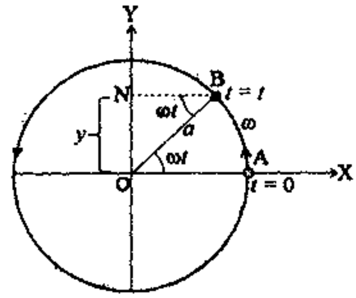
$$\therefore \omega t + 2\pi = \omega t + \omega T \qquad \Rightarrow T = \frac{2\pi}{\omega}$$

(iv) Frequency: The number of complete oscillation or periodic motion executed by a body per second is called frequency. It can also be define as the reciprocal of time period. It is denoted by f and is expressed in s^{-1} or hertz.

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$\omega = 2\pi f$ is called angular frequency. It is expressed in rad s^{-1}.

(v) Phase: The phase of the vibrating particle denotes the state of motion of the particle at any instant. The state of motion at any instant means the velocity, acceleration, force etc of the body at that instant. In the equation $y = a \sin \omega t$ the term ωt is known as the phase of the vibrating particle. Note that if the phase is ωt, the particle starts from the mean position i.e., the initial phase is zero.



(vi) Epoch: The phase of the vibrating particle at the commencement of its motion is called epoch or initial phase.

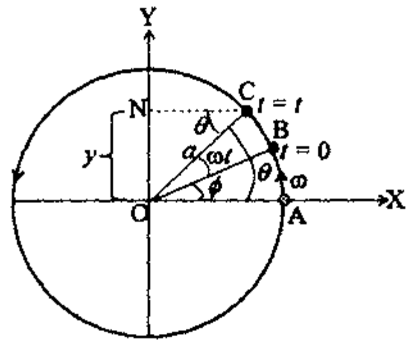
Let the particle is at the point B at time $t = 0$, and moves to point C with uniform angular velocity ω, in time t as shown in the figure.

So the time-displacement equation for the vibrating particle is

$$y = a \sin \theta$$

$$\because \theta = \omega t + \phi \text{ is the phase}$$

$$\therefore y = a \sin(\omega t + \phi)$$



Hence the particle leads the position of zero phase by ϕ. The initial phase or epoch is ϕ

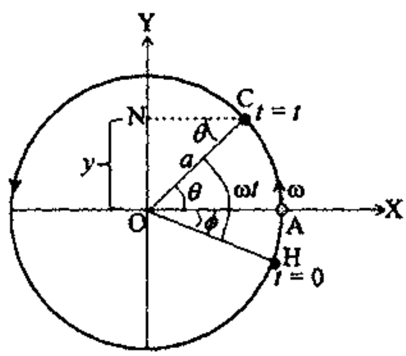
Let the particle is at the point H at time $t = 0$, and moves to point C with uniform angular velocity ω, in time t as shown in the figure.

So the time-displacement equation for the vibrating particle is

$$y = a \sin \theta$$

$$\because \theta = \omega t - \phi \text{ is the phase}$$

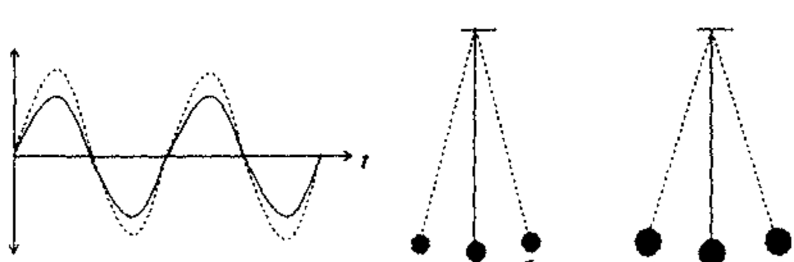
$$\therefore y = a \sin(\omega t - \phi)$$



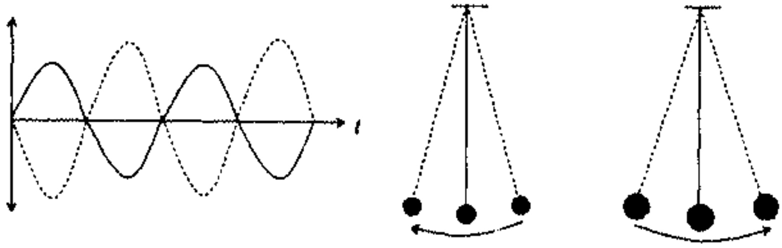
Hence the particle lags behind the position of zero phase by ϕ. The initial phase or epoch is $-\phi$

(vii) Phase difference:

(a) If two vibrating particles executing SHM having the same time period, and both cross their respective mean positions at the same time in the same direction, they are said to be in phase.



(b) If the two vibrating particles cross their respective mean position at the same time but in the opposite direction, they are said to be out of phase i.e., they have a phase difference of π.



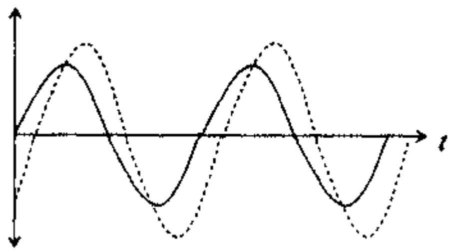
(c) If the vibrating motions are represented by equations

$y_1 = a \sin \omega t$ and

$y_2 = b \sin (\omega t - \phi)$

Then the phase difference between their phase angles is equal to the phase difference between the two motions.

$\therefore$ phase difference $= (\omega t - \phi) - \omega t = -\phi$ negative sign indicates that the second motion lags behind the first.



(d) If the vibrating motions are represented by equations

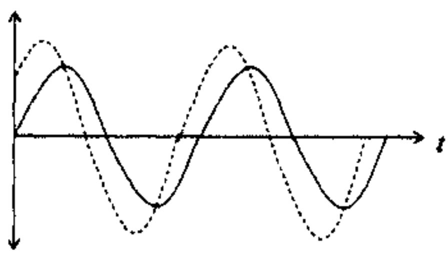
$y_1 = a \sin \omega t$ and

$y_2 = b \sin (\omega t + \phi)$

Then the phase difference between their phase angles is equal to the phase difference between the two motions.

$\therefore$ phase difference $= (\omega t + \phi) - \omega t = +\phi$ positive.

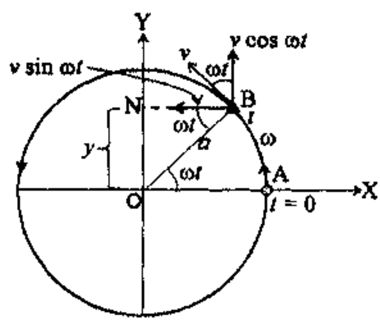
Here the second motion leads the first motion.



(viii) Velocity in SHM: The rate of change of displacement is the velocity of the vibrating particle.

$$\because y = a \sin \omega t$$

$$\frac{dy}{dt} = v = \frac{d}{dt}(a \sin \omega t) = a\omega \cos \omega t$$



$$v = a\omega\sqrt{1-\sin^2 \omega t} = a\omega\sqrt{1-\left(\frac{y}{a}\right)^2} = a\omega\sqrt{1-\frac{y^2}{a^2}}$$

$$v = a\omega\sqrt{\frac{a^2-y^2}{a^2}} \qquad \Rightarrow v = \omega\sqrt{a^2-y^2}$$

(a) When the particle is at mean position, i.e., $y = 0$, velocity is $a\omega$ and is maximum.
$v = \pm a\omega$ is called velocity amplitude.

(b) When the particle is in the extreme position, i.e., $y = \pm a$, the velocity is zero.

(ix) Acceleration in SHM: The rate of change of velocity is the acceleration of the vibrating particle.

$$\because y = a \sin \omega t$$

$$\frac{d^2y}{dt^2} = accel^n = \frac{d}{dt}\left(\frac{d}{dt}a\sin\omega t\right) = \frac{d}{dt}(a\omega\cos\omega t) = -a\omega^2\sin\omega t$$

$$\therefore accel^n = -\omega^2 y$$

The negative sign indicates that the acceleration is always opposite to the direction of displacement and is directed towards the centre.

(a) When the particle is at the mean position i.e., $y = 0$, the acceleration is zero.

(b) When the particle is at the extreme position i.e., $y = \pm a$, acceleration is $\mp a\omega^2$ which is called as acceleration amplitude.

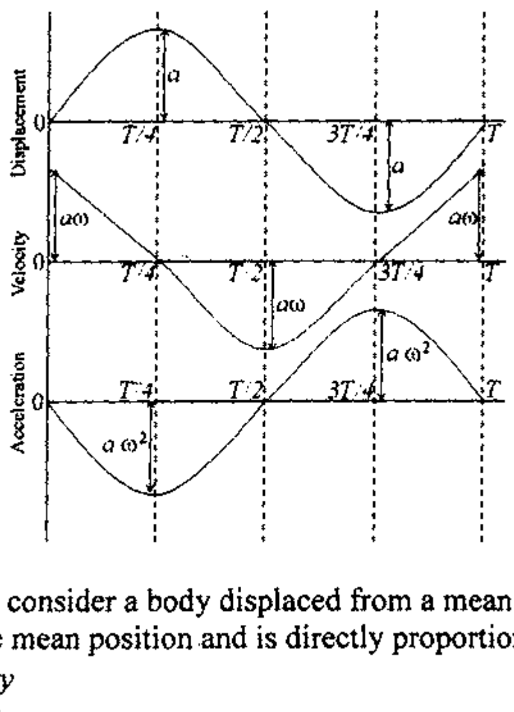
(c) The differential equation of simple harmonic motion is $\frac{d^2y}{dt^2} + \omega^2 y = 0$

Using the above equations, the values of displacement, velocity and acceleration for the SHM are given in the table below.

Table - Displacement, Velocity and Acceleration

Time	ωt	Displacement $y = a\sin\omega t$	Velocity $v = a\omega\cos\omega t$	Acceleration $accel^n = -\omega^2 y$
$t = 0$	0	0	$a\omega$	0
$t = T/4$	$\pi/2$	$+a$	0	$-a\omega^2$
$t = T/2$	π	0	$-a\omega$	0
$t = 3T/4$	$3\pi/2$	$-a$	0	$a\omega^2$
$t = T$	2π	0	$a\omega$	0

(x) Graphical representation of SHM: Graphical representation of displacement, velocity and acceleration of a particle vibrating simple harmonically with respect to time t is shown in the figure.



Dynamics of SHM: Let us consider a body displaced from a mean position. The restoring force brings the body to the mean position and is directly proportional to the displacement.

i.e., $-F \propto y$

or $F = -k\,y$

Where k is called the force constant or the spring constant. It is expressed in N m^{-1}.

From Newton's second law, $F = m \times accel^n$

$m \times accel^n = -k\,y \qquad$ Or $accel^n = -\frac{k}{m} y$ ----------(1)

From definition of SHM, $accel^n = -\omega^2 y$

$$\therefore -\omega^2 y = -\frac{k}{m} y \qquad \Rightarrow \omega^2 = \frac{k}{m} \qquad \text{Or } \omega = \sqrt{\frac{k}{m}}$$

Therefore the period of SHM is

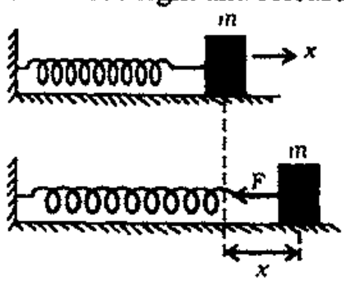
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\text{inertial factor}}{\text{spring factor}}}$$

From equation (1) we have by neglecting the negative sign

$$\frac{m}{k} = \frac{y}{accel^n}$$

$$Also \;\; T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y}{accel^n}} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

Horizontal oscillations of spring: Consider a mass m is attached to one end of a spring and the other end is fixed to a support as shown in the figure. The body is placed on a smooth horizontal surface. Let the body be displaced through a distance x towards right and released. It will oscillate about its mean position.



Restoring force $F = -\,k\,x$.

From Newton's second law, we know that $F = m\,a$

$$\therefore m\,a = -k\,x$$

$$a = -\frac{k}{m} x$$

From definition of SHM, $a = -\,\omega^2 x$

$$\therefore -\omega^2 x = -\frac{k}{m} x \qquad \Rightarrow \omega^2 = \frac{k}{m} \qquad \text{Or } \omega = \sqrt{\frac{k}{m}}$$

Therefore the time period of SHM is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

The frequency is

$$f = \frac{1}{T} = \frac{1}{2\pi\sqrt{\frac{m}{k}}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Note: If we consider the displacement $y = a\sin\omega t$, then the velocity and acceleration will be $v = a\omega\cos\omega t$ and $accel^n = -\omega^2 y$ respectively.

If we consider the displacement $x = a\cos\omega t$, then the velocity and acceleration will be $v = -a\omega\sin\omega t$ and $accel^n = -\omega^2 x$ respectively.

Vertical oscillations of spring: Figure (a) shows a light spring suspended vertically from a rigid support in a relaxed position. When a mass 'm' is attached to the spring as in figure (b), the spring is extended by a small length l such that the restoring force F_1 exerted by the spring is equal to the weight mg. Since the mass m is in equilibrium, the sum of the forces on it is zero i.e.,

$$-kl + mg = 0$$

If the mass is displaced downward by a small distance y, then the restoring force exert by the spring is $F_2 = -k(l + y)$ as shown in figure (c). Now the net force on mass m is

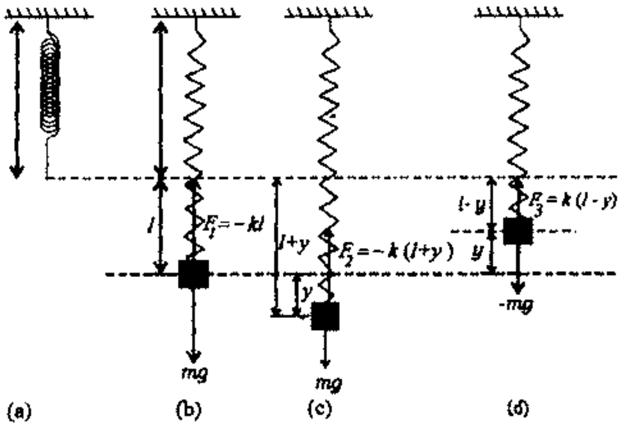
$$F = -k(l + y) + mg = -kl - ky + mg = -ky \qquad (\because -kl + mg = 0)$$

If the mass is displaced upward by a small distance y, then the restoring force exert by the spring is $F_3 = k(l - y)$ as shown in figure (d). Now the net force on mass m is

$$F = k(l - y) + (-mg) = kl - ky - mg = -ky \qquad (\because -kl + mg = 0 \quad \text{Or } kl - mg = 0)$$

Here the net force is always proportional to the displacement of the body from its equilibrium position and so the motion is simple harmonic.

According to Newton's second law, $F = ma$ so that



$$m\,a = -k\,y$$

$$\text{or } a = -\frac{k}{m}y$$

From definition of SHM, $a = -\omega^2 y$

$$\therefore -\omega^2 y = -\frac{k}{m}y \qquad \Rightarrow \omega^2 = \frac{k}{m}$$

$$\text{Or } \omega = \sqrt{\frac{k}{m}}$$

Therefore the time period of SHM is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

The frequency is

$$f = \frac{1}{T} = \frac{1}{2\pi\sqrt{\frac{m}{k}}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

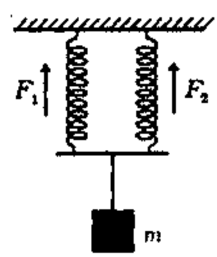
Springs in parallel: Two springs of spring factors k_1 and k_2 are suspended from a rigid support as shown in the figure. A load m is attached to the combination. Let y be the increase in length for both the springs but their restoring forces are different.

If F_1 and F_2 are the restoring forces i.e.,

$$F_1 = -k_1 y \qquad F_2 = -k_2 y$$

$\therefore$ Total restoring force $= (F_1 + F_2) = -(k_1 + k_2)\,y$

So, time period of the body is given by



$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

If $k_1 = k_2 = k$ then

$$\therefore T = 2\pi\sqrt{\frac{m}{2k}}$$

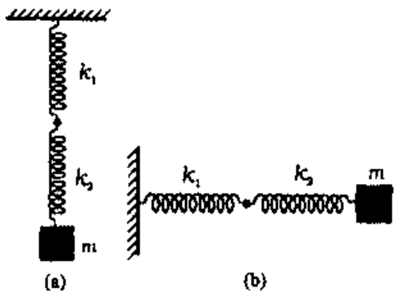
And Frequency $f=\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

Springs in series: Two springs are connected in series in two different ways. This arrangement is shown in figure (a) and (b).
In this system when the combination of two springs is displaced to a distance y, it produces extension y_1 and y_2 in two springs of force constants k_1 and k_2.

$\therefore \quad F=-k_1 y_1, \qquad F=-k_2 y_2$

Where F is the restoring force.
Total extension y in the series combination is

$$y=y_1+y_2=-\frac{F}{k_1}-\frac{F}{k_2}=-\left[\frac{1}{k_1}+\frac{1}{k_2}\right]F$$



$$y=-\left[\frac{k_1+k_2}{k_1k_2}\right]F \qquad \Rightarrow F=-\left[\frac{k_1k_2}{k_1+k_2}\right]y$$

$$\because F=-ky$$

$$\therefore k=\frac{k_1k_2}{k_1+k_2}$$

So, time period of the body is given by

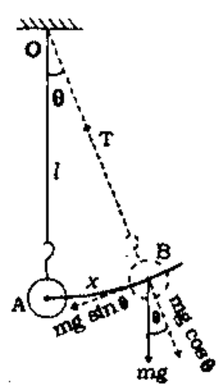
$$T=2\pi\sqrt{\frac{m}{\frac{k_1k_2}{k_1+k_2}}} \qquad \text{Or } T=2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$$

If $k_1=k_2=k$ then

$$T=2\pi\sqrt{\frac{2km}{k^2}}=2\pi\sqrt{\frac{2m}{k}}$$

And Frequency $f=\frac{1}{2\pi}\sqrt{\frac{k}{2m}}$

Oscillations of a simple pendulum: A simple pendulum consists of massless and inelastic thread whose one end is fixed to a rigid support and a small bob of mass m is suspended from the other end of the thread. Let l be the length of the pendulum. When the bob is slightly displaced and released, it oscillates about its equilibrium position. The figure shows the displaced position of the pendulum. Suppose the thread makes an angle θ with the vertical. The distance of the bob from the equilibrium position A is AB. At B, the weight mg acts vertically downwards. This force is resolved into two components.



(i) The component $mg\cos\theta$ is balanced by the tension in the thread acting along the length, towards the fixed point O.
(ii) $mg\sin\theta$ which is unbalanced, acts perpendicular to the length of the thread. This force tends to restore the bob to the mean position. If the amplitude of oscillation is small, then the path of the bob is a straight line.

Restoring force $F=-mg\sin\theta$

If the angular displacement is small $\sin\theta\approx\theta$

$$F=-mg\theta$$

$$\text{But } \theta = \frac{x}{l} \qquad \therefore F = -mg\frac{x}{l}$$

Comparing this equation with Newton's second law, $F = ma$ we get,

$$ma = -mg\frac{x}{l} \qquad \text{Or } a = -g\frac{x}{l}$$

Negative sign indicates that the motion of simple pendulum is SHM.

We know that $a = -\omega^2 x$

$$\therefore -\omega^2 x = -g\frac{x}{l} \qquad \text{Or } \omega^2 = \frac{g}{l} \qquad \text{Or } \omega = \sqrt{\frac{g}{l}}$$

$$\therefore \text{Time period is } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

$$\text{Frequency is } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

Second's pendulum: A simple pendulum whose time period is two seconds is called second's pendulum. To find the length of the second's pendulum

$$\because T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{Or } T^2 = 4\pi^2\frac{l}{g} \qquad \text{Or } l = \frac{T^2 g}{4\pi^2}$$

$$\text{Or } l = \frac{2^2 \times (9.8)}{4 \times (3.14)^2} \qquad \therefore l = 0.993m = 99.3cm$$

Total energy in SHM: The total energy (E) of an oscillating particle is equal to the sum of its kinetic energy and potential energy if conservative force acts on it.

Kinetic Energy:

Kinetic energy of the particle of mass m is

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\omega\sqrt{a^2 - y^2}\right)^2 \qquad [\because v = \omega\sqrt{a^2 - y^2}]$$

$$\therefore E_K = \frac{1}{2}m\,\omega^2(a^2 - y^2)$$

Potential Energy:

From definition of SHM $F = -ky$

Work done by the restoring force during the small displacement dy is

$$dW = -F.dy = -(-ky)\,dy = ky\,dy$$

$\therefore$ Total work done for the displacement y is

$$W = \int_0^y ky\,dy = k\int_0^y y\,dy = k\left[\frac{y^2}{2}\right]_0^y = k\left[\frac{y^2}{2} - \frac{0}{2}\right] = \frac{1}{2}ky^2$$

$$\because k = \omega^2 m$$

$$\therefore W = \frac{1}{2}m\omega^2 y^2 \qquad \Rightarrow E_P = \frac{1}{2}m\omega^2 y^2$$

Total Energy:

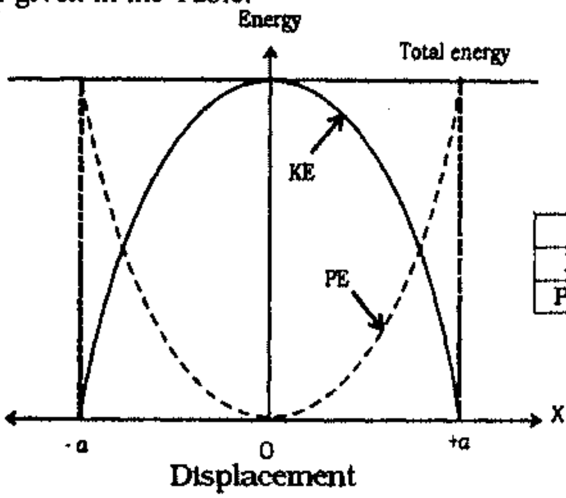
$$E = E_K + E_P = \frac{1}{2}m\omega^2(a^2 - y^2) + \frac{1}{2}m\omega^2 y^2$$

$$\therefore E = \frac{1}{2}m\omega^2 a^2$$

We know that $\omega = 2\pi f$

$$\therefore E = \frac{1}{2}m(2\pi f)^2 a^2 \qquad \text{or} \quad E = 2m\pi^2 f^2 a^2$$

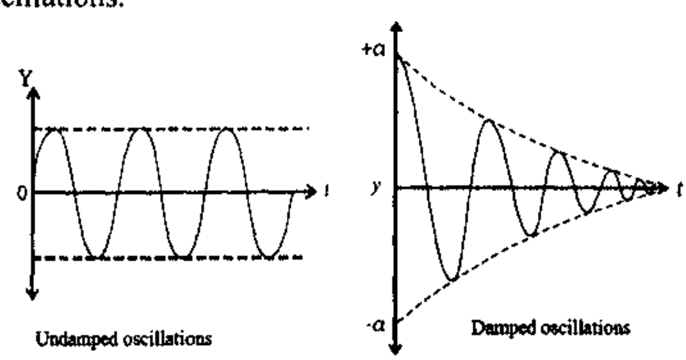
Graphical representation of energy: The variation of energy of an oscillating particle with the displacement can be represented in a graph as shown in the figure. The values of E_K and E_P in terms of E for different values of y are given in the Table.



Table

y	0	a/2	a	-a/2	-a
Kinetic Energy	E	3E/4	0	3E/4	0
Potential Energy	0	E/4	E	E/4	E

Undamped oscillation: The oscillations whose amplitude remains constant with time are called undamped oscillations.



Damped oscillations: The oscillations whose amplitude goes on decreasing with time are called damped oscillations.

Free oscillations: When a body vibrates with its own natural frequency, it is said to execute free oscillations.

Forced oscillations: When a vibrating body is maintained in the state of vibration by a periodic force of frequency (f) other than its natural frequency of the body, the vibrations are called forced vibrations.

Resonant oscillations: If the frequency of the external periodic force is equal to the natural frequency of oscillation of the system, then the amplitude of oscillation will be large and this is known as resonance.

Waves: The patterns, which move without the real physical transfer or flow of matter as a whole, are called waves. For example, while we speak, the sound moves outward from us, without any flow of air from one part of the medium to another.

Wave motion: Wave motion is a form of disturbance which travels through a medium due to the repeated periodic motion of the particles of the medium about their mean position. The motion is transferred continuously from one particle to its neighbouring particle.

Characteristics of wave motion:

(1) Wave motion is a form of disturbance travelling in the medium due to the periodic motion of the particles about their mean position.
(2) It is necessary that the medium should possess elasticity and inertia.
(3) All the particles of the medium do not receive the disturbance at the same instant.
(4) The wave velocity is different from the particle velocity. The velocity of a wave is constant for a given medium, whereas the velocity of the particles goes on changing and it becomes maximum in their mean position and zero in their extreme positions.
(5) During the propagation of wave motion, there is transfer of energy from one particle to another without any actual transfer of the particles of the medium.
(6) The waves undergo reflection, refraction, diffraction and interference.

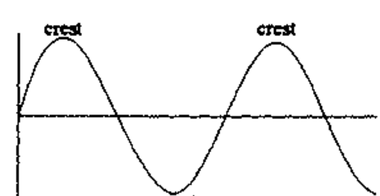
Types of waves: There are mainly three types of waves
(1) Mechanical waves: Mechanical waves obey Newton's laws and they exist in material media.

(2) Electromagnetic waves: Unlike the mechanical waves, electromagnetic waves do not require any medium for their propagation.

(3) Matter waves: The waves that are associated with moving electrons, protons, neutrons, other particles, and even with atoms and molecules are called matter waves.

Mechanical wave motion: There are two types of mechanical wave motion

(1) transverse wave motion: When the constituents of a medium oscillate perpendicular to the direction of wave propagation, then the wave is called transverse waves. Examples of transverse waves are waves produced by plucked strings of guitar, violin and electromagnetic waves.



Transverse waves travel in the form of crests and troughs. The maximum displacement of the particle in the positive direction i.e., above its mean position is called crest and maximum displacement of the particle in the negative direction i.e., below its mean position is called trough.

(2) longitudinal wave motion: When the constituents of a medium oscillate along the direction of wave propagation, then the wave is called longitudinal wave. Example of longitudinal waves are sound waves in fluids (liquids and gases).

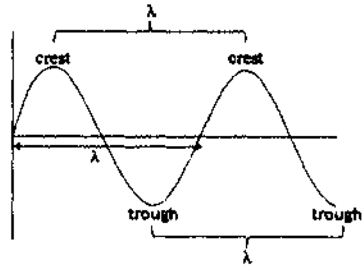
When a longitudinal wave travels through a medium, it produces compressions (C) and rarefactions (R).

Important terms used in wave motion:

(i) Wavelength (λ): The distance travelled by a wave during which a particle of the medium completes one vibration is called wavelength.

Or

Wavelength may also be defined as the distance between two successive crests or troughs in transverse waves, or the distance between two successive compressions or rarefactions in longitudinal waves.

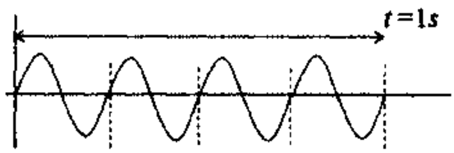


(ii) Time period (*T*): The time period of a wave is the time taken by the wave to travel a distance equal to its wavelength.

(iii) Frequency (f): It is defined as the number of waves produced in one second. It can also be defined as the reciprocal of the time period.

$$\text{i.e., } f = \frac{1}{T}$$

In the figure, the frequency $f = 4$.



(iv) Velocity of the wave (v): The distance travelled by a wave in a medium in one second is called the velocity of propagation of the wave in that medium. If v represents the velocity of propagation of the wave, it is given by

$$v = \frac{\text{distance travelled by wave}}{\text{time taken}}$$

$$v = \frac{\lambda}{T} = \lambda \cdot \frac{1}{T} = \lambda f$$

Or Wave velocity = Wavelength × Frequency

Comparison between transverse and longitudinal waves:

Transverse waves	Longitudinal waves
The particles of the medium vibrate perpendicular to the direction in which the wave advances.	The particles of the medium vibrate in the same direction in which the wave advances.
It is formed of crests and troughs	It is formed in a series of compressions and rarefactions
It can propagate only in solids and at the surface of liquid	It can propagate in all types of media (solid, liquid and gas)

Progressive wave: When a wave moves from one point of medium to another point, it is called travelling or progressive wave.

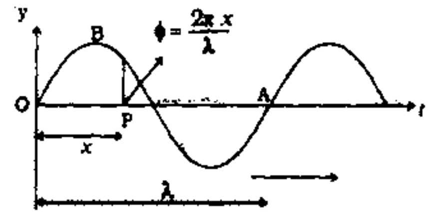
Displacement relation for a progressive wave: Let us assume that a progressive wave travels from the origin O along the positive direction of X axis, i.e., from left to right. The displacement of a particle at a given instant is

$$y = a \sin \omega t$$

where a is the amplitude of the vibration of the particle and $\omega = 2\pi f$.
The displacement of the particle P at a distance x from O at a given instant is given by

$$y = a \sin(\omega t - \phi)$$

If the two particles are separated by a distance λ, they will differ by a phase of 2π



i.e., $\lambda = 2\pi$

$$\text{Or } 1 = \frac{2\pi}{\lambda}$$

$$\text{Or } x = \frac{2\pi}{\lambda} x$$

$$\text{Or } \phi = \frac{2\pi}{\lambda} x$$

$$\text{Or } y = a \sin\left(\omega t - \frac{2\pi}{\lambda} x\right)$$

$$\because \omega = 2\pi f = 2\pi\left(\frac{v}{\lambda}\right) = \frac{2\pi v}{\lambda}$$

$$\text{Or } y = a \sin\left(\frac{2\pi v}{\lambda} t - \frac{2\pi}{\lambda} x\right)$$

$$\text{Or } y = a \sin\frac{2\pi}{\lambda}(v t - x) \text{------------(1)}$$

If the wave travels in opposite direction, i.e., from right to left, the equation becomes.

$$y = a \sin\frac{2\pi}{\lambda}(v t + x) \text{------------(2)}$$

$$\text{Again } \because v = \frac{\lambda}{T}$$

Equation (1) can be written as

$$y = a \sin\frac{2\pi}{\lambda}\left(\frac{\lambda}{T} t - x\right)$$

$$\text{Or } y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \text{------------(3)}$$

If the wave travels in opposite direction, i.e., from right to left, the equation becomes.

$$y = a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) \text{------------(4)}$$

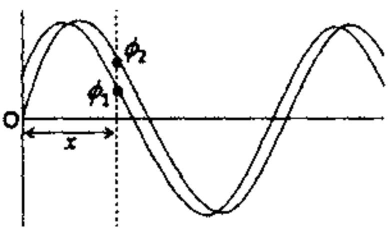
Characteristics of progressive wave:

(1) Each particle of the medium executes vibration about its mean position. The disturbance progresses onward from one particle to another.
(2) The particles of the medium vibrate with same amplitude about their mean positions.
(3) Each successive particle of the medium performs a motion similar to that of its predecessor along the propagation of the wave, but later in time.
(4) The phase of every particle changes from 0 to 2π.

(5) No particle remains permanently at rest. Twice during each vibration, the particles are momentarily at rest at extreme positions, different particles attain the position at different time.
(6) Transverse progressive waves are characterised by crests and troughs. Longitudinal waves are characterised by compressions and rarefactions.
(7) There is a transfer of energy across the medium in the direction of propagation of progressive wave.
(8) All the particles have the same maximum velocity when they pass through the mean position.
(9) The displacement, velocity and acceleration of the particle separated by $m\lambda$ are the same, where m is an integer.

Variation of phase with time: The phase changes continuously with time at a constant distance.
At a given distance x from O let ϕ_1 and ϕ_2 be the phase of a particle at time t_1 and t_2 respectively.

$$\phi_1 = 2\pi\left(\frac{t_1}{T} - \frac{x}{\lambda}\right) \quad \text{and} \quad \phi_2 = 2\pi\left(\frac{t_2}{T} - \frac{x}{\lambda}\right)$$



$$\phi_2 - \phi_1 = 2\pi\left(\frac{t_2}{T} - \frac{x}{\lambda}\right) - 2\pi\left(\frac{t_1}{T} - \frac{x}{\lambda}\right)$$

$$\phi_2 - \phi_1 = 2\pi\frac{t_2}{T} - 2\pi\frac{t_1}{T}$$

$$\phi_2 - \phi_1 = 2\pi\left(\frac{t_2}{T} - \frac{t_1}{T}\right)$$

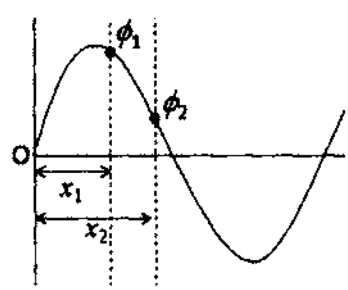
$$\phi_2 - \phi_1 = \frac{2\pi}{T}(t_2 - t_1)$$

$$\Delta\phi = \frac{2\pi}{T}\Delta t$$

This is the phase change $\Delta\phi$ of a particle in time interval Δt. If $\Delta t = T$, $\Delta\phi = 2\pi$. This shows that after a time period T, the phase of a particle becomes the same.

Variation of phase with distance: At a given time t phase changes periodically with distance x. Let ϕ_1 and ϕ_2 be the phase of two particles at distance x_1 and x_2 respectively from the origin at a time t.

$$\phi_1 = 2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right) \quad \text{and} \quad \phi_2 = 2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right)$$



$$\phi_2 - \phi_1 = -\frac{2\pi}{\lambda}(x_2 - x_1)$$

$$\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$$

The negative sign indicates that the forward points lag in phase when the wave travels from left to right.
When $\Delta x = \lambda$, $\Delta\phi = 2\pi$, the phase difference between two particles having a path difference λ is 2π.

Reflection of transverse waves:

(i) Reflection at rigid support: When a transverse wave is reflected at the rigid support, a phase change of 180^0 takes place in the displacement.

(ii) Reflection at free boundary: When a transverse wave is reflected at the free boundary, no phase change takes place in the displacement.

Superposition principle: When two waves travel in a medium simultaneously in such a way that each wave represents its separate motion, then the resultant displacement at any point at any time is equal to the vector sum of the individual displacements of the waves.

If $|\vec{Y}_1| = a$ and $|\vec{Y}_2| = a$ are the displacements at a point, then the resultant displacement is given by

$$\vec{Y} = \vec{Y}_1 + \vec{Y}_2$$

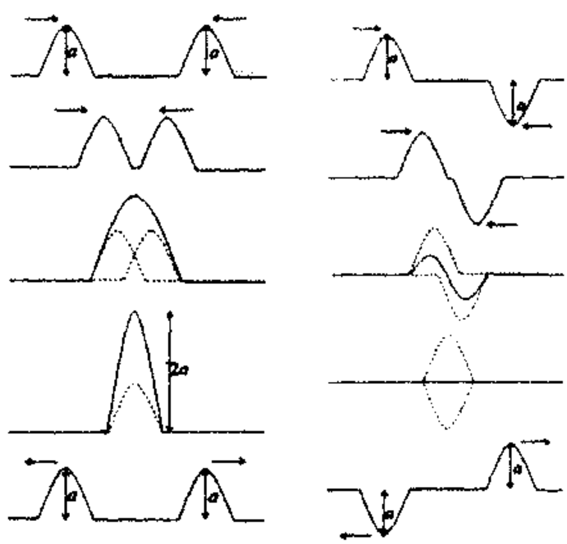
Figure 1

$$|\vec{Y}| = |\vec{Y}_1 + \vec{Y}_2| = |\vec{Y}_1| + |\vec{Y}_2| = a + a = 2a$$

Figure 2

$$|\vec{Y}| = |\vec{Y}_1 + \vec{Y}_2| = |\vec{Y}_1 + (-\vec{Y}_1)| = |\vec{Y}_1 - \vec{Y}_1| = |\vec{Y}_1| - |\vec{Y}_1| = a - a = 0$$

The principle of superposition of waves is applied in wave phenomena such as interference, beats and stationary waves.



(1) (2)

Stationary (Standing) waves: When two progressive waves of same amplitude and wavelength travelling along a straight line in opposite directions superimpose on each other, stationary waves are formed.

Let us consider a progressive wave of amplitude a and wavelength λ travelling in the positive direction of X axis.

$$y_1 = a\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)$$

This wave is reflected from a free end and it travels in the negative direction of X axis, then

$$y_2 = a\sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$$

According to principle of superposition, the resultant displacement is

$$y = y_1 + y_2$$

$$y = a\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)+a\sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$$

$$y = a\left[\sin\left(\frac{2\pi t}{T}-\frac{2\pi x}{\lambda}\right)+\sin\left(\frac{2\pi t}{T}+\frac{2\pi x}{\lambda}\right)\right]$$

$$y = a\left[2\sin\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi x}{\lambda}\right)\right]$$

$$\text{Or } y = \left[2a\cos\left(\frac{2\pi x}{\lambda}\right)\right]\sin\left(\frac{2\pi t}{T}\right)$$

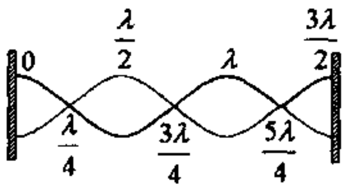
This is the equation of a stationary wave.

Each particle vibrates in a SHM with an amplitude $\left|2a\cos\left(\frac{2\pi x}{\lambda}\right)\right|$. The amplitudes are not equal for all the particles. In particular, there are points where the amplitude $\left|2a\cos\left(\frac{2\pi x}{\lambda}\right)\right|=0$. These points are called nodes, and this will be the case when

$$\cos\left(\frac{2\pi x}{\lambda}\right)=0$$

$$\therefore \frac{2\pi x}{\lambda}=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}\ldots=\left(n+\frac{1}{2}\right)\pi$$

$$Or\quad x=\left(n+\frac{1}{2}\right)\frac{\lambda}{2}$$



Where n is an integer.

Again there are points where the amplitude is maximum i.e., $\left|2a\cos\left(\frac{2\pi x}{\lambda}\right)\right|=2a$. Such points are called antinodes. This will be the case when

$$\cos\left(\frac{2\pi x}{\lambda}\right)=1$$

$$\therefore \frac{2\pi x}{\lambda}=0,\pi,2\pi,\ldots=n\pi$$

$$Or\quad x=n\left(\frac{\lambda}{2}\right)$$

The distance between any two successive antinodes or nodes is equal to $\frac{\lambda}{2}$, and the distance between an antinode and a node is $\frac{\lambda}{4}$.

Vibration of a string fixed at both ends: Suppose a string of length L is kept fixed at the ends $x = 0$ and $x = L$

Waves going to the positive direction of X interfere to give a resultant waves

$$y_1 = a\sin(kx - \omega t)$$

Waves going to the negative direction of X interfere to give a resultant waves

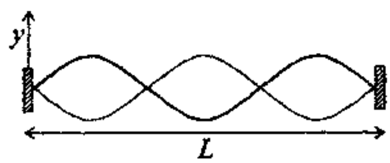
$$y_2 = a\sin(kx + \omega t)$$

The resultant displacement of the particle is

$$y = y_1 + y_2 = a\sin(kx - \omega t) + a\sin(kx + \omega t)$$

$$y = a\left[\sin(kx - \omega t) + \sin(kx + \omega t)\right]$$

$$y = 2a\sin kx\cos\omega t$$



$$\text{Or}\quad y = 2a\sin\left(\frac{2\pi}{\lambda}x\right)\cos\omega t$$

The boundary conditions are

(i) at $x = 0,\ \ y = 0$

(ii) at $x = L,\ \ y = 0$

$$\therefore 0 = 2a\sin\left(\frac{2\pi}{\lambda}L\right)\cos\omega t$$

$$\text{Or}\quad \sin\frac{2\pi}{\lambda}L = 0$$

$$\text{Or}\quad \frac{2\pi}{\lambda}L = n\pi$$

$$\text{Or}\quad L = \frac{n\lambda}{2}$$

First mode of vibration: The first mode of vibration occurs for $n = 1$, the corresponding wavelength λ_1 and frequency f_1 are given by

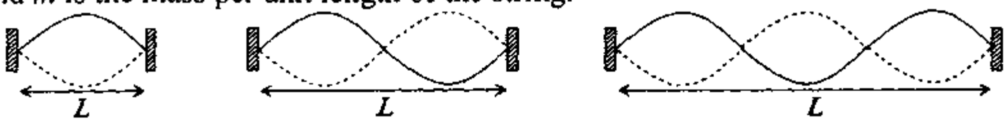
$$1\times\left(\frac{\lambda_1}{2}\right) = L \qquad\qquad \Rightarrow \lambda_1 = 2L$$

$$f_1 = \frac{v_1}{\lambda_1} = \frac{v}{\lambda_1}$$

Note that the velocity v is the same for all wavelength.

$$\because v = \sqrt{\frac{T}{m}} \qquad\qquad \Rightarrow f_1 = \frac{v}{\lambda_1} = \frac{\sqrt{\frac{T}{m}}}{2L} = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

The frequency v_1 is called the fundamental frequency or first harmonic, T is the tension in th string and m is the mass per unit length of the string.



Second mode of vibration: The second mode of vibration occurs for $n = 2$, the corresponding wavelength λ_2 and frequency f_2 are given by

$$2\times\left(\frac{\lambda_2}{2}\right)=L \qquad \Rightarrow \lambda_2 = L$$

$$f_2=\frac{v_2}{\lambda_2}=\frac{v}{\lambda_2}$$

The velocity v is the same for all wavelength.

$$\because v=\sqrt{\frac{T}{m}} \qquad \Rightarrow f_2=\frac{1}{L}\sqrt{\frac{T}{m}}=\frac{2}{2L}\sqrt{\frac{T}{m}}$$

$$\therefore f_2 = 2f_1$$

The frequency f_2 is called the first overtone or second harmonic.

Third mode of vibration: The third mode of vibration occurs for $n = 3$, the corresponding wavelength λ_3 and frequency f_3 are given by

$$3\times\left(\frac{\lambda_3}{2}\right)=L \qquad \Rightarrow \lambda_3=\frac{2L}{3}$$

$$f_3=\frac{v_3}{\lambda_3}=\frac{v}{\lambda_3}$$

The velocity v is the same for all wavelength.

$$\because v=\sqrt{\frac{T}{m}} \qquad \Rightarrow f_3=\frac{3}{2L}\sqrt{\frac{T}{m}}$$

$$\therefore f_3 = 3f_1$$

The frequency f_3 is called the second overtone or third harmonic.
The frequency of n^{th} harmonic is thus $f_n = nf_1$

Position of nodes:
In the first mode of vibration, the number of nodes = $0, L$

In the second mode of vibration, the number of nodes = $0, \frac{L}{2}, \frac{2L}{2}$

In the third mode of vibration, the number of nodes = $0, \frac{L}{3}, \frac{2L}{3}, \frac{3L}{3}$

In the n^{th} mode of vibration, the number of nodes $= 0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n} \dots \frac{nL}{n}$

Position of antinodes:

In the first mode of vibration, the number of antinodes = $\frac{L}{2}$

In the second mode of vibration, the number of antinodes = $\frac{L}{4}, \frac{3L}{4}$

In the third mode of vibration, the number of antinodes = $\frac{L}{6}, \frac{3L}{6}, \frac{5L}{6}$

In the n^{th} mode of vibration, the number of nodes $= \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \frac{7L}{2n} \dots \frac{(2n-1)L}{2n}$

Characteristics of stationary waves:

(1) The waveform remains stationary.
(2) Nodes and antinodes are formed alternately.
(3) The points where displacement is zero are called nodes and the points where the displacement is maximum are called antinodes.
(4) Pressure changes are maximum at nodes and minimum at antinodes.
(5) All the particles except those at the nodes, execute simple harmonic motions of same period.
(6) Amplitude of each particle is not the same, it is maximum at antinodes decreases gradually and is zero at the nodes.
(7) The velocity of the particles at the nodes is zero. It increases gradually and is maximum a the antinodes.
(8) There is no transfer of energy. All the particles of the medium pass through their mean position simultaneously twice during each vibration.
(9) Particles in the same segment vibrate in the same phase and between the neighbouring segments, the particles vibrate in opposite phase.

Standing waves in a pipe closed at one end:

Modes of vibration: If the reference origin is taken at a node, the equation of the stationary wave is given by

$$y = 2a\sin kx\cos\omega t$$

Or $$y = 2a\sin\left(\frac{2\pi}{\lambda}x\right)\cos\left(\frac{2\pi}{T}t\right)$$

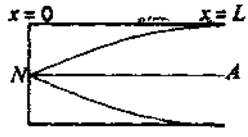
At the closed end of the pipe, i.e., at $x = 0$, $y = 0$, $\sin kx = 0$, a node is formed.
At an open end of the pipe of length L, y is maximum, $\sin kx$ is maximum, (i.e., $\sin kx = 1$), an antinode is formed.

$$\therefore \sin kL = 1$$

Or $$\frac{2\pi}{\lambda}L = (2n-1)\frac{\pi}{2}$$

Or $$\lambda = \frac{4L}{(2n-1)}$$

First mode of vibration: The first mode of vibration occurs for $n = 1$ and the corresponding wavelength λ_1 and frequency f_1 are given by

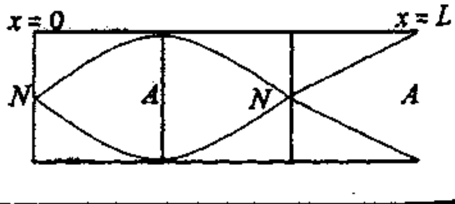


$$\lambda_1 = \frac{4L}{(2\times1-1)} = 4L$$

$$f_1 = \frac{v_1}{\lambda_1} = \frac{v}{\lambda_1} = \frac{v}{4L}$$

Where v is the velocity of sound in air.
The frequency f_1 is called the fundamental frequency or first harmonic.

Second mode of vibration: The second mode of vibration occurs for $n = 2$ and the corresponding wavelength λ_2 and frequency f_2 are given by



$$\lambda_2 = \frac{4L}{(2\times2-1)} = \frac{4L}{3}$$

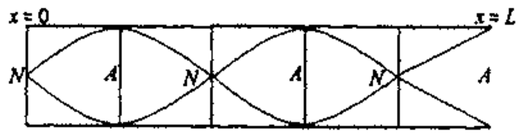
$$f_2 = \frac{v_2}{\lambda_2} = \frac{v}{\lambda_2} = \frac{3v}{4L}$$

$$\text{Or } f_2 = 3f_1$$

Where v is the velocity of sound in air.
The frequency f_2 is called the first overtone. Its frequency is three times the fundamental frequency. Therefore it is called third harmonic.

Third mode of vibration: The third mode of vibration occurs for $n = 3$ and the corresponding wavelength λ_3 and frequency f_3 are given by

$$\lambda_3 = \frac{4L}{(2\times3-1)} = \frac{4L}{5}$$



$$f_3 = \frac{v_3}{\lambda_3} = \frac{v}{\lambda_3} = \frac{5v}{4L}$$

$$\text{Or } f_3 = 5f_1$$

Where v is the velocity of sound in air.
The frequency f_3 is called the second overtone. Its frequency is five times the fundamental frequency. Therefore it is called fifth harmonic.
The frequency of n^{th} harmonic is $f_n = (2n-1)f_1$

Standing waves in an open pipe:

Modes of vibration: If the reference origin is taken at an antinode, the equation of the stationary wave is given by

$$y = 2a\cos kx \sin \omega t$$

$$\text{Or } y = 2a\cos\left(\frac{2\pi}{\lambda}x\right)\sin\left(\frac{2\pi}{T}t\right)$$

The amplitude $2a\cos kx$ must be maximum (i.e., $\cos kx = 1$) at the ends, because antinodes are formed there.

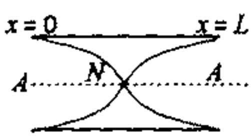
$$\cos kL = 1$$

$$\text{Or } \frac{2\pi}{\lambda}L = n\pi$$

$$\text{Or } \lambda = \frac{2L}{n}$$

First mode of vibration: The first mode of vibration occurs for $n = 1$ and the corresponding wavelength λ_1 and frequency f_1 are given by

$$\lambda_1 = \frac{2L}{1} = 2L$$

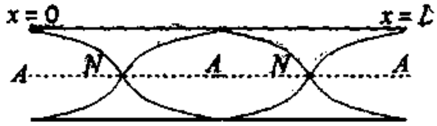


$$f_1 = \frac{v_1}{\lambda_1} = \frac{v}{\lambda_1} = \frac{v}{2L}$$

Where v is the velocity of sound in air.
The frequency f_1 is called the fundamental frequency or first harmonic.

Second mode of vibration: The second mode of vibration occurs for $n = 2$ and the corresponding wavelength λ_2 and frequency f_2 are given by

$$\lambda_2 = \frac{2L}{2} = L$$



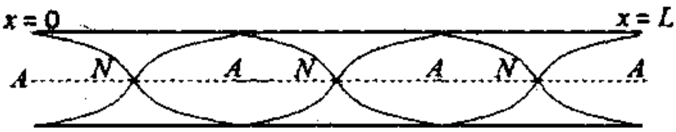
$$f_2 = \frac{v_2}{\lambda_2} = \frac{v}{\lambda_2} = \frac{v}{L}$$

$$\text{Or } f_2 = 2\left(\frac{v}{2L}\right) = 2f_1$$

Where v is the velocity of sound in air.
The frequency f_2 is called the first overtone. Its frequency is twice the fundamental frequenc
Therefore it is called the second harmonic.

Third mode of vibration: The third mode of vibration occurs for $n = 3$ and the corresponding wavelength λ_3 and frequency f_3 are given by

$$\lambda_3 = \frac{2L}{3}$$



$$f_3 = \frac{v_3}{\lambda_3} = \frac{v}{\lambda_3} = \frac{3v}{2L}$$

$$\text{Or } f_3 = 3\left(\frac{v}{2L}\right) = 3f_1$$

Where v is the velocity of sound in air.
The frequency f_3 is called the second overtone. Its frequency is three times the fundamental frequency. Therefore it is called the third harmonic.
The frequency of n^{th} harmonic is $f_n = n\, f_1$

Closed and open pipes compared:

One end open, one closed:

$\lambda_n = \dfrac{4L}{(2n-1)}$ where $n = 1,2,3,...$ Or $\lambda_n = \dfrac{4L}{n}$ where n is odd integer

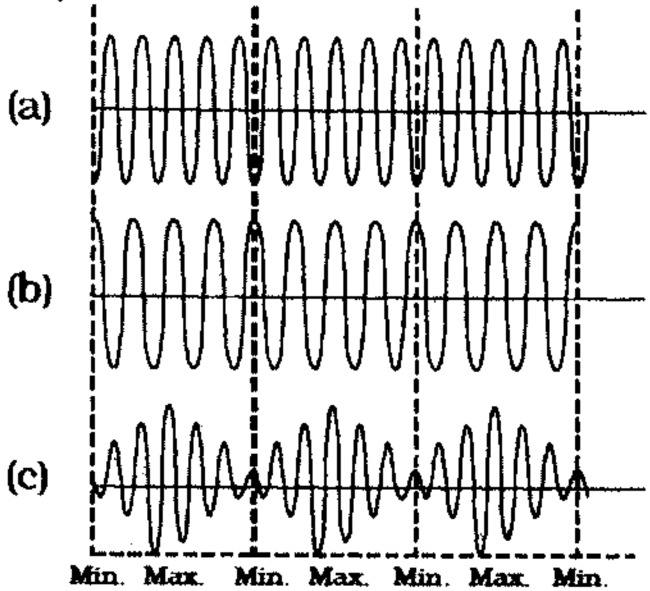
$f_n = (2n-1)\left(\dfrac{v}{4L}\right)$ where $n = 1,2,3,...$ Or $f_n = \dfrac{nv}{4L}$ where n is odd integer

Both ends open:

$\lambda_n = \dfrac{2L}{n}$ where $n = 1,2,3,...$

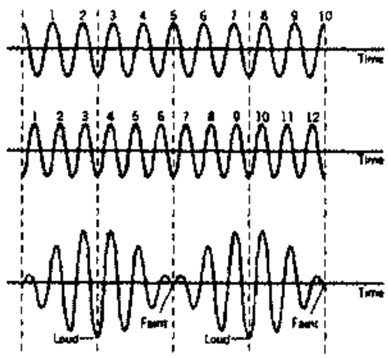
$f_n = \dfrac{nv}{2L}$ where $n = 1,2,3,...$

Beats: The phenomenon of waxing and waning of sound due to interference of two sound waves of nearly equal frequencies are called beats.



Beat frequency: The frequency of the beats is the number of intense sounds heard per second and it is the difference between the two sound frequencies. For example, when two tuning forks of frequencies $f_1 = 12Hz$ and $f_2 = 10Hz$ are sounded together, then

Beat frequency $f_b = f_1 - f_2 = (12-10)\,Hz = 2$ beats/s



Mathematical treatment of beats: Let us consider two waves of slightly different frequencies f_1 and f_2 ($f_1 \sim f_2 < 10$) having equal amplitude travelling in a medium in the same direction.

At time $t = 0$, both waves travel in same phase.

The equations of the two waves are

$$y_1 = a\sin\omega_1 t = a\sin 2\pi\, f_1 t$$

$$y_2 = a\sin\omega_2 t = a\sin 2\pi\, f_2 t$$

When the two waves superimpose, the resultant displacement is given by

$$y = y_1 + y_2$$

$$y = a\sin 2\pi\, f_1 t + a\sin 2\pi\, f_2 t = a\,(\sin 2\pi\, f_1 t + \sin 2\pi\, f_2 t)$$

$$y = 2a\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\,\sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

Substituting $2a\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t = A$, and $\frac{f_1 + f_2}{2} = f$ we have

$$y = A\sin 2\pi\, f\, t$$

This represents a simple harmonic wave of frequency f and amplitude A which changes with time.

Uses of beats:

(1) The phenomenon of beats is useful in tuning two vibrating bodies in unison. For example, a sonometer wire can be tuned in unison with a tuning fork by observing the beats. When an

excited tuning fork is kept on the sonometer and if the sonometer wire is also excited, beats are heard, when the frequencies are nearly equal. If the length of the wire is adjusted carefull so that the number of beats gradually decreases to zero, then the two are said to be in unison Most of the musical instruments are made to be in unison based on this method.

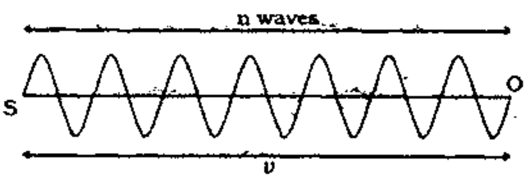
(2) The frequency of a tuning fork can be found using beats. A standard tuning fork of frequency N is excited along with the experimental fork. If the number of beats per second is n, then the frequency of experimental tuning fork is $N + n$. The experimental tuning fork is then loaded with a little bees' wax, thereby decreasing its frequency. Now the observations are repeated. If the number of beats increases, then the frequency of the experimental tuning fork is $N - n$, and if the number of beats decreases its frequency is $N + n$.

Doppler effect: The phenomenon of the apparent change in the frequency of sound due to the relative motion between the source of sound and the observer is called Doppler effect. The apparent frequency due to Doppler effect for different cases can be deduced as follows.

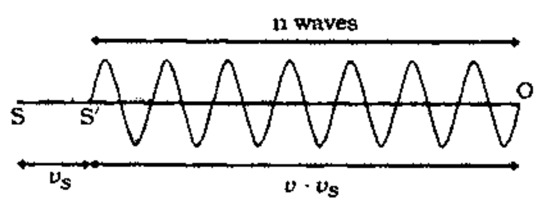
(1) Both source and observer at rest: Suppose S and O are the positions of the source and the observer respectively.
If in one second, n waves produced by the source travel a distance SO, then the frequency is n, and if v is the speed of the wave, then SO is equal to v.

$$\text{wavelength} = \frac{\text{speed of the wave}}{\text{frequency}}$$

$$Or\ \lambda = \frac{v}{n}$$



(2) When the source moves towards the stationary observer: If the source moves with a velocity v_s towards the stationary observer, then after one second, the source will reach S′, such that SS′ = v_s. Now n waves emitted by the source will occupy a distance of $(v - v_s)$ only as shown in the figure. Therefore the apparent wavelength of the sound is



$$\lambda' = \frac{v - v_s}{n}$$

The apparent frequency is

$$n' = \frac{v}{\lambda'} = v\left(\frac{n}{v - v_s}\right) = \left(\frac{v}{v - v_s}\right)n$$

As $n' > n$, the pitch of the sound appears to increase.

(3) When the source moves away from the stationary observer: If the source moves awa from the stationary observer with velocity v_s, the apparent frequency will be given by

$$n' = \frac{v}{\lambda'} = v\left(\frac{n}{v + v_s}\right) = \left(\frac{v}{v + v_s}\right)n$$

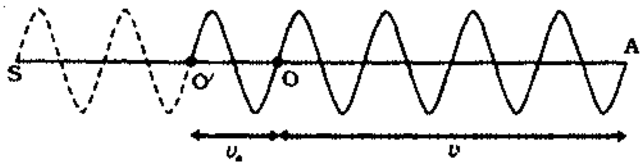
As $n' < n$, the pitch of the sound appears to decrease.

(4) When the observer moves towards the stationary source: Suppose the observer is moving towards the stationary source with velocity v_o. After one second the observer will reach the point O′ such that OO′ = v_o. The number of waves crossing the observer will be n

waves in the distance OA in addition to the number of waves in the distance OO′ which is equal to v_o/λ as shown in the figure. Therefore, the apparent frequency of sound is

$$n' = n + \frac{v_o}{\lambda} = n + v_o\frac{n}{v} = n\left(1 + \frac{v_o}{v}\right)$$

$$\therefore n' = \left(\frac{v + v_o}{v}\right)n$$



As $n' > n$, the pitch of the sound appears to increase.

(5) When the observer moves away from the stationary source: The apparent frequency of sound is

$$n' = n - \frac{v_o}{\lambda} = n - v_o\frac{n}{v} = n\left(1 - \frac{v_o}{v}\right)$$

$$\therefore n' = \left(\frac{v - v_o}{v}\right)n$$

As $n' < n$, the pitch of sound appears to decrease.

Note : If the source and the observer move along the same direction, the equation for apparent frequency is

$$n' = \left(\frac{v - v_o}{v - v_s}\right)n$$

ELECTROSTATICS

Frictional Electricity: The property of rubbed substances due to which they attract light objects is called Frictional Electricity or static Electricity

The rubbed substances which show this property of attraction are said to have become electrified or electrically charged.

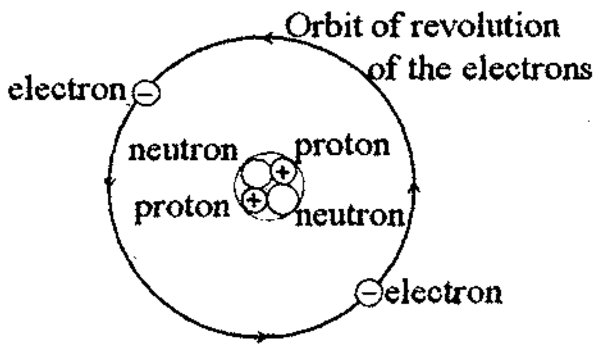
Note:

(1) Every object contains equal amount of the two kinds of charges, positive charge and negative charge. With such an equality or balance of charges, the object is said to be electrically neutral or neutral body.

(2) An object is said to be charged, when it has a charge imbalance i.e., the number of positive and negative charges are not equal.

Electric charge: Electric charge is an intrinsic property of elementary particles (electrons, protons, etc.,) which give rise to electric force between various objects.

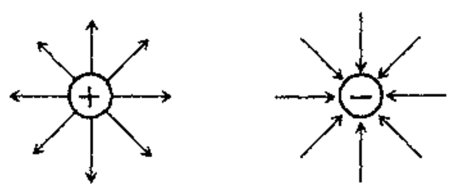
The SI unit of electric charge is coulomb (C) and it is a scalar quantity.



Note:

(1) Electron always has a negative charge ($-e$)
(2) Neutron has no charge. It is a neutral particle.
(3) Proton always has a positive charge ($+e$)
An electric charge is represented by a symbol e, whose value is $\pm 1.6 \times 10^{-19}$ coulomb (C)
(4) The symbol of electron is not e or $-e$. Instead it is e^{-}.

Two kinds of electric charges: There are only two kind of electric charges. The positive charge ($+e$) and the negative charge ($-e$).



Fundamental law of Electrostatics: Like charges repel and unlike charges attract each other.

Positive and negative charges:

(1) If a glass rod is rubbed with a silk cloth, it acquires positive charge while the silk cloth acquires an equal amount of negative charge.
(2) If an ebonite rod is rubbed with fur, it becomes negatively charged, while the fur acquire equal amount of positive charge.

Or

(1) The charge developed on a glass rod when robbed with silk is called positive charge.
(2) The charge developed on an ebonite rod when robbed with fur is called negative charge.

Electrostatics: The branch of physics that deals with the study of the charges at rest is known as electrostatics.

Two kinds of charges developed on rubbing

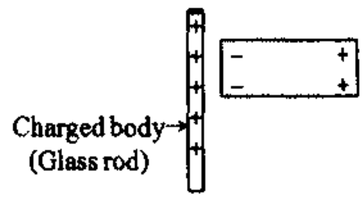
Positive Charge	Negative Charge
Glass rod	Silk cloth
Flannel or cat skin	Ebonite rod
Woolen cloth	Amber rod
Woolen coat	Plastic seat
Woolen carpet	Rubber shoes

Conductors and Insulators:

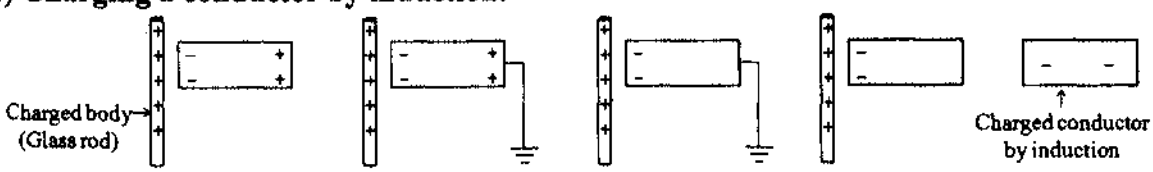
Conductors: Substances through which electric charges can flow easily are called conductors. Metals, human and animal bodies are conductors.

Insulators: Substances through which electric charges cannot flow easily are called insulators. Glass, diamond, porcelain, plastic, nylon, wood, mica etc are insulators.

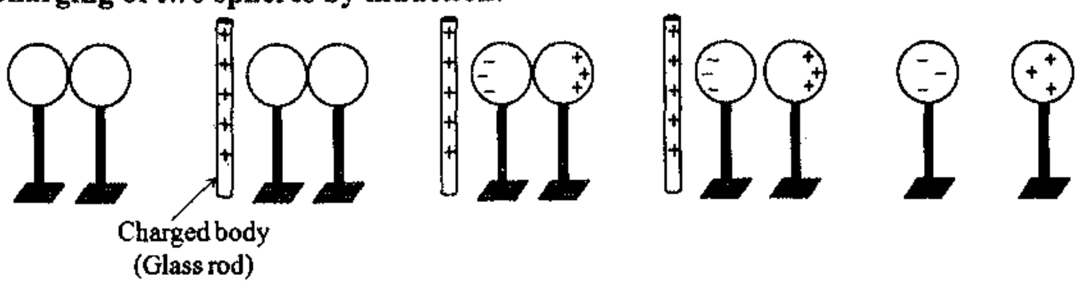
Electrostatic Induction: Electrostatic Induction is the phenomenon of temporary electrification of a conductor in which opposite charges appears at its closer end and similar charges appear at its farther end in the presence of a nearby charged body



(1) Charging a conductor by induction:



(2) Charging of two spheres by induction:



Basic properties of electric charge: Electric charge has the following three basic properties

(1) Additivity

(2) Quantisation

(3) Conservation

(1) Additivity: Additivity of electric charge means that the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system

If a system contain charges $q_1, q_2, q_3, \ldots q_n$

Then its total charge is $Q = q_1 + q_2 + q_3 + \ldots + q_n$

(2) Quantisation: The quantisation of electric charge means that the total charge (q) of a body is always an integral multiple of a basic quantum of charge (e) i.e.,

$$q = ne, \qquad \text{where } n = 0, \pm1, \pm2, \pm3, \ldots$$

Note:

(1) If $n = 0$, the body is a neutral body. If $n < 0$, the body is negatively charged. If $n > 0$, the body is positively charged.

(2) The smallest value of electric charge is $e = 1.6 \times 10^{-19}$ C

(3) The quantisation of charge shows that charge is discrete and not of continuous nature.

(3) Conservation:

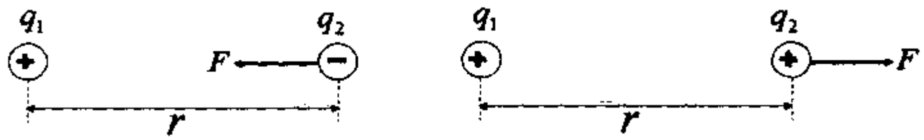
(a) The total charge of an isolated system remains constant.

(b) The electric charges can neither be created nor destroyed, they can only be transferred from one body to another.

Coulomb's law of electric force: Coulomb's law states that the force of attraction or repulsion between two stationary point charges is

(1) Directly proportional to the product of the magnitudes of the two charges and

(2) Inversely proportional to the square of the distance between them. This force acts along the line joining the two charges



If two point charges q_1 and q_2 are separated by a distance r, then the force F of attraction or repulsion between them is

$$F \propto q_1 q_2 \qquad \text{and} \qquad F \propto \frac{1}{r^2}$$

$$\therefore F \propto \frac{q_1 q_2}{r^2} \qquad \text{or} \qquad F = k\frac{q_1 q_2}{r^2}$$

where k is a constant of proportionality, called electro-static force constant.

For the two charges located in free space and in SI unit, we have $F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$

where $k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9\, Nm^2C^{-2}$ and $\varepsilon_o = 8.854 \times 10^{-12}\, C^2N^{-1}m^{-2}\,(Fm^{-1})$ is called permittivity of free space.

Dimensions of ε_o is $[\varepsilon_o] = \frac{[AT][AT]}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]$

Note:

(1) For the two charges located in free space and in cgs unit, we have $F = \frac{q_1 q_2}{r^2}$

where $k = 1\, dyne\, cm^2 statC^{-2}$

(2) In electrostatic cgs system, the unit of charge is known as electrostatic unit of charge (e.s.u of charge) or statcoulomb (stat C)

$$1\,C = 3 \times 10^9 \text{statC}$$

(3) In electromagnetic cgs system, the unit of charge is known as electromagnetic unit of charge (e.m.u of charge) or abcoulomb.

$$1\,C = 0.1 \text{ e.m.u}$$

(4) Coulomb's law of electrostatics is valid only if $r > 10^{-15} m$ (the size of nucleus)
(5) Strictly speaking Coulomb's law applies only to point charges.
(6) If $q_1 q_2 > 0$ the force is positive and repulsive.
(7) If $q_1 q_2 < 0$ the force is negative and attractive.
(8) The permittivity of free space (ε_o) is determined experimentally.

Permittivity (ε_o): Permittivity is a measure of how an electric field affects the medium and how it is affected by a medium.

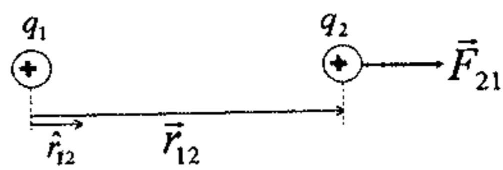
Definition of 1C from Coulomb's Law: 1C is that charge which when placed 1m from an equal and similar charge in vacuum (or air) repels it with a force of $9 \times 10^9 N$

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} = 9\times10^9\, Nm^2C^{-2}\,\frac{(1C)(1C)}{(1m)^2} = 9\times10^9\, N$$

Note: 1coulomb is very large unit of charge. So charges are produced experimentally in the range between pico-coulomb (pC) and micro-coulomb (μC)

$$1pC = 10^{-12}C \qquad \text{and} \qquad 1\mu C = 10^{-6}C$$

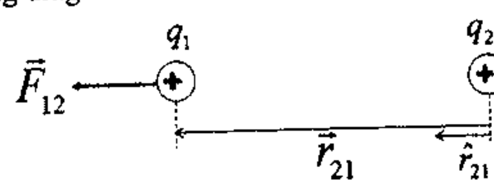
Coulomb's law in vector form: As shown in the figure, consider two positive point charges q_1 and q_2 place in vacuum at a distance r from each other.



In vector form, Coulomb's law may be express as

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_{12}|^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{(\vec{r}_{12})^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Here we see that $\vec{F}_{21}$ and $\hat{r}_{12}$ are in the same direction.
Now consider the following diagram



Coulomb's law may be express as

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_{21}|^2}\hat{r}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{(\vec{r}_{21})^2}\hat{r}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

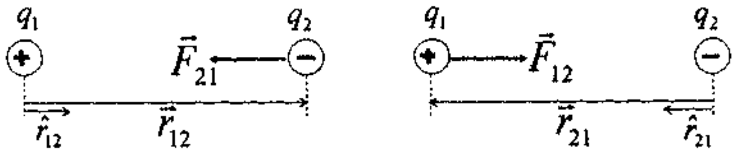
Here we see that $\vec{F}_{12}$ and $\hat{r}_{21}$ are in the same direction.
But the unit vectors are opposite to each other i.e., $\hat{r}_{12} = -\hat{r}_{21}$

$$\therefore \vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}(-\hat{r}_{21}) = -\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}_{21} = -\vec{F}_{12}$$

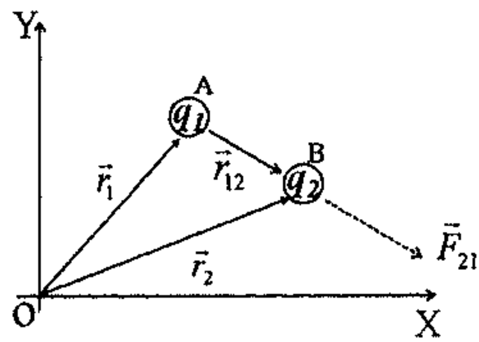
Note:
(1) $\vec{F}_{21}$ is the force on q_2 due to q_1 and $\vec{F}_{12}$ is the force on q_1 due to q_2.

(2) The two charges exert equal and opposite force on each other, so Coulomb's electrostatic force obey Newton's third law of motion.
(3) As the electrostatic force act along the line joining the centres of two charges, so it is a central force.
(4) Electrostatic force is a conservative force i.e., work done by the electrostatic force over a closed path is zero
(5) Electrostatic force acts over an enormous range of separation (r), from $r = 10^{-15}\,m$ to $r = 10^{8}\,m$ to a high degree of accuracy.

Do it yourself: As shown in the diagram, show that $\vec{F}_{21} = -\vec{F}_{12}$



Forces between two charges in terms of their position vectors: Let two point charges q_1 and q_2 be located at points A and B in vacuum or free space as shown in the figure respectively. Let $\vec{r}_1$ and $\vec{r}_2$ are position vectors of points A and B, where O is the origin of the Cartesian coordinate system XY.



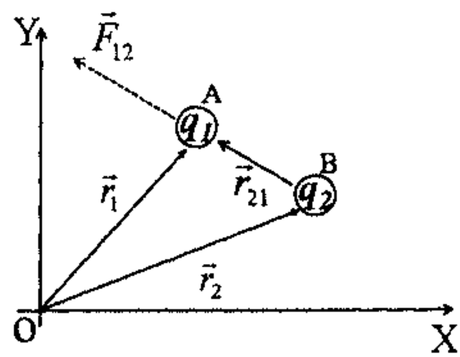
Using triangular law of vectors we have

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

Coulomb's law may be express as

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_{12}|^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_{12}|^3}\vec{r}_{12}$$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$



Again, in the opposite diagram we have

$$\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$$

Coulomb's law may be express as

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_{21}|^2}\hat{r}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

So we have

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}\{-(\vec{r}_1 - \vec{r}_2)\} = -\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) = -\vec{F}_{12}$$

Note:

(1) $\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{\vec{r}_{12}}{r_{12}} = \frac{\vec{r}_{12}}{r}$ $\quad\Rightarrow \vec{r}_{12} = |\vec{r}_{12}|\hat{r}_{12} = r_{12}\hat{r}_{12} = r\,\hat{r}_{12}$

(2) $\hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|} = \frac{\vec{r}_{21}}{r_{21}} = \frac{\vec{r}_{21}}{r}$ $\Rightarrow \vec{r}_{21} = |\vec{r}_{21}|\,\hat{r}_{21} = r_{21}\hat{r}_{21} = r\,\hat{r}_{21}$

(3) $|\vec{r}_1 - \vec{r}_2|^3 = |\vec{r}_2 - \vec{r}_1|^3$

(4) $|\vec{r}_{12}| \neq (\vec{r}_{12})$. $|\vec{r}_{12}|$ is the magnitude of $\vec{r}_{12}$. $(\vec{r}_{12})$ is a vector quantity. But $|\vec{r}_{12}|^2 = (\vec{r}_{12})^2$

Dielectric constant (κ) or Relative permittivity (ε_r): Dielectric constant (κ) or Relative permittivity (ε_r) may be defined as the ratio of the force between two charges placed some distance apart in free space to the force between the same two charges when they are placed the same distance apart in the given medium.

Electrostatic force of q_1 and q_2 in vacuum or air at a distance r from each other is

$$F_{air} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

Electrostatic force of q_1 and q_2 in a medium (water) at a distance r from each other is

$$F_{water} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1 q_2}{r^2}$$

$$\therefore \frac{F_{air}}{F_{water}} = \frac{\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1 q_2}{r^2}}$$

Or $\frac{F_{air}}{F_{water}} = \varepsilon_r$ (relative permittivity)

Note: $\frac{F_{air}}{F_{water}} = \varepsilon_r$ Or $F_{air} = \varepsilon_r \cdot F_{water}$

We see that $F_{air} > F_{water}$

In vacuum $\varepsilon_r = 1$

In air $\varepsilon_r = 1.00054 \approx 1$

In water $\varepsilon_r = 80$

Relation between (ε_o) and (ε_r):

Electrostatic force of q_1 and q_2 in vacuum or air at a distance r from each other is

$$F_{air} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

Electrostatic force of q_1 and q_2 in a medium (water) at a distance r from each other is

$$F_{water} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$$

$$\therefore \frac{F_{air}}{F_{water}} = \frac{\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}} = \frac{\varepsilon}{\varepsilon_o}$$

But $\frac{F_{air}}{F_{water}} = \varepsilon_r$ (relative permittivity)

$$\therefore \frac{\varepsilon}{\varepsilon_o} = \varepsilon_r \qquad \Rightarrow \varepsilon = \varepsilon_o \varepsilon_r$$

ε is called the absolute permittivity

ε_o is called the permittivity

ε_r is called the relative permittivity

Note: In air or vacuum $\varepsilon = \varepsilon_o$. So ε_o is also known as absolute permittivity.

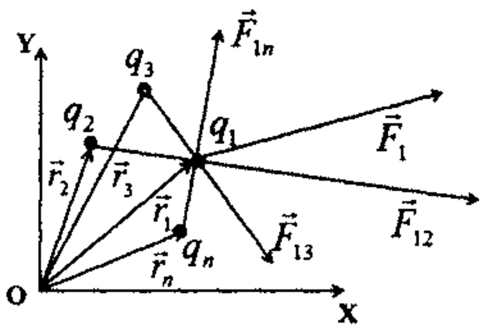
Comparing Electrostatic and Gravitational forces

Newton's law of Gravitation	Coulomb's law
Only attractive force	Attractive or repulsive force
Force due to mass interaction	Force due to charge interaction
The force is a long–range force.	The force is a short–range force.
The equation of the gravitational force is $F_g = G\frac{m_1 m_2}{r^2}$	The equation of the electrostatics force is $F_e = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$

Forces between multiple charges – The superposition principle:

The principle of superposition is to calculate the electric force experienced by a charge q_1 due to other charges $q_2, q_3, ... q_n$.

The total force on a given charge (q_1) is the vector sum of the forces exerted on it due to all other charges. Thus



$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} ... + \vec{F}_{1n}$$

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + ... \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{n1}^2}\hat{r}_{n1}$$

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{21}^3}\vec{r}_{21} + \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_3}{r_{31}^3}\vec{r}_{31} + ... \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{n1}^3}\vec{r}_{n1}$$

$$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_o}\left[\frac{q_2}{r_{21}^3}\vec{r}_{21} + \frac{q_3}{r_{31}^3}\vec{r}_{31} + ... \frac{q_2}{r_{n1}^3}\vec{r}_{n1}\right]$$

$$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_o}\sum_{i=2}^{i=n}\frac{q_i}{r_{i1}^3}\vec{r}_{i1}$$

In terms of position vectors,

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) + \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_3}{|\vec{r}_1 - \vec{r}_3|^3}(\vec{r}_1 - \vec{r}_3) + ... \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_n}{|\vec{r}_1 - \vec{r}_n|^3}(\vec{r}_1 - \vec{r}_n)$$

$$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_o}\left[\frac{q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) + \frac{q_3}{|\vec{r}_1 - \vec{r}_3|^3}(\vec{r}_1 - \vec{r}_3) + ... \frac{q_n}{|\vec{r}_1 - \vec{r}_n|^3}(\vec{r}_1 - \vec{r}_n)\right]$$

$$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_o}\sum_{i=2}^{i=n}\frac{q_i}{|\vec{r}_1 - \vec{r}_i|^3}(\vec{r}_1 - \vec{r}_i)$$

Electrostatic force due to continuous charge distribution:

(1) Linear charge distribution: When the charges are distributed linearly, the total charge dq over a small length dl is given by

$$dq = \lambda\, dl$$

where λ is the linear charge density, i.e., the charge per unit length.

The force on a charge q_o due to a charge element dq where r is the distance between them is

$$dF = \frac{1}{4\pi\varepsilon_o}\frac{q_o dq}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_o \lambda\, dl}{r^2}$$

Therefore total force on q_o is $F = \frac{q_o}{4\pi\varepsilon_o}\int_{l=o}^{l=L}\frac{\lambda\, dl}{r^2}$

(2) Surface charge distribution: When the charges are distributed uniformly over a surface, the total charge dq over a small surface ds is given by

$$dq = \sigma\, ds$$

where σ is the surface charge density, i.e., the charge per unit area.

The force on a charge q_o due to a charge element dq is

$$dF = \frac{1}{4\pi\varepsilon_o}\frac{q_o dq}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_o \sigma\, ds}{r^2}$$

Therefore total force on q_o is $F = \frac{q_o}{4\pi\varepsilon_o}\int_{s=o}^{s=S}\frac{\sigma\, ds}{r^2}$

(3) Volume charge distribution: When the charges are distributed uniformly over a volume, the total charge dq over a small volume dv is given by

$$dq = \rho\, dv$$

where ρ is the volume charge density, i.e., the charge per unit volume.

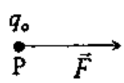
The force on a charge q_o due to a charge element dq is

$$dF = \frac{1}{4\pi\varepsilon_o}\frac{q_o dq}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_o \rho\, dv}{r^2}$$

Therefore total force on q_o is $F = \frac{q_o}{4\pi\varepsilon_o}\int_{v=o}^{v=V}\frac{\rho\, dv}{r^2}$

Electric field: The electric field $\vec{E}$ at a point is defined as the force experienced by a unit positive test charge placed at that point, without disturbing the position of source charge.

Suppose a test charge q_o experiences a force $\vec{F}$ at the point P.



Then the electric field $\vec{E}$ at that point is $\vec{E} = \frac{\vec{F}}{q_o}$

Or

The electric field $\vec{E}$ at a point is also defined as the electrostatic force per unit test charge acting on a vanishingly small positive test charge placed at that point. Hence

$$\vec{E} = \lim_{q_o \to 0} \frac{\vec{F}}{q_o}$$

The SI unit of electric field is NC^{-1}. It is equivalent to Vm^{-1}. It is a vector quantity.

The dimensions for $\vec{E}$ can be determined as follows

$$[E] = \frac{[F]}{[q_o]} = \frac{[MLT^{-2}]}{[AT]} = [MLT^{-3}A^{-1}]$$

Note:

(1) Electric field is also called electric field intensity or electric field strength.

(2) The electric field is defined more accurately as $q_o \to 0$. However, the minimum value of q_o is $1.6 \times 10^{-19} C$

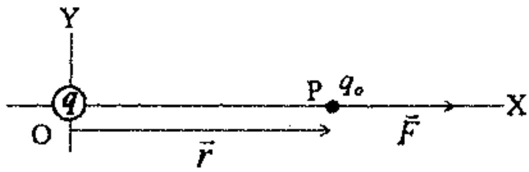
Electric field due to a point charge: Consider a point charge q placed at the origin O. We want to find the electric field at a point P at a distance r from O. According to Coulomb's law the force on charge q_o is

$$\vec{F} = \frac{1}{4\pi\varepsilon_o} \frac{qq_o}{\vec{r}^2} \hat{r} = \frac{1}{4\pi\varepsilon_o} \frac{qq_o}{r^2} \hat{r}$$

The electric field at the point P is

$$\vec{E} = \frac{\vec{F}}{q_o} = \frac{\frac{1}{4\pi\varepsilon_o} \frac{qq_o}{r^2} \hat{r}}{q_o}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_o} \frac{q}{r^2} \hat{r}$$



The magnitude of $\vec{E}$ is

$$|\vec{E}| = E = \frac{1}{4\pi\varepsilon_o} \frac{q}{r^2}$$

Note:

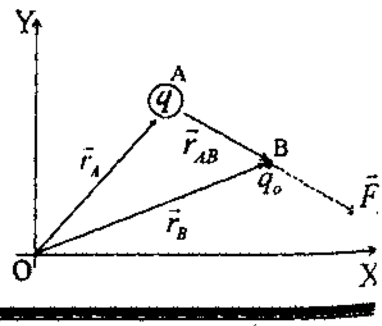
(1) Clearly $E \propto \frac{1}{r^2}$. This means that at all points on the spherical surface drawn around the point charge, the magnitude of $\vec{E}$ is same and does not depends on the direction of $\vec{r}$. Such a field is called spherically symmetric or radial field.

(2) The magnitude of $\vec{E}$ depends upon the magnitude of charge q which produces the electric field and not on the value of the test charge q_o.

Electric field due to a point charge (In terms of position vectors): Consider a point charge q located at A. It is desired to find the electric field at B due to charge q. As shown in the figure,

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$

According to Coulomb's law, the force on charge q_o is



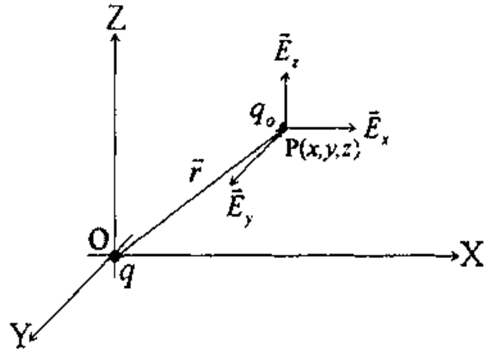
$$\vec{F}_{BA} = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{|\vec{r}_{AB}|^2}\hat{r}_{AB} = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{|\vec{r}_{AB}|^3}\vec{r}_{AB}$$

$$\vec{F}_{BA} = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{|\vec{r}_B - \vec{r}_A|^3}(\vec{r}_B - \vec{r}_A)$$

The electric field at the point B is $\vec{E}_{BA} = \dfrac{\vec{F}_{BA}}{q_o} = \dfrac{\frac{1}{4\pi\varepsilon_o}\frac{qq_o}{|\vec{r}_B - \vec{r}_A|^3}(\vec{r}_B - \vec{r}_A)}{q_o}$

Or $\vec{E}_{BA} = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q}{|\vec{r}_B - \vec{r}_A|^3}(\vec{r}_B - \vec{r}_A)$

Electric field due to a point charge (In terms of rectangular components): Consider a point charge q placed at O, the origin of the coordinate system as shown in the figure. We are to find the rectangular components of the electric field intensity $\vec{E}$ at any point P(x,y,z).



$$\vec{r} = \hat{i}\,x + \hat{j}\,y + \hat{k}\,z$$

$$\vec{r}\cdot\vec{r} = (\hat{i}\,x + \hat{j}\,y + \hat{k}\,z)\cdot(\hat{i}\,x + \hat{j}\,y + \hat{k}\,z)$$

$$(\vec{r})^2 = r^2 = x^2 + y^2 + z^2$$

$$r = \sqrt{x^2 + y^2 + z^2} = (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$r^3 = (x^2 + y^2 + z^2)^{\frac{3}{2}}$$

Electric field at P is

$$\vec{E} = \frac{\vec{F}}{q_o} = \frac{\frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}\hat{r}}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^3}\vec{r}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}(\hat{i}\,x + \hat{j}\,y + \hat{k}\,z)$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_o}\frac{(\hat{i}\,x + \hat{j}\,y + \hat{k}\,z)}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} = \frac{q}{4\pi\varepsilon_o}\left[\frac{\hat{i}\,x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{\hat{j}\,y}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{\hat{k}\,z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}\right]$$

$$\vec{E} = \hat{i}\frac{1}{4\pi\varepsilon_o}\frac{qx}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \hat{j}\frac{1}{4\pi\varepsilon_o}\frac{qy}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \hat{k}\frac{1}{4\pi\varepsilon_o}\frac{qz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \text{ -------- (1)}$$

If E_x, E_y and E_z are the components of $\vec{E}$ along the three co-ordinate axes, then

$$\vec{E} = \hat{i}\,E_x + \hat{j}\,E_y + \hat{k}\,E_z \text{ -------- (2)}$$

Comparing equation (1) and equation (2) we have

$$E_x = \frac{1}{4\pi\varepsilon_o}\frac{qx}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$E_y = \frac{1}{4\pi\varepsilon_o}\frac{qy}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$E_z = \frac{1}{4\pi\varepsilon_o}\frac{qz}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

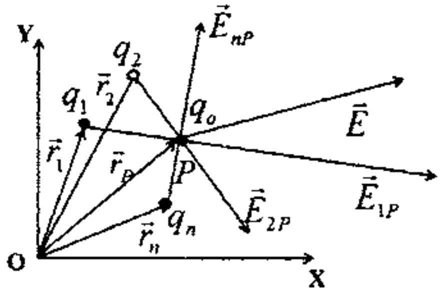
Or $\vec{E} = \vec{E}_x + \vec{E}_y + \vec{E}_z$

Electric field due to a system of point charges: The total Electric field at a point P due to all charges is.

$$\vec{E} = \vec{E}_{1P} + \vec{E}_{2P} \ldots + \vec{E}_{nP}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{(\vec{r}_{1P})^3}\vec{r}_{1P} + \frac{1}{4\pi\varepsilon_o}\frac{q_2}{(\vec{r}_{2P})^3}\vec{r}_{2P} + \ldots\frac{1}{4\pi\varepsilon_o}\frac{q_1}{(\vec{r}_{nP})^3}\vec{r}_{nP}$$

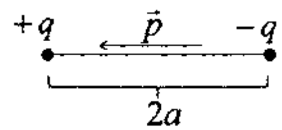
$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{i=n}\frac{q_i}{(\vec{r}_{iP})^3}\vec{r}_{iP}$$



In terms of position vectors

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{i=n}\frac{q_i}{|\vec{r}_P - \vec{r}_i|^3}(\vec{r}_P - \vec{r}_i)$$

Electric dipole: A pair of equal and opposite charges separated by a small distance is called an electric dipole. Examples are HCl, H_2O, etc

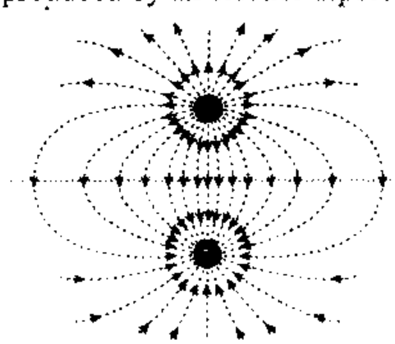


Electric dipole moment ($\vec{p}$): The dipole moment for a pair of opposite charges of magnitude 'q' is defined as the magnitude of the charge times the distance between them.

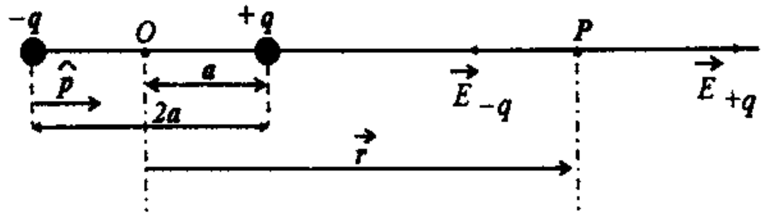
$$\vec{p} = q \times 2\vec{a}$$

Note: The direction of the electric dipole moment is toward the positive charge.

Dipole field: The electric field produced by an electric dipole is called a dipole field.



Electric field at an axial point of a dipole: Consider an electric dipole consisting of charg $-q$ and $+q$ separated by a small distance $2a$ in free space. Let P be a point on the axial line o a dipole at a distance r from the centre O of the dipole. We want to find the field intensity at P due to the dipole.



Electric field due to charge $-q$ at point P is

$$\vec{E}_{-q} = -\frac{1}{4\pi\varepsilon_o}\frac{q}{(r+a)^2}\hat{p}$$

Electric field due to charge $+q$ at point P is

$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-a)^2}\hat{p}$$

Hence the resultant electric field at point P is

$$\vec{E}_{axial} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+a)^2}\hat{p}$$

$$\vec{E}_{axial} = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$\vec{E}_{axial} = \frac{q}{4\pi\varepsilon_o}\left[\frac{(r+a)^2-(r-a)^2}{(r^2-a^2)^2}\right]\hat{p}$$

$$\vec{E}_{axial} = \frac{q}{4\pi\varepsilon_o}\left[\frac{r^2+a^2+2ar-r^2-a^2+2ar}{(r^2-a^2)^2}\right]\hat{p}$$

$$\vec{E}_{axial} = \frac{q}{4\pi\varepsilon_o}\frac{4ar}{(r^2-a^2)^2}\hat{p}$$

$$\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_o}\frac{2(q\times 2a)r}{(r^2-a^2)^2}\hat{p}$$

$$\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_o}\frac{2pr}{(r^2-a^2)^2}\hat{p}$$

Where $p = q\times 2a$ is the dipole moment

For $r >> a$, a^2 can be neglected compared to r^2

$$\therefore \vec{E}_{axial} = \frac{1}{4\pi\varepsilon_o}\frac{2pr}{r^4}\hat{p}$$

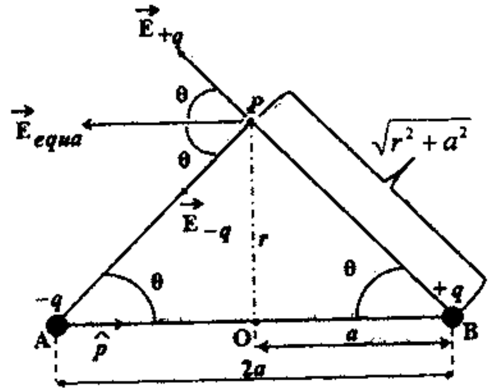
Or $$\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_o}\frac{2p}{r^3}\hat{p}$$

Note:

(1) The direction of the resultant field $\vec{E}_{axial}$ is along the dipole axis. i.e., along $\hat{p}$

(2) For a short dipole $\vec{E}_{axial} \propto \frac{1}{r^3}$

Electric field at an equatorial point of a dipole: Consider an electric dipole consisting of charges $-q$ and $+q$ separated by a small distance $2a$ in free space. Let P be a point on the equatorial line of a dipole at a distance r from the centre O of the dipole. We want to find the field intensity at P due to the dipole.



Electric field due to charge $-q$ at point P is

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(AP)^2} \text{ along PA}$$

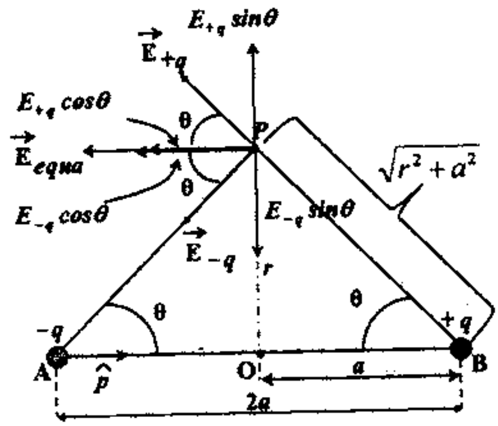
$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2+a^2} \text{ along PA}$$

Electric field due to charge $+q$ at point P is

$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(BP)^2} \text{ along BP}$$

$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2+a^2} \text{ along BP}$$

Clearly $|\vec{E}_{-q}| = |\vec{E}_{+q}| = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2+a^2}$



The components of $\vec{E}_{-q}$ are

(1) $\vec{E}_{-q}\sin\theta$ perpendicular to the dipole axis.

(2) $\vec{E}_{-q}\cos\theta$ parallel to the dipole axis.

The components of $\vec{E}_{+q}$ are

(1) $\vec{E}_{+q}\sin\theta$ perpendicular to the dipole axis.

(2) $\vec{E}_{+q}\cos\theta$ parallel to the dipole axis.

The components perpendicular to the dipole axis will cancel out, while the components parallel to the dipole axis add up. So the total electric field $\vec{E}_{equa}$ is opposite to $\vec{p}$.

$$\therefore \vec{E}_{equa} = -(E_{-q}\cos\theta + E_{+q}\cos\theta)\hat{p}$$

$$\vec{E}_{equa} = -2E_{-q}\cos\theta\,\hat{p}$$

$$\vec{E}_{equa} = -2\left(\frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+a^2)}\right)\frac{a}{\sqrt{r^2+a^2}}\,\hat{p}$$

$$\vec{E}_{equa} = -\frac{1}{4\pi\varepsilon_o}\frac{q\times 2a}{(r^2+a^2)^{\frac{3}{2}}}\,\hat{p}$$

$$\vec{E}_{equa} = -\frac{1}{4\pi\varepsilon_o}\frac{p}{(r^2+a^2)^{\frac{3}{2}}}\,\hat{p}$$

Where $p = q\times 2a$ is the dipole moment

For $r >> a$, a^2 can be neglected compared to r^2

$$\vec{E}_{equa} = -\frac{1}{4\pi\varepsilon_o}\frac{p}{r^3}\,\hat{p}$$

Note:

(1) The direction of the resultant field $\vec{E}_{equa}$ is opposite to the dipole moment.

(2) For a short dipole $\vec{E}_{equa} \propto \frac{1}{r^3}$ and also $\vec{E}_{axial} \propto \frac{1}{r^3}$

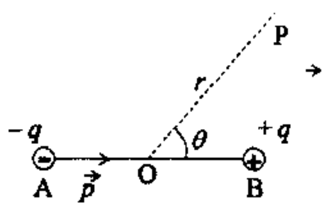
(3) For a single charge $\vec{E} \propto \frac{1}{r^2}$

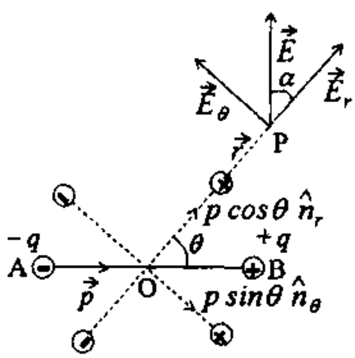
(4) For a linearly charged wire $\vec{E} \propto \frac{1}{r}$.

(5) For a uniformly charged plane sheet $\vec{E}$ does not depend on r.

(6) $|\vec{E}_{axial}| = 2|\vec{E}_{equa}|$

Electric field at any general point (r,θ): Consider a point P at a distance $\vec{r}$ from the mid point O of a small dipole AB and OP makes an angle θ with the direction of the dipole moment $\vec{p}$ as shown in the figure. To calculate the electric field at point P due to a dipole, we can resolve the dipole moment $\vec{p}$ in to two components, $p\cos\theta\,\hat{n}_r$ along $\vec{r}$ and $p\cos\theta\,\hat{n}_\theta$ perpendicular to $\vec{r}$.





The electric field at P due to a dipole of dipole moment $p\cos\theta\,\hat{n}_r$ is

$$\vec{E}_r = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}\hat{n}_r$$

$$\text{Or}\quad E_r = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

The electric field at P due to a dipole of dipole moment $p\cos\theta\,\hat{n}_\theta$ is

$$\vec{E}_\theta = -\frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}\hat{n}_\theta$$

$$\text{Or}\quad E_\theta = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

The resultant electric field at P is

$$E = \sqrt{(E_r)^2 + (E_\theta)^2} = \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}\right)^2 + \left(\frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}\right)^2}$$

$$E=\frac{1}{4\pi\varepsilon_0}\frac{\sqrt{4p^2\cos^2\theta+p^2\sin^2\theta}}{r^3}=\frac{1}{4\pi\varepsilon_0}\frac{p\sqrt{4\cos^2\theta+\sin^2\theta}}{r^3}$$

$$E=\frac{1}{4\pi\varepsilon_0}\frac{p\sqrt{3\cos^2\theta+\cos^2\theta+\sin^2\theta}}{r^3}$$

$$\therefore E=\frac{1}{4\pi\varepsilon_0}\frac{p\sqrt{3\cos^2\theta+1}}{r^3}$$

The direction of electric field intensity due to the dipole is

$$\tan\alpha=\frac{E_\theta}{E_r}=\frac{\frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}}{\frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}}=\frac{1}{2}\tan\theta$$

Or $\quad \alpha=\tan^{-1}\left(\frac{1}{2}\tan\theta\right)$

Note:

(1) When $\theta=0^0$,

$$E=\frac{1}{4\pi\varepsilon_0}\frac{p\sqrt{3\cos^2 0^0+1}}{r^3}=\frac{1}{4\pi\varepsilon_0}\frac{p\sqrt{3+1}}{r^3}=\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

$$\alpha=\tan^{-1}\left(\frac{1}{2}\tan 0^0\right)=\tan^{-1}(0)=0$$

Therefore $\vec{E}_{axial}$ is parallel to the dipole moment $\vec{p}$

(2) When $\theta=90^0$,

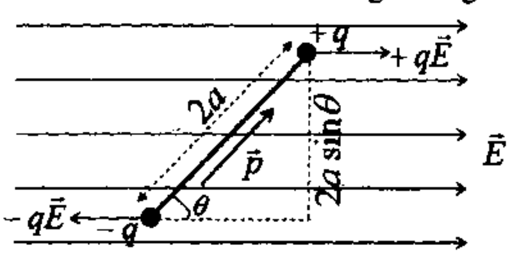
$$E=\frac{1}{4\pi\varepsilon_0}\frac{p\sqrt{3\cos^2 90^0+1}}{r^3}=\frac{1}{4\pi\varepsilon_0}\frac{p\sqrt{0+1}}{r^3}=\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

$$\alpha=\tan^{-1}\left(\frac{1}{2}\tan 90^0\right)=\tan^{-1}(\infty)=90^0$$

Therefore $\vec{E}_{equa}$ is anti-parallel to the dipole moment $\vec{p}$.

(3) Electric field E due to a dipole varies with respect to distance as $E\propto\frac{1}{r^3}$

Torque on a dipole in a uniform electric field: Consider an electric dipole consisting of charges $+q$ and $-q$ and of length $2a$ placed in a uniform electric field $\vec{E}$ making an angle θ with it. It has a dipole moment of magnitude



$$p=q\times 2a$$

Force on charge $+q$ is $\vec{F}=+q\vec{E}$

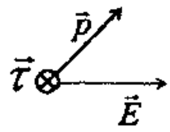
Force on charge $-q$ is $\vec{F}=-q\vec{E}$

$\therefore \vec{F}_{Total}=+q\vec{E}-q\vec{E}=0$

But the torque $\tau\neq 0$

Torque = either force × perpendicular distance between the two forces.

$$\tau = qE \times 2a\sin\theta$$
$$\tau = (q \times 2a)\, E\sin\theta$$
$$\tau = p\, E\sin\theta$$



In vector form, torque can be written as $\vec{\tau} = \vec{p} \times \vec{E}$

Special Cases:

(1) When $\theta = 90^0$, $\tau = pE$ (torque is maximum)

(2) When $\theta = 0$, $\tau = 0$ (torque is minimum)

Note:

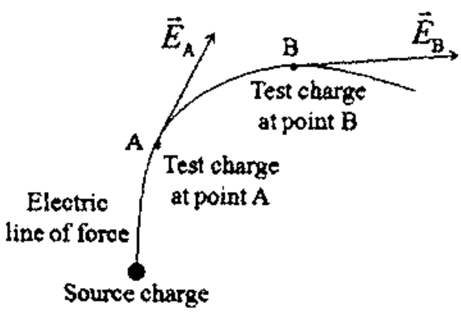
(1) If $E = 1$ unit, $\theta = 90^0$ then $\tau = p$

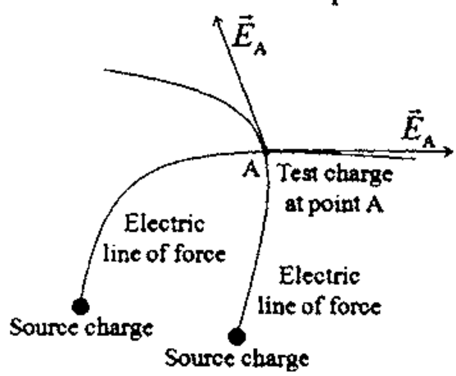
The dipole moment may be defined as the torque acting on an electric dipole, placed perpendicular to a uniform electric field of unit strength.

(2) In a non-uniform electric field, a dipole experiences a non zero force and non zero torque. In a special case when the dipole moment is parallel or antiparallel to the field, the dipole experiences a zero torque and a non zero force.

(3) The concept of electric dipole is used in the study of the effect of electric field on an insulator, and in the study of radiation of energy from an antenna.

Electric lines of force: An electric line of force maybe defined as the curve along which a small positive charge would tend to move when free to do so in an electric field and the tangent to which at any point gives the direction of the electric field at that point.





Note: No two lines of force (electric field line) can cross each other. If they intersect as shown in the figure above, then there will be two tangents at the point of intersection and hence two directions of the electric field at the same point, which is not possible.

Properties of the electric lines of force:

(1) Lines of force start from positive charge and terminate at negative charge.

(2) Lines of force never intersect.

(3) The tangent to a line of force at any point gives the direction of the electric field $\vec{E}$ at that point.

(4) The lines of force are continuous smooth curves without any break.

(5) The lines of force are always normal to the surface of a conductor on which the charges are in equilibrium.

(6) The lines of force do not pass through a conductor because the electric field inside a conductor is zero.

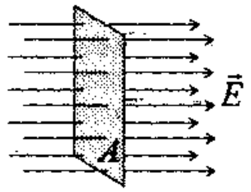
(7) The relative closeness of the lines of force gives a measure of the strength of the electric field in any region. The lines of force are
(i) close together in a strong field
(ii) far apart in a weak field
(iii) parallel and equally spaced in a uniform field.

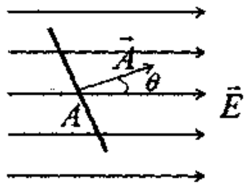
Area vector: The direction of a planar area vector is specified by the normal to the plane.

The length of the vector $d\vec{S}$ represents the magnitude dS of the area element.

Electric flux: The number of electric field lines crossing the surface normally is known as the electric flux.

If an electric field $\vec{E}$ passes through an area A, then the electric flux through this area is





$\phi_E = \vec{E} \cdot \vec{A} = E\,A\cos\theta$

where θ is the angle between the area vector and the uniform field $\vec{E}$.

Electric flux is a scalar quantity. Its SI unit is Nm^2C^{-1} or Vm

Special Cases:

(1) If the angle θ between the area vector $\vec{A}$ and the uniform field $\vec{E}$ is 0^0, then the flux through this area is

$$\phi_E = \vec{E} \cdot \vec{A} = E\,A\cos\theta$$

$$\phi_E = E\,A\cos 0^0 = E\,A \text{ (The flux is maximum and positive)}$$

(2) If the angle θ between the area vector $\vec{A}$ and the uniform field $\vec{E}$ is 90^0, then the flux through this area is

$$\phi_E = \vec{E} \cdot \vec{A} = E\,A\cos\theta$$

$$\phi_E = E\,A\cos 90^0 = 0 \text{ (The flux is zero i.e., no lines of force pass through the area)}$$

(3) If the angle θ between the area vector $\vec{A}$ and the uniform field $\vec{E}$ is 180^0, then the flux through this area is

$$\phi_E = \vec{E} \cdot \vec{A} = E\,A\cos\theta$$

$$\phi_E = E\,A\cos 180^0 = -E\,A \text{ (The flux is minimum and negative)}$$

Note: The greater the magnitude of electric field $\vec{E}$ the greater is the electric flux.

Gauss's Law: Gauss's law states that the total flux through a closed surface is $\frac{1}{\varepsilon_o}$ times the net charge enclosed by the closed surface.

Mathematically, it can be expressed as $\phi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_o}$

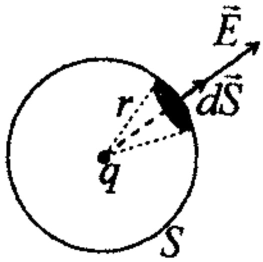
Suppose the surface S is a sphere of radius r centred on q. Then surface S is a Gaussian surface.

The flux through an area dS is

$$d\phi_E = \vec{E}\cdot d\vec{S} = EdS\cos 0^0 = E\,dS$$

The total flux through the surface S is

$$\phi_E = \oint_{\phi_E} d\phi_E = \oint_S E\,dS = E\oint_S dS$$



Electric field at any point on S is

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

And $\oint_S dS = 4\pi r^2$

$$\therefore \phi_E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}(4\pi r^2)$$

Or $\phi_E = \dfrac{q}{\varepsilon_o}$

This is Gauss's law.

Note:

(1) In Gauss's law, q is the net charge enclosed by the Gaussian surface. Not the total charge.

(2) If the net charge enclosed by the closed surface is zero, then flux through it is also zero

$$\phi_E = \frac{q}{\varepsilon_o} = \frac{0}{\varepsilon} = 0$$

(3) The location of charge or charges inside the closed surface does not matter.

(4) The shape of the surface does not matter provided it is a closed surface enclosing the charge or charges.

(5) If the medium surrounding the charge has a relative permittivity ε_r, then $\phi_E = \dfrac{q}{\varepsilon_o\varepsilon_r} = \dfrac{q}{\varepsilon}$

where $\varepsilon = \varepsilon_o\varepsilon_r$ is the absolute permittivity.

(6) The charges situated outside the closed surface make no contribution to the total electric flux over the surface.

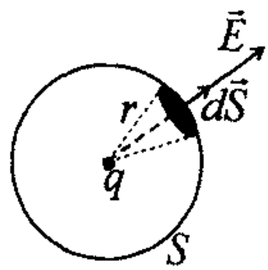
(7) Gauss's law is true only if inverse square law for electric force between point charges is true.

(8) Gauss's law simply says that the number of field lines crossing a closed surface depends only on the enclosed charge.

Application of Gauss's law: Gauss' law is great, especially when there is symmetry. But it does not always tell, what is $\vec{E}$ at any given point in space. So it is always true, but not always helpful for problem solving.

Gaussian surface: Any hypothetical closed surface enclosing a charge is called the Gaussian surface of that charge.

Coulomb's law from Gauss's law: Consider a charge q located at the centre of a sphere S of radius r. Then the surface S of the sphere is the Gaussian surface.



The flux through an area dS is

$$d\phi_E = \vec{E}\cdot d\vec{S} = EdS\cos 0^0 = E\,dS$$

The net flux through the surface S is

$$\phi_E = \oint_{\phi_E} d\phi_E = \oint_S E\, dS = E \oint_S dS = E \times 4\pi r^2 \text{ ------- } (1)$$

Again Gauss's law is given by $\phi_E = \dfrac{q}{\varepsilon_o}$ ------- (2)

From equation (1) and equation (2) we get

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_o}$$

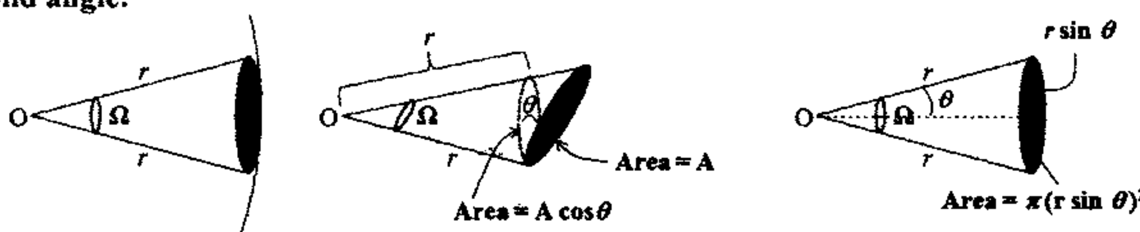
$$\text{Or } E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

$$\because F = q_o\, E$$

$$\therefore F = q_o \cdot \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

This proves the Coulomb's law.

Solid angle:



(1) (2) (3)

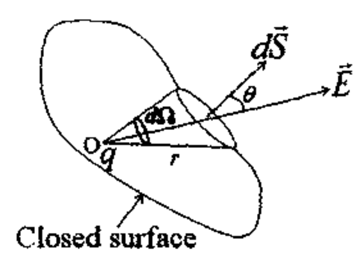
Let A be the area element (circle) on the surface of the sphere of radius r and Ω is the solid angle as shown in the figure (1). Let the radius of the circle of area A be ($r \sin\theta$) shown in figure (3).

In figure (1) $\Omega = \dfrac{A}{r^2}$

In figure (2) $\Omega = \dfrac{A\cos\theta}{r^2}$

In figure (3) $\Omega = \dfrac{\pi(r\sin\theta)^2}{r^2} = \dfrac{\pi r^2 \sin^2\theta}{r^2} = \pi \sin^2\theta$

Gauss's law from Coulomb's law:



According to Coulomb's law, the force on a charge q_o at a distance r from the source charge q is $F = \dfrac{1}{4\pi\varepsilon_o}\dfrac{qq_o}{r^2}$

The electric field is $E = \dfrac{F}{q_o} = \dfrac{\frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}}{q_o} = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q}{r^2}$

The electric flux through a small surface dS is $d\phi_E = \vec{E}\cdot d\vec{S} = E\, dS\cos\theta$

Or $d\phi_E = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q}{r^2}\, dS\cos\theta$

Or $d\phi_E = \frac{q}{4\pi\varepsilon_o}\frac{dS\cos\theta}{r^2}$

But $d\Omega = \frac{dS\cos\theta}{r^2}$ is the solid angle subtended by a small area dS at the point O.

$$d\phi_E = \frac{q}{4\pi\varepsilon_o}d\Omega$$

The total flux through the whole area is

$$\phi_E = \frac{q}{4\pi\varepsilon_o}\int d\Omega$$

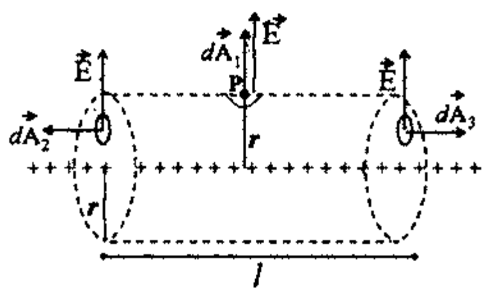
$$\phi_E = \frac{q}{4\pi\varepsilon_o}4\pi$$

$$\therefore \phi_E = \frac{q}{\varepsilon_o}$$

This proves the Gauss's law.

Field at a point due to an infinitely long charged wire: Consider an infinitely long straight wire of charge, having charged density λ. We want to derive an expression for the electric field at a point P, distance r from it.

Let l be the length and r be the radius of the cylindrical Gaussian surface. The charge enclosed by the Gaussian surface is $q = \lambda l$



The total flux is given by

$$\phi_E = \oint_A \vec{E}\cdot d\vec{A} = \int_{A_1}\vec{E}\cdot d\vec{A}_1 + \int_{A_2}\vec{E}\cdot d\vec{A}_2 + \int_{A_3}\vec{E}\cdot d\vec{A}_3$$

$$\phi_E = \int_{A_1} E\,dA_1\cos 0^0 + \int_{A_2} E\,dA_2\cos 90^0 + \int_{A_3} E\,dA_3\cos 90^0$$

$$\phi_E = E\int_{A_1} dA_1 + 0 + 0 = E\int_{A_1} dA_1$$

$$\phi_E = E\times 2\pi\, rl \text{ ---------------- } (1)$$

The Gauss's law is $\phi_E = \frac{q}{\varepsilon_o} = \frac{\lambda l}{\varepsilon_o}$ ---------------- (2)

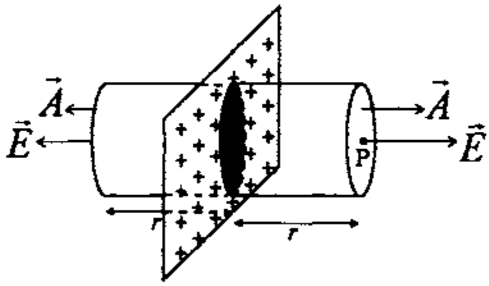
Equating equation (1) and equation (2) we have

$$E\times 2\pi\, rl = \frac{\lambda l}{\varepsilon_o}$$

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_o r}$$

Thus the electric field of a line charge is inversely proportional to the distance from the line charge i.e., $E \propto \frac{1}{r}$

Field at a point due to a uniformly charged infinite plane sheet: Consider an infinite plane sheet of charge having a uniform charge density σ. Let P be a point at a distance r from it. We have to calculate the electric field intensity at P.



The total flux is

$$\phi_E = \vec{E}\cdot\vec{A} + \vec{E}\cdot\vec{A}$$

$$\phi_E = E\,A\cos 0^0 + E\,A\cos 0^0$$

$$\phi_E = E\,A + E\,A = 2\,E\,A \text{ ---------------- (1)}$$

Charge enclosed by the Gaussian surface is

$$q = \sigma\,A$$

The Gauss's law is $\phi_E = \dfrac{q}{\varepsilon_o} = \dfrac{\sigma\,A}{\varepsilon_o}$ ---------------- (2)

Equating equation (1) and equation (2) we have

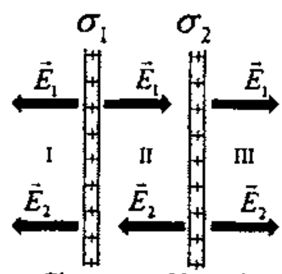
$$2\,E\,A = \frac{\sigma\,A}{\varepsilon_o}$$

$$\therefore E = \frac{\sigma}{2\,\varepsilon_o}$$

The magnitude of $\vec{E}$ remains the same and does not depend on the distance from the charged sheet.

Electric field of two positively charged parallel plates: The figure shows two thin parallel plane sheets having uniform charge densities σ_1 and σ_2 with

$\sigma_1 > \sigma_2 > 0$



In the region I, the total electric field at any point is

$$\vec{E}_I = \vec{E}_1 + \vec{E}_2$$

Or $E_I = E_1 + E_2 = \dfrac{\sigma_1}{2\varepsilon_o} + \dfrac{\sigma_2}{2\varepsilon_o}$

Or $E_I = \dfrac{1}{2\varepsilon_o}(\sigma_1 + \sigma_2)$. The direction of the field is to the left.

In the region II, the total electric field at any point is

$$\vec{E}_{II} = \vec{E}_1 + (-\vec{E}_2) = \vec{E}_1 - \vec{E}_2$$

Or $E_{II} = E_1 - E_2 = \dfrac{\sigma_1}{2\varepsilon_o} - \dfrac{\sigma_2}{2\varepsilon_o}$

Or $E_{II} = \dfrac{1}{2\varepsilon_o}(\sigma_1 - \sigma_2)$. The direction of the field is to the right.

In the region III, the total electric field at any point is

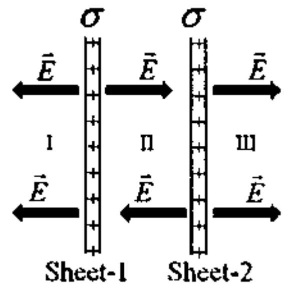
$$\vec{E}_{III} = \vec{E}_1 + \vec{E}_2$$

Or $E_{III} = E_1 + E_2 = \dfrac{\sigma_1}{2\varepsilon_o} + \dfrac{\sigma_2}{2\varepsilon_o}$

Or $E_{III} = \frac{1}{2\varepsilon_o}(\sigma_1 + \sigma_2)$. The direction of the field is to the right.

Special Cases:

If $\sigma_1 = \sigma_2 = \sigma$ then $E_1 = E_2 = E$

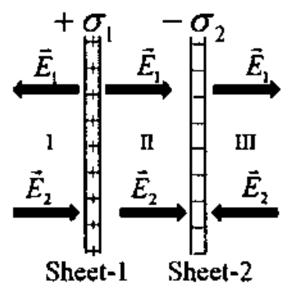


In region I, $E_I = \frac{1}{2\varepsilon_o}(\sigma + \sigma) = \frac{2\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o}$.

In region II, $E_{II} = \frac{1}{2\varepsilon_o}(\sigma - \sigma) = \frac{1}{2\varepsilon_o}(0) = 0$.

In region III, $E_{III} = \frac{1}{2\varepsilon_o}(\sigma + \sigma) = \frac{2\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o}$.

Electric field of two oppositely charged parallel plates: The figure shows two thin parallel plane sheets having uniform charge densities $+\sigma_1$ and $-\sigma_2$ with $|+\sigma_1| > |-\sigma_2|$.



In the region I, the total electric field at any point is

$$\vec{E}_I = \vec{E}_1 + (-\vec{E}_2) = \vec{E}_1 - \vec{E}_2$$

Or $E_I = E_1 - E_2 = \frac{\sigma_1}{2\varepsilon_o} - \frac{\sigma_2}{2\varepsilon_o}$

Or $E_I = \frac{1}{2\varepsilon_o}(\sigma_1 - \sigma_2)$. The direction of the field is to the left.

In the region II, the total electric field at any point is

$$\vec{E}_{II} = \vec{E}_1 + \vec{E}_2$$

Or $E_{II} = E_1 + E_2 = \frac{\sigma_1}{2\varepsilon_o} + \frac{\sigma_2}{2\varepsilon_o}$

Or $E_{II} = \frac{1}{2\varepsilon_o}(\sigma_1 + \sigma_2)$. The direction of the field is to the right.

In the region III, the total electric field at any point is

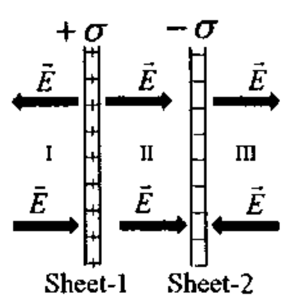
$$\vec{E}_{III} = \vec{E}_1 + (-\vec{E}_2) = \vec{E}_1 - \vec{E}_2$$

Or $E_{III} = E_1 - E_2 = \frac{\sigma_1}{2\varepsilon_o} - \frac{\sigma_2}{2\varepsilon_o}$

Or $E_{III} = \frac{1}{2\varepsilon_o}(\sigma_1 - \sigma_2)$. The direction of the field is to the right.

Special Cases:

If $+\sigma_1 = +\sigma$ and $-\sigma_2 = -\sigma$ then $E_1 = E_2 = E$



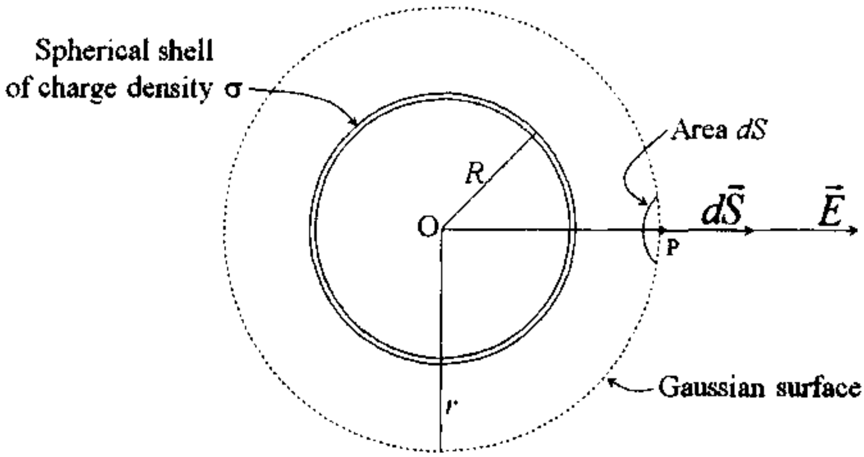
In region I, $E_I = \frac{1}{2\varepsilon_o}(\sigma - \sigma) = \frac{1}{2\varepsilon_o}(0) = 0$.

In region II, $E_{II} = \frac{1}{2\varepsilon_o}(\sigma + \sigma) = \frac{2\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o}$.

In region III, $E_{III} = \frac{1}{2\varepsilon_o}(\sigma - \sigma) = \frac{1}{2\varepsilon_o}(0) = 0$.

Field due to a uniformly charged thin spherical shell:

(1) When point P lies outside the spherical shell ($r >> R$):



Total charge q on the shell of radius R is

$$q = 4\pi R^2 \sigma$$

So the net charges enclosed by the Gaussian surface is

$$q = 4\pi R^2 \sigma.$$

Flux through the Gaussian surface is

$$\phi_E = \oint \vec{E} \cdot d\vec{S} = \oint E\, dS \cos 0^0 = E\oint dS = E \times 4\pi r^2 \text{ -------- (1)}$$

By Gauss's law we have $\phi_E = \frac{q}{\varepsilon_o}$ -------- (2)

Equating equation (1) and equation (2) gives

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

For points outside the shell, the field due to a uniformly charged shell is as if the entire charge of the shell is concentrated at its centre.

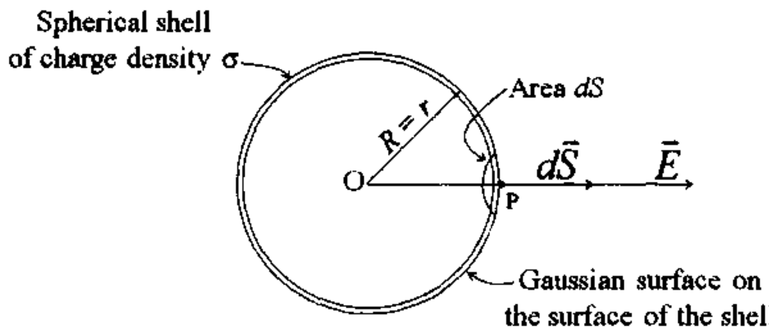
Note:

(1) The electric field $E = \frac{1}{4\pi\varepsilon_o}\frac{4\pi R^2 \sigma}{r^2}$ where $r >> R$

(2) The electric field $E = \frac{1}{4\pi\varepsilon_o}\frac{4\pi R^2 \sigma}{R^2} = \frac{\sigma}{\varepsilon_o}$ when $r = R$

(3) The electric field $E = 0$ when $r < R$

(2) When point P lies on the spherical shell ($r = R$):



Flux through the Gaussian surface is

$$\phi_E = \oint \vec{E} \cdot d\vec{S} = \oint E\, dS \cos 0^0 = E \oint dS = E \times 4\pi R^2 \text{ ------- (1)}$$

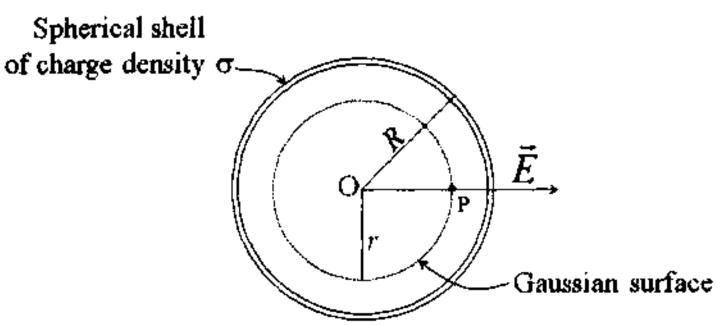
By Gauss's law we have $\phi_E = \frac{q}{\varepsilon_o} = \frac{4\pi R^2\, \sigma}{\varepsilon_o}$ ------- (2)

Equating equation (1) and equation (2) gives

$$E \times 4\pi R^2 = \frac{4\pi R^2\, \sigma}{\varepsilon_o}$$

$$E = \frac{\sigma}{\varepsilon_o}$$

(3) When point P lies inside the spherical shell ($r < R$):



Flux through the Gaussian surface is $\phi_E = E \times 4\pi\, r^2$ ------- (1)

By Gauss's law we have $\phi_E = \frac{q}{\varepsilon_o} = \frac{0}{\varepsilon_o} = 0$ ------- (2)

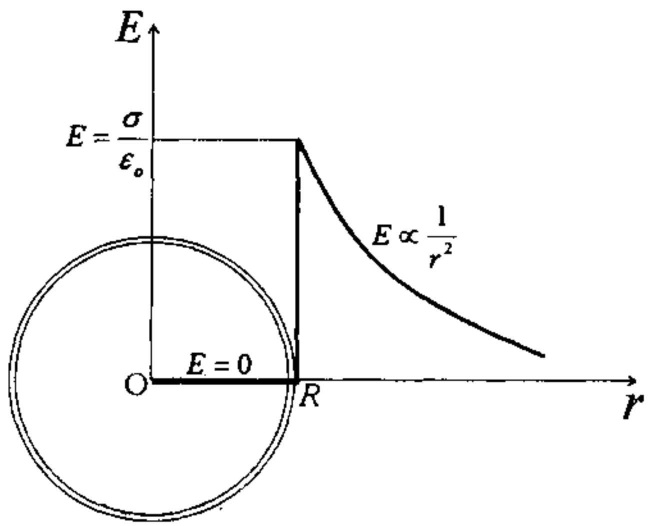
Equating equation (1) and equation (2) gives

$$E \times 4\pi\, r^2 = 0$$

$$\Rightarrow E = 0$$

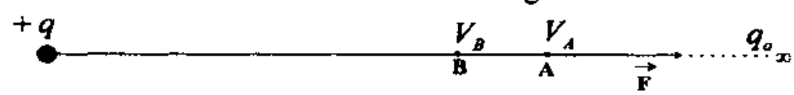
Hence electric field due to a uniformly charged spherical shell is zero at all points inside the shell.

Variation of E with distance r



Electric potential: Electric potential at a point is defined as the amount of work done in moving a unit positive charge from infinity to that point against the electrostatic force. i.e.,

$$\text{Electric potential} = \frac{\text{Work done}}{\text{Charge}}$$



Electric potential at A is $V_A = \frac{W_{\infty A}}{q_o}$

Electric potential at B is $V_B = \frac{W_{\infty B}}{q_o}$

The SI unit of electric potential is volt (V), and it is a scalar quantity.

The dimensional formula for electric potential is $[V] = \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$

Note:

(1) The electric potential of a point charge at infinity is zero.

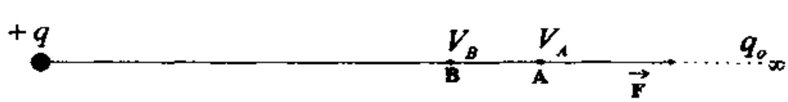
(2) Units of electrostatic potential $V = \frac{J}{C} = JC^{-1}$ or $V = \frac{Nm}{C} = NmC^{-1}$

(3) Electric potential = Electrostatic potential

(4) Electric field = Electrostatic field

(5) Electric force = Electrostatic force

Electric potential difference: The potential difference between two points in an electric field may be defined as the amount of work done in moving a unit positive charge from one point to the other against the electrostatic force.



Electric potential at A is $V_A = \frac{W_{\infty A}}{q_o}$

Electric potential at B is $V_B = \frac{W_{\infty B}}{q_o}$

Electric potential difference is $V_B - V_A = \frac{W_{\infty B}}{q_o} - \frac{W_{\infty A}}{q_o} = \frac{W_{\infty B} - W_{\infty A}}{q_o} = \frac{W_{AB}}{q_o}$

Or $V_B - V_A$ is equal to the work done in moving a unit charge q_o from point A to point B.

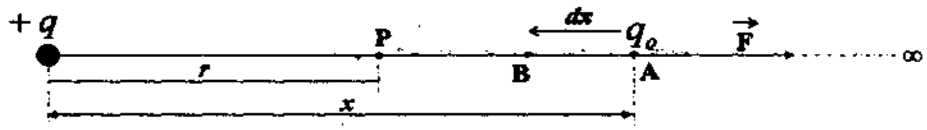
Note:

(1) $V_B > V_A$ i.e., Potential at B is greater than potential at A.

(2) Electric potential difference = Potential difference

(3) The potential difference $V_B - V_A$ is the work per unit charge necessary to move a test charge at constant speed from *A* to *B*.

Electric potential due to a single point charge:



The Coulomb's law of electrostatic force acting on charge q_o at A is

$$F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{x^2}$$

The small work done in moving a charge q_o from A to B through a small displacement *dx*, at constant speed against the electrostatic force is

$$dW = \vec{F}\cdot d\vec{x} = Fdx\cos\theta = Fdx\cos 180^0 = -Fdx$$

The total work done in moving a charge q_o from ∞ to the point P will be

$$W = \int dW = -\int_{\infty}^{r} Fdx = -\int_{\infty}^{r}\frac{1}{4\pi\varepsilon_o}\frac{qq_o}{x^2}dx = -\frac{qq_o}{4\pi\varepsilon_o}\int_{\infty}^{r} x^{-2}dx$$

$$W = -\frac{qq_o}{4\pi\varepsilon_o}\left[-\frac{1}{x}\right]_{\infty}^{r} = \frac{qq_o}{4\pi\varepsilon_o}\left[\frac{1}{x}\right]_{\infty}^{r} = \frac{qq_o}{4\pi\varepsilon_o}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r}$$

Hence the work done in moving a unit test charge from infinity to the point P, or the electric potential at point P is

$$V = \frac{W}{q_o} = \frac{\frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r}}{q_o}$$

Or $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$

Note:

(1) For q positive, V is positive i.e., $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$ and $V \propto \frac{1}{r}$

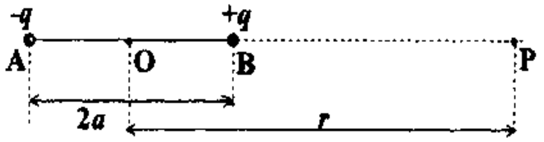
(2) If q is negative, V is negative i.e., $V = -\frac{1}{4\pi\varepsilon_o}\frac{q}{r}$ and $V \propto -\left(\frac{1}{r}\right)$.

(3) In a medium of dielectric constant κ, $V = \frac{1}{4\pi\varepsilon_o \kappa} \frac{q}{r}$

(4) At $r = \infty$, $V = 0$

(5) At equal distances from the point charge $+q$, V is the same. Thus the electric potential due to a point charge is spherically symmetric.

Electric potential due to a dipole (axial): Consider an electric dipole consisting of two point charges $-q$ anu $+q$ and separated by a distance $2a$. Let P be a point on the axis of the dipole at a distance r from its centre O.



Electric potential at P due to $-q$ is

$$V_{-q} = \frac{1}{4\pi\varepsilon_o} \frac{-q}{r+a} = -\frac{1}{4\pi\varepsilon_o} \frac{q}{r+a}$$

Electric potential at P due to $+q$ is $V_{+q} = \frac{1}{4\pi\varepsilon_o} \frac{q}{r-a}$

Electric potential at P due to the dipole is $V = V_{-q} + V_{+q} = -\frac{1}{4\pi\varepsilon_o} \frac{q}{r+a} + \frac{1}{4\pi\varepsilon_o} \frac{q}{r-a}$

$$V = \frac{q}{4\pi\varepsilon_o} \left[\frac{1}{r-a} - \frac{1}{r+a} \right]$$

$$V = \frac{q}{4\pi\varepsilon_o} \left[\frac{(r+a)-(r-a)}{(r-a)(r+a)} \right]$$

$$V = \frac{q}{4\pi\varepsilon_o} \left[\frac{r+a-r+a}{r^2-a^2} \right] = \frac{q}{4\pi\varepsilon_o} \left[\frac{2a}{r^2-a^2} \right]$$

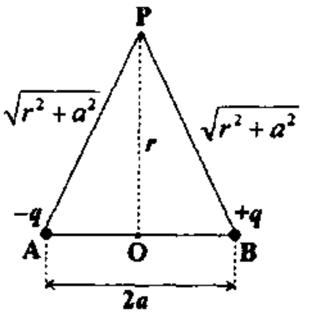
$$V = \frac{1}{4\pi\varepsilon_o} \frac{q \times 2a}{r^2-a^2}$$

$$V = \frac{1}{4\pi\varepsilon_o} \frac{p}{r^2-a^2}$$

For a short dipole, $a^2 << r^2$, so

$$V = \frac{1}{4\pi\varepsilon_o} \frac{p}{r^2}$$

Electric potential due to a dipole (equatorial): Consider an electric dipole consisting of two point charges $-q$ and $+q$ and separated by a distance $2a$. Let P be a point on the perpendicular bisector of the dipole at distance r from its centre O.



Electric potential at P due to $-q$ is

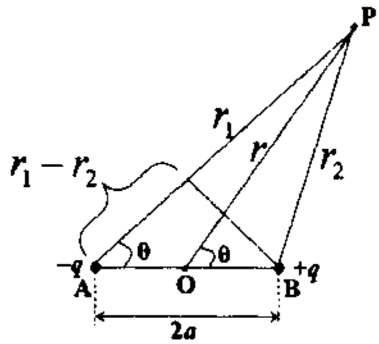
$$V_{-q} = \frac{1}{4\pi\varepsilon_o} \frac{-q}{\sqrt{r^2+a^2}} = -\frac{1}{4\pi\varepsilon_o} \frac{q}{\sqrt{r^2+a^2}}$$

Electric potential at P due to $+q$ is $V_{+q} = \frac{1}{4\pi\varepsilon_o} \frac{q}{\sqrt{r^2+a^2}}$

Electric potential at P due to the dipole is $V = V_{-q} + V_{+q} = -\frac{1}{4\pi\varepsilon_o}\frac{q}{\sqrt{r^2+a^2}} + \frac{1}{4\pi\varepsilon_o}\frac{q}{\sqrt{r^2+a^2}}$

$$\therefore V = 0$$

Electric potential due to a dipole at any general point:
Consider an electric dipole consisting of two point charges $-q$ and $+q$ and separated by a distance $2a$. We wish to determine the potential at any point P from the centre O of the dipole.



Suppose $AP = r_1, OP = r, BP = r_2$ and $\angle POB = \theta$.

The net potential at P due to the dipole is

$$V = V_{-q} + V_{+q} = -\frac{1}{4\pi\varepsilon_o}\frac{q}{r_1} + \frac{1}{4\pi\varepsilon_o}\frac{q}{r_2}$$

$$V = V_{-q} + V_{+q} = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{q}{4\pi\varepsilon_o}\left[\frac{r_1 - r_2}{r_1 r_2}\right]$$

If the point P lies far away from the dipole, then

$$r_1 - r_2 = 2a\cos\theta \quad \text{and} \quad r_1 r_2 \approx r^2$$

$$\therefore V = \frac{q}{4\pi\varepsilon_o}\left[\frac{2a\cos\theta}{r^2}\right] = \frac{1}{4\pi\varepsilon_o}\frac{q\times 2a\cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{p\cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{p\,r\cos\theta}{r^3} = \frac{1}{4\pi\varepsilon_o}\frac{\vec{p}\cdot\vec{r}}{r^3} = \frac{1}{4\pi\varepsilon_o}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

Note: At large distance, the dipole potential falls off as $\frac{1}{r^2}$ while the potential due to a single charge falls off as $\frac{1}{r}$

Special cases:

(1) When the point P lies on the axial line of the dipole on the side of $+q$, then $\theta = 0^0$.

$$V = \frac{1}{4\pi\varepsilon_o}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{p\cos 0^0}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{p}{r^2}$$

(2) When the point P lies on the axial line of the dipole on the side of $-q$, then $\theta = 180^0$.

$$V = \frac{1}{4\pi\varepsilon_o}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{p\cos 180^0}{r^2} = -\frac{1}{4\pi\varepsilon_o}\frac{p}{r^2}$$

(3) When the point P lies on the equatorial line of the dipole then $\theta = 90^0$.

$$V = \frac{1}{4\pi\varepsilon_o}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{p\cos 90^0}{r^2} = 0$$

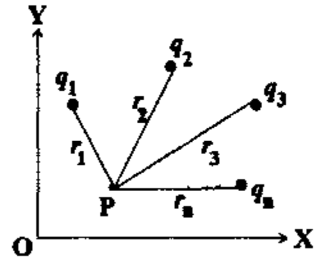
Electric potential due to a system of point charges: As shown in the figure, suppose n point charges $q_1, q_2, q_3, \ldots q_n$ lie at distances $r_1, r_2, r_3, \ldots r_n$ from a point P.

Electric potential at P due to charge q_1 is $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$

Electric potential at P due to charge q_2 is $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$

Electric potential at P due to charge q_3 is $V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$

Electric potential at P due to charge q_n is $V_n = \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n}$



The net potential at P is

$$V = V_1 + V_2 + V_3 + \ldots + V_n$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3} + \ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots + \frac{q_n}{r_n}\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{i=n}\frac{q_i}{r_i}$$

Note: If $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_n$ are the position vectors of n point charges, the electric potential at a point whose position vector is $\vec{r}$, would be

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{i=n}\frac{q_i}{|\vec{r} - \vec{r}_i|}$$

Electric potential due to a continuous charge distribution:

(1) When the charge is distributed uniformly along a line L, $dq = \lambda dL$, where λ is the line charge density.

$$\therefore V_L = \frac{1}{4\pi\varepsilon_o}\int_L \frac{\lambda dL}{r}$$

$$\text{Or } V_L = \frac{1}{4\pi\varepsilon_o}\int_L \frac{\lambda dL}{|\vec{r} - \vec{r}_i|}$$

(2) When the charge is distributed continuously over an area $S, dq = \sigma\, dS$, where σ is the surface charge density.

$$\therefore V_S = \frac{1}{4\pi\varepsilon_o}\int_S \frac{\sigma\, dS}{r}$$

$$\text{Or } V_S = \frac{1}{4\pi\varepsilon_o}\int_S \frac{\sigma\, dS}{|\vec{r} - \vec{r}_i|}$$

(3) When the charge is distributed continuously in a volume $V, dq = \rho\, dV$, where ρ is the volume charge density.

$$\therefore V_V = \frac{1}{4\pi\varepsilon_o}\int_V \frac{\rho\, dV}{r}$$

$$\text{Or } V_V = \frac{1}{4\pi\varepsilon_o}\int_V \frac{\rho\, dV}{|\vec{r} - \vec{r}_i|}$$

The electron-volt (eV): One electron-volt is the amount of energy gained by an electron when accelerated through a potential difference of 1 V.

$$1eV = 1.6\times10^{-19}J$$

Electric potential due to a uniformly charged thin spherical shell:

(1) When the point P lies outside the shell: We know that for a uniformly charged spherical shell, the electric field outside the shell is as if the entire charge is concentrated at the centre. Hence electric potential at a point outside the shell is

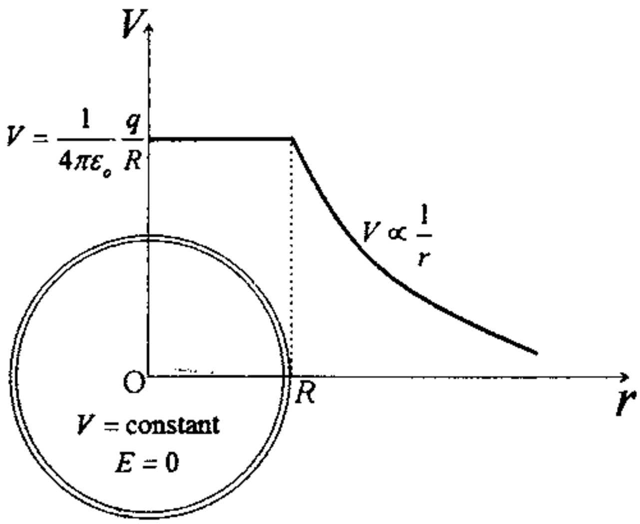
$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} \qquad (r > R)$$

(2) When the point P lies on the surface of the shell: Here $r = R$. Hence the potential on the surface of the shell is

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{R} \qquad (r = R)$$

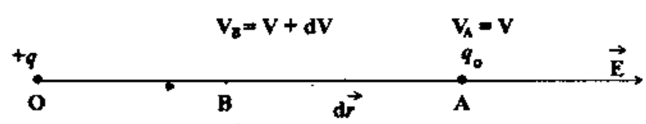
(3) When the point P lies inside the shell: The electric field at any point inside the shell is zero. Hence the electric potential due to uniformly charged spherical shell is constant everywhere inside the shell. (Not yet done)

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{R} \qquad (r < R)$$



Relation between electric field and electric potential: Consider the electric field due to charge $+q$ located at the origin O.

Let A and B be two adjacent points separated by distance dr. The two points are so closed that electric field $\vec{E}$ between them remains almost constant. Let V and V + dV be the potentials at the two points.



The work done to move a test charge from A to B is

$$W = \vec{F}\cdot d\vec{r} = Fdr\cos180^0 = -Fdr$$

$$W = -q_oEdr \text{ ----------------- } (1)$$

Also $W = q_o(V_B - V_A) = q_o\{(V + dV) - V\}$

$$W = q_o(V + dV - V) = q_odV \text{ ----------------- } (2)$$

Equating equation (1) and equation (2) we have

$$-q_o E dr = q_o dV$$

$$E = -\frac{dV}{dr}$$

In vector form $\vec{E} = -\frac{dV}{d\vec{r}}$ or $dV = -\vec{E}\cdot d\vec{r}$

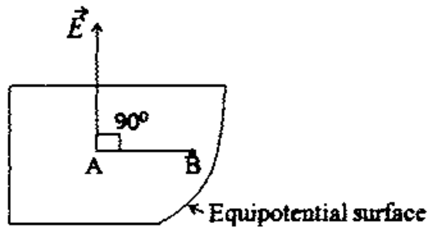
This is called the potential gradient. The negative sign shows that the direction of the electric field is in the direction of decreasing potential.

Note: Electric potential is a scalar quantity while potential gradient is a vector quantity.

Equipotential surfaces: Any surface that has same electric potential at every point on it is called an equipotential surface.

Properties of equipotential surfaces:

(1) No work is done in moving a test charge over an equipotential surface .



$$W_{AB} = q_o(V_B - V_A)$$

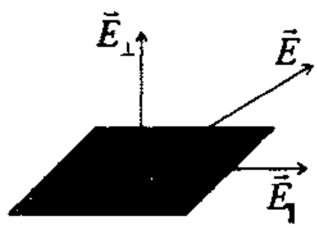
As the surface is equipotential,

$$V_B = V_A$$

$$V_B - V_A = 0$$

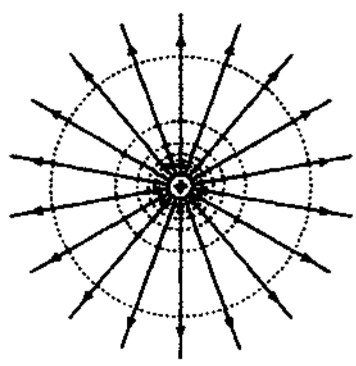
$$\therefore W_{AB} = q_o(0) = 0$$

(2) Electric field is always normal to the equipotential surface at every point.



If $\vec{E}$ is not normal to the equipotential surface, $\vec{E}_{\parallel} \neq 0$. So to move a test charge against this component, a work would have to be done. Hence the electric field $\vec{E}$ must be normal to the equipotential surface at every point.

(3) Equipotential surfaces are closer together in the region of strong field and farther apart in the region of weak field.



$$E = -\frac{dV}{dr}$$

$$\text{Or } dr = -\frac{dV}{E}$$

When dV is constant

$$dr \propto \frac{1}{E}$$

(4) No two equipotential surfaces can intersect each other. If they intersect, then there will be two values of electric potential at the point of intersection, which is impossible.

Electric potential energy: The electric potential energy of the system of point charges may be defined as the amount of work done in assembling the charges at their locations by bringing them in, from infinity.

Note:

(1) If a positive test charge q_o moves from point A to point B against the electrostatic field of the source charge q, then the external work done is positive while the electrostatic work done is negative.

$$V_B - V_A = +\frac{W_{AB}^{ext}}{q_o} = -\frac{W_{AB}^{elec}}{q_o}$$

$$q_o V_B - q_o V_A = +W_{AB}^{ext} = -W_{AB}^{elec}$$

$$U_B - U_A = +W_{AB}^{ext} = -W_{AB}^{elec}$$

where U_B and U_A are the potential energy at B and A respectively and $U_B > U_A$

(2) If a positive test charge q_o moves from point B to point A along the electrostatic field of the source charge q, then the external work done is negative while the electrostatic work done is positive.

$$V_B - V_A = -\frac{W_{AB}^{ext}}{q_o} = +\frac{W_{AB}^{elec}}{q_o}$$

$$q_o V_B - q_o V_A = -W_{AB}^{ext} = +W_{AB}^{elec}$$

$$U_B - U_A = -W_{AB}^{ext} = +W_{AB}^{elec}$$

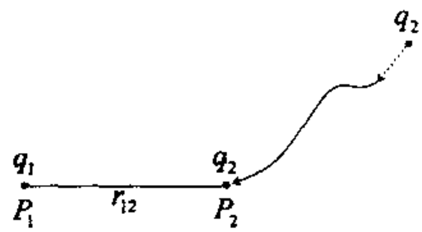
where U_B and U_A are the potential energy at B and A respectively and $U_B > U_A$

Electric potential energy of two-charge system:

Suppose a point charge q_1 is at rest at a point P_1 in space.

It takes no work to bring the charge q_1 because there is no field yet to work against.

$$\therefore W_1 = 0$$



Electric potential due to q_1 at a point P_2 at distance r_{12} from P_1 will be

$$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_{12}}$$

By bringing charge q_2 from infinity to point P_2, the work required is

$$W_2 = V_1 \times q_2 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}}$$

Hence the electrostatic potential energy U of the system is

$$U = W_1 + W_2 = 0 + \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}}$$

$$\therefore U = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}}$$

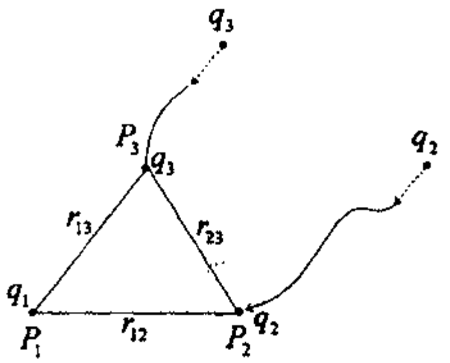
Note:

(1) For positive potential energy (i.e., U positive or $q_1 q_2 > 0$), a positive amount of work has to be done against this force to bring the charges from infinity to finite position.

(2) For negative potential energy (i.e., U negative or $q_1 q_2 < 0$), a positive amount of work has to be done against this force to take the charges from the given locations to infinity.

Electric potential energy of a system of three point charges: As shown in the figure, the electric potential at P_3 due to q_1 and q_2 is

$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_{13}}$ and $V_2 = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r_{23}}$ respectively.



Now by bringing a third charge q_3 from infinity to the point P_3, the work required is

$$W_3 = V_1 \times q_3 + V_2 \times q_3 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_o}\frac{q_2 q_3}{r_{23}} = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

Hence the electrostatic potential energy U of the system is

$$U = W_1 + W_2 + W_3 = 0 + \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

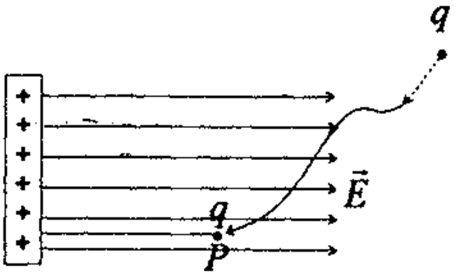
The electrostatic potential energy U for a system of n charges is

$$U = \frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_o}\sum_{\substack{i=j=1 \\ i\neq j}}^{i=j=n}\frac{q_i q_j}{r_{ij}}$$

Electric potential energy in an external field: Electric potential energy in an external field can be defined as the potential energy of a unit positive charge at that point.

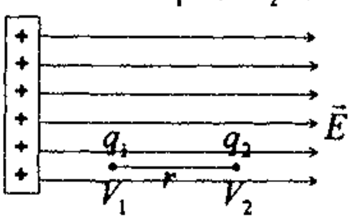
Let the electric potential at a point P in a uniform electric field $\vec{E}$ is V

Therefore the electric potential energy at that point is $U = V \times q$



Potential energy of a system of two point charges in an electric field: Let V_1 and V_2 be the electric potential of the field $\vec{E}$ at the points where q_1 and q_2 are located. Then potential energy of the two charges in the field $\vec{E}$ is

$$U = V_1 \times q_1 + V_2 \times q_2 + \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r}$$



where $V_1 \times q_1$ is the interaction between q_1 and $\vec{E}$,

$V_2 \times q_2$ is the interaction between q_2 and $\vec{E}$,

$\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r}$ is the interaction between q_1 and q_2.

Electric potential energy of an electric dipole in uniform electric field: Consider an electric dipole placed in a uniform electric field $\vec{E}$ with its dipole moment $\vec{p}$ making an angle θ with the field. Two equal and opposite forces $+q\vec{E}$ and $-q\vec{E}$ act on its two ends. The two forces form a couple. The torque exerted by the couple will be

$$\tau = pE\sin\theta$$

If the dipole is rotated through a small angle $d\theta$ against the torque acting on it, then the small work done is

$$dW = \tau\, d\theta = pE\sin\theta\, d\theta$$

The total work done in rotating the dipole from angle θ_1 with $\vec{E}$ to θ_2 will be

$$W = \int dW = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$

$$W = pE[-\cos\theta]_{\theta_1}^{\theta_2} = -pE[\cos\theta_2 - \cos\theta_1]$$

$$W = pE[\cos\theta_1 - \cos\theta_2]$$

This work done is stored as the potential energy U of the dipole.

$$\therefore U = pE[\cos\theta_1 - \cos\theta_2]$$

If initially the dipole is oriented perpendicular to the direction of the field, then $\theta_1 = 90^0$. When brought to some orientation making an angle θ with the field, then $\theta_2 = \theta$. The potential energy of the dipole will be

$$U = pE[\cos 90^0 - \cos\theta] = -pE\cos\theta$$

Or $U = -\vec{p}\cdot\vec{E}$

Special cases:

(1) When $\theta = 0^0$, $\quad U = -pE\cos 0^0 = -pE$

In this position, the dipole has its minimum potential energy and hence it is in stable equilibrium.

(2) When $\theta = 90^0$, $\quad U = -pE\cos 90^0 = 0$

In this position, the potential energy of the dipole is zero.

(3) When $\theta = 180^0$, $\quad U = -pE\cos 180^0 = +pE$

In this position, the dipole has its maximum potential energy and hence it is in unstable equilibrium.

Note: If we hold the dipole perpendicular to the electric field and bring it from infinity into the field, then the work done on charge $+q$ by the external agent is equal to the work done on charge $-q$. The net work done on the dipole will be zero and hence its potential energy is zero.

Conductors: Those substances which permit the flow of charge through them are called conductors. The conductors may be classified into two categories (1) metallic conductors and (2) ionic conductors.

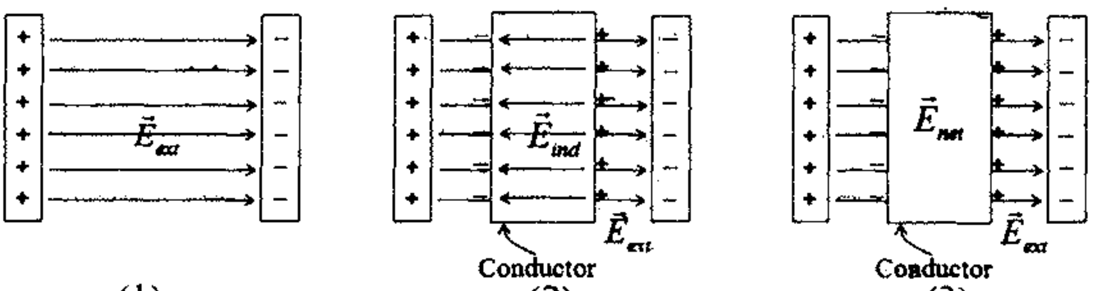
(1) metallic conductors: In metallic conductors (examples copper, silver etc), there are a large number of free electrons

(2) ionic conductors or electrolytes: In ionic conductors (examples solution of NaCl, KOH etc), a large number of positive and negative ions are present due to ionisation.

Insulators: Those materials which do not permit the flow of charge through them are called insulators or dielectrics (examples glass, mica, paper etc).

Behaviour of conductors in electrostatic field:

(1) Net electrostatic field is zero in the interior of a conductor.



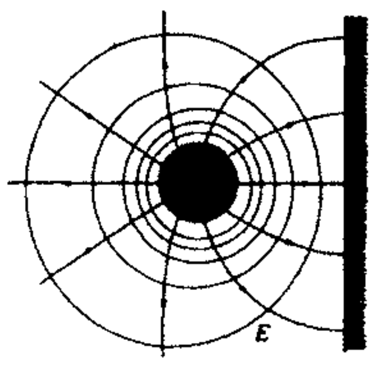
(1) (2) (3)

When a conductor is placed in an electric field $\vec{E}_{ext}$, induced electric field $\vec{E}_{ind}$ set up by the induced charges becomes equal and opposite to the field $\vec{E}_{ext}$. Thus the net field inside the conductor is

$$\vec{E}_{net} = \vec{E} = \vec{E}_{ext} + \vec{E}_{ind} = \vec{E}_{ext} - \vec{E}_{ext} = 0$$

Note: $\because \vec{E}_{net} = 0$ inside the conductor, the medium inside the conductor completely blocks the passage of field. So absolute permittivity $\varepsilon = \varepsilon_o \varepsilon_r = \infty$.

(2) Just outside the surface of a charged conductor, electric field is normal to the surface.



If the electric field is not normal to the surface, it will have a component tangential to the surface which will immediately cause the flow of charges, producing surface currents. But no such currents can exist under static conditions.

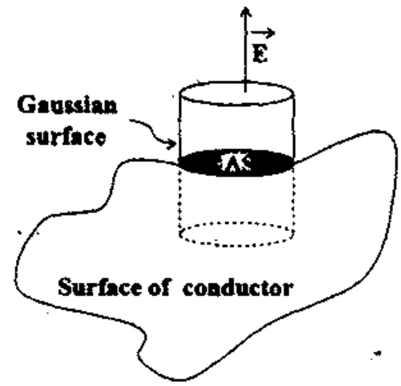
(3) Electric field at the surface of a charged conductor is proportional to the surface charge density.

Electric field is zero inside the conductor and just outside, it is normal to the surface.

The electric flux through the pill box is $\phi_E = E\,\Delta S$ ------- (1)

Charge enclosed by the pill box is $q = \sigma\,\Delta S$

By Gauss's theorem $\phi_E = \frac{q}{\varepsilon_o} = \frac{\sigma\,\Delta S}{\varepsilon_o}$ ------- (2)



Equating equation (1) and equation (2), gives

$$E\,\Delta S = \frac{\sigma\,\Delta S}{\varepsilon_o}$$

$$E = \frac{\sigma}{\varepsilon_o}$$

$$E \propto \sigma$$

(4) Potential is constant within and on the surface of a conductor.

Inside a conductor $E = 0$

$\because E = -\frac{dV}{dr} \qquad \Rightarrow \frac{dV}{dr} = 0$

$\therefore V =$ constant.

The electric field is always perpendicular to the equipotential surface, and also perpendicular to the surface of the charged conductor. Thus V is constant on the surface, and the surface of the conductor is an equipotential surface.

Note: If the conductor is charged, there exist an electric field normal to its surface. This indicates that the potential on the surface will be different from the potential at a point just outside the surface.

(5) The net charge in the interior of a conductor is zero and any excess charge resides at its surface.

We know that the electric flux is $\phi_E = \int_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_o}$

$\because E = 0$ inside the conductor

$\therefore q = 0$

Hence there is no charge in the interior of the conductor and the entire excess charge q must reside at the surface of the conductor.

(6) Electric field is zero in the cavity of a hollow charged conductor.

$\because E \propto \sigma$

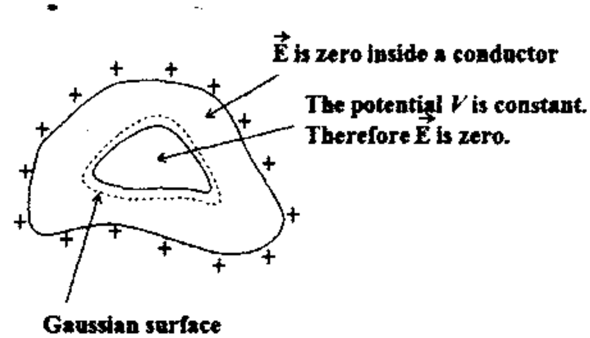
As $E = 0$

$\therefore \sigma = 0$

And $q = 0$

So there is no charge in the surface of the cavity.

Again electric flux through the Gaussian surface is $\phi_E = E\,S = \frac{q}{\varepsilon_o}$



$\therefore E\,S = \dfrac{0}{\varepsilon_o}$

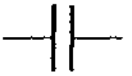
$E = 0$

Hence the electric field must be zero at every point inside the cavity.

Electrostatic shielding: The phenomenon of making a region free from any electric field is called electrostatic shielding. It is based on the fact that electric field vanishes inside the cavity of a hollow cylinder.

Such a field free region is called a Faraday cage.

Capacitor: A capacitor is a device that is capable of storing charge.



Electrical capacitance of a conductor: The electrical capacitance of a conductor is the measure of its ability to hold electric charge.
If a charge q is put on an insulated conductor, it increases its potential by V. Thus

$$q \propto V$$

$$\text{Or } q = CV$$

The constant C is called the capacitance of the capacitor.

$$\therefore C = \frac{q}{V}$$

The capacitance of the capacitor may also be defined as the amount of charge required to raise its potential by unity.

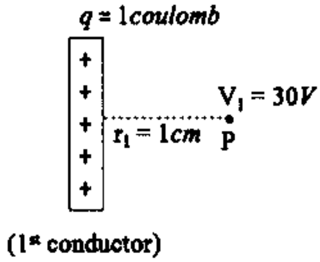
The capacitance depends on:
(1) size and shape of the conductor,
(2) nature (permittivity) of the surrounding medium,
(3) presence of the other conductors in its neighbourhood.

The SI unit of capacitance is farad (F). The capacitance of the capacitor is $1F$ if the addition of a charge of $1C$ to it, increases its potential by $1V$.

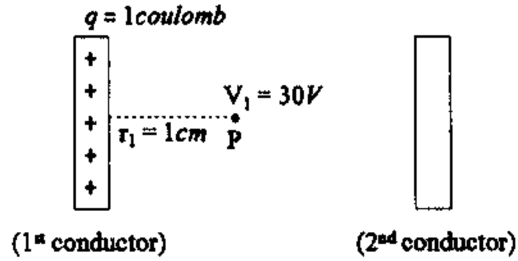
The unit of capacitance is $1F = \dfrac{1C}{1V} = \dfrac{1C}{1\dfrac{J}{C}} = \dfrac{1C^2}{1J} = \dfrac{1(As)^2}{1Nm}$

$\therefore$ The dimensions of capacitance $[C] = \dfrac{[A^2T^2]}{[MLT^{-2}L]} = [M^{-1}L^{-2}T^4A^2]$

Concept of capacitance by example: Let the charges stored on the 1st conductor is 1*coulomb*, P is a point in space where the electric potential can be calculated and the 2nd conductor is introduced latter.



(1)

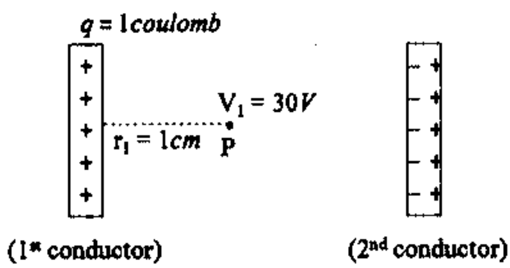


(2)

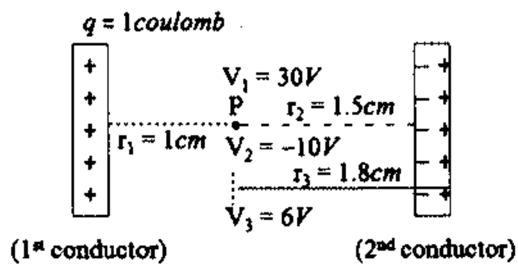
In figure (1): The potential at the point P distance $r_1 = 1cm$ from the plane of positive charges is $V_1 = 30V$. The capacitance of the 1st conductor is

$$C = \frac{q}{V} = \frac{1coulomb}{30volt} = 0.03F$$

In figure (2): The 2nd conductor is introduced.



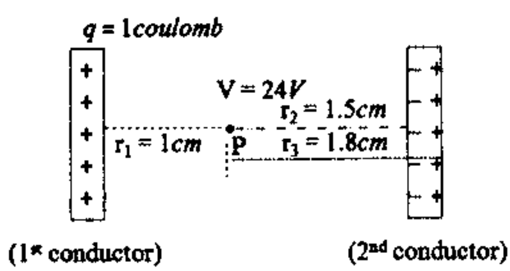
(3)



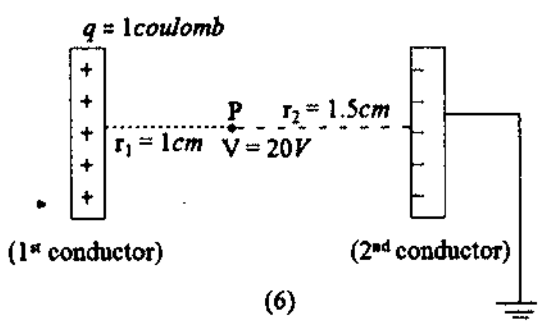
(4)

In figure (3): By induction, the negative charges come closer to the 1st conductor, while the positive charges move farther away.

In figure (4): The potential at the point P distance $r_2 = 1.5cm$ from the plane of negative charges is $V_2 = -10V$. The potential at the point P distance $r_3 = 1.8cm$ from the plane of positive charges is $V_3 = 6V$.



(5)



(6)

In figure (5): The net potential at the point P is $V = 30V - 10V + 6V = 24V$. The capacitance of the 1st conductor now is

$$C = \frac{q}{V} = \frac{1coulomb}{24volt} = 0.04F$$

In figure (6): If the 2^{nd} conductor is earthed as shown in the figure, all the positive charges has been neutralised by the negative charges from the ground. So the potential at P is $V = 30V - 10V = 20V$. The capacitance of the 1^{st} conductor now is

$$C = \frac{q}{V} = \frac{1coulomb}{20volt} = 0.05F$$

Note:

(1) The capacitance of a conductor increases in the presence of another conductor.
(2) As the potential V decreases, the capacitance C increases.

Electrical capacitance of an isolated spherical conductor: The electric potential at any point on the surface of the spherical conductor of radius R is

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

$\therefore$ The capacitance is $C = \frac{q}{V} = \frac{q}{\frac{1}{4\pi\varepsilon_o}\frac{q}{R}} = 4\pi\varepsilon_o R$

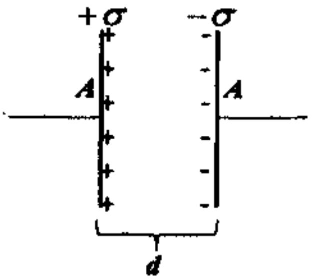
Clearly the capacitance of a spherical conductor is proportional to its radius. i.e., $C \propto R$

Note:

(1) The formula $C = 4\pi\varepsilon_o R$ is valid for both hollow and solid spherical conductors.
(2) $1farad$ is very large unit of capacitance.
(3) It is not possible to have a single isolated conductor of very large capacitance.

Parallel plate capacitor: Parallel plate capacitor is an arrangement of two parallel conducting plates of equal area separated by air medium or any other insulating medium such as paper, mica, glass, wood, ceramic, etc.

Let A be the area of each plate, d is the distance between the two plates, $\pm\sigma$ is the uniform surface charge densities on the two plates and $\pm\sigma A$ is the total charge on each plate.



Potential difference between the plates is $V = Ed = \frac{\sigma}{\varepsilon_o}d$

Where $E = \frac{\sigma}{\varepsilon_o}$ is the electric field in the inner region between the two capacitor plates.

Capacitance of the parallel plate capacitor is

$$C = \frac{q}{V} = \frac{\sigma A}{\frac{\sigma d}{\varepsilon_o}} = \sigma A \times \frac{\varepsilon_o}{\sigma d}$$

Or $C = \frac{\varepsilon_o A}{d}$

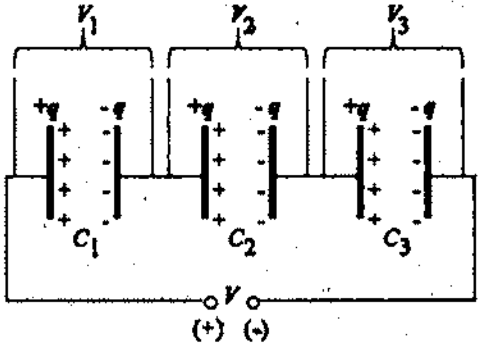
The capacitance of parallel plate capacitor depends on the following factors

(1) Area of the plates ($C \propto A$)

(2) Distance between the plates $\left(C \propto \frac{1}{d}\right)$

(3) Permittivity of the medium between the plates ($C \propto \varepsilon$) i.e., $C = \frac{\varepsilon_o \varepsilon_r A}{d}$

Combination of capacitors in series: The figure shows three capacitors of capacitances C_1, C_2 and C_3 connected in series. A potential difference V is applied across the combination.



The potential differences across the various capacitors are

$$V_1 = \frac{q}{C_1},\ V_2 = \frac{q}{C_2} \text{ and } V_3 = \frac{q}{C_3}$$

For the series circuit, we have

$$V = V_1 + V_2 + V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \text{-------- (1)}$$

If C_S is the equivalent capacitance of the series combination, then

$$C_S = \frac{q}{V} \qquad \text{Or} \quad \frac{1}{C_S} = \frac{V}{q} \text{-------- (2)}$$

From equation (1) and equation (2), we get

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For a series combination of n capacitors, we can write

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

Note:

(1) If two capacitors of capacitances C_1 and C_2 are connected in series, then the equivalent capacitance C_S is $C_S = \frac{C_1 C_2}{C_1 + C_2}$

(2) The charge on each capacitors is same.

(3) The equivalent capacitance is smaller than the smallest individual capacitance.

Combination of capacitors in parallel: The figure shows three capacitors of capacitances C_1, C_2 and C_3 connected in parallel. A potential difference V is applied across the combination.

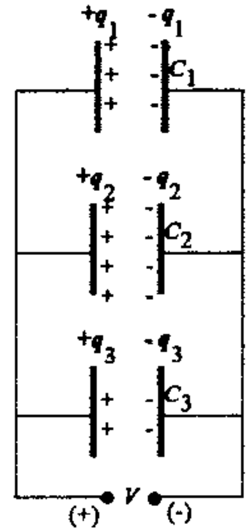
All the capacitors have the same potential difference V but different charges given by

$$q_1 = C_1 V,\ \ q_2 = C_2 V \ \text{ and } \ q_3 = C_3 V$$

Total charge stored in the combination is

$$q = q_1 + q_2 + q_3 = C_1V + C_2V + C_3V = V\left[C_1 + C_2 + C_3\right]$$

$$\frac{q}{V} = C_1 + C_2 + C_3 \text{ ------- (1)}$$



If C_p is the equivalent capacitance of the parallel combination, then

$$C_p = \frac{q}{V} \text{ -------- (2)}$$

From equation (1) and equation (2), we get

$$C_p = C_1 + C_2 + C_3$$

For a parallel combination of n capacitors, we can write

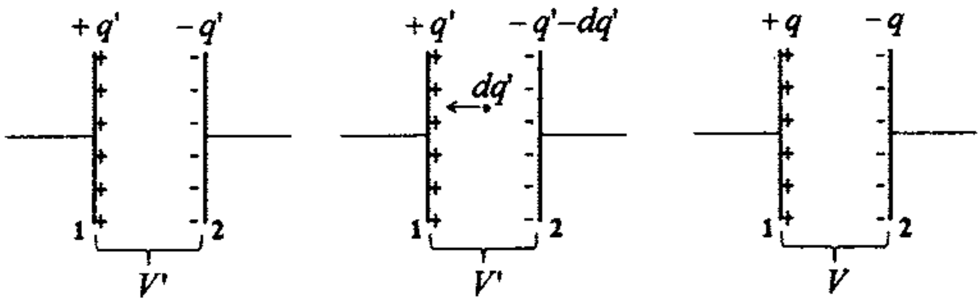
$$C_p = C_1 + C_2 + C_3 + ... + C_n$$

Note:

(1) When capacitors are connected in parallel, the equivalent capacitance is larger than the largest individual capacitance.

(2) The potential difference across each capacitor is same.

Energy stored in a capacitor: The work done in charging the capacitor is stored as its electrical potential energy.



Suppose at any stage of charging, the potential difference is $V' = \frac{q'}{C}$

If a small charge dq' is further transferred, the work done is

$$dW = V'dq' = \frac{q'}{C}dq'$$

The total work done in transferring a charge q from plate 2 to plate 1 will be

$$W = \int dW = \int_0^q \frac{q'}{C}dq' = \left[\frac{(q')^2}{2C}\right]_0^q = \frac{1}{2}\frac{q^2}{C}$$

This work done is stored as electrical potential energy U of the capacitor.

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}qV$$

Note: The energy supplied by the battery is qV, but the energy stored in the electric field is $\frac{1}{2}qV$. The rest $\frac{1}{2}qV$ of the energy is dissipated as heat in the conducting wires and battery itself.

Energy stored in a series combination of capacitors: For series combination, q is constant.

Total energy is $U=\frac{1}{2}\frac{q^2}{C}=\frac{q^2}{2}\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+...\right]=\frac{1}{2}\frac{q^2}{C_1}+\frac{1}{2}\frac{q^2}{C_2}+\frac{1}{2}\frac{q^2}{C_3}+...$

$$U=U_1+U_2+U_3+...$$

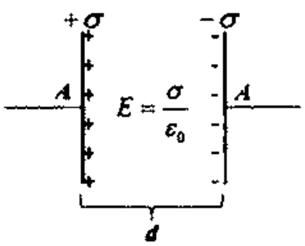
Energy stored in a parallel combination of capacitors: For parallel combination, V is constant.

Total energy is $U=\frac{1}{2}CV^2=\frac{1}{2}[C_1+C_2+C_3+...]V^2=\frac{1}{2}C_1V^2+\frac{1}{2}C_2V^2+\frac{1}{2}C_3V^2+...$

$$U=U_1+U_2+U_3+...$$

Hence total energy is additive both in series and parallel combinations of capacitors.

Energy density of an electric field: The presence of an electric field in a capacitor implies stored energy.



The capacitance of a parallel plate capacitor is $C=\frac{\varepsilon_0 A}{d}$

Charge on either plate is $Q=\sigma A=(\varepsilon_0 E)A$ $\qquad \left[\because E=\frac{\sigma}{\varepsilon_0}\right]$

The energy stored in the capacitor is

$$U=\frac{Q^2}{2C}=\frac{(\varepsilon_0 E\,A)^2}{2\cdot\frac{\varepsilon_0 A}{d}}=\frac{\varepsilon_0^2E^2A^2}{\frac{2\varepsilon_0 A}{d}}=\frac{\varepsilon_0^2E^2A^2d}{2\varepsilon_0 A}$$

$$\text{Or}\quad U=\frac{\varepsilon_0E^2Ad}{2}$$

$$\text{Or}\quad u=\frac{U}{Ad}=\frac{1}{2}\varepsilon_0E^2$$

Here u is the energy density of the electric field and Ad is the volume inside the two plates of the capacitor

Dielectrics: A dielectric is an insulating material in which all the electrons are tightly bound to the nucleus of the atom. There are no free electrons to carry current. Ebonite, mica and oil are few examples of dielectrics. The electrons are not free to move under the influence of an external field.

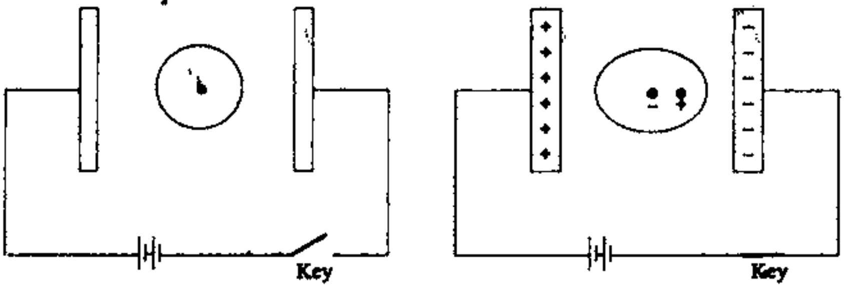
Polar molecule of a dielectrics: A molecule in which the centre of mass of positive charges does not coincide with the centre of mass of negative charges is called a polar molecule. Examples: Water, HCl, CO, etc

Note: Molecules of polar dielectrics possess permanent dipole moments.

Non-polar molecule of a dielectrics: A molecule in which the centre of mass of positive charges coincide with the centre of mass of negative charges is called a non-polar molecule. Examples: H_2, N_2, O_2, CO_2, etc

Note: Molecules of non-polar dielectrics do not possess permanent dipole moments.

Polarisation of a non-polar molecule in an external electric field: If a non polar molecule is placed in an electric field, the centre of charges get displaced. The molecule is then said to be polarised and the arrangement becomes an electric dipole. The molecules of the material are said to have induced dipole moments.



Behaviour of a dielectric of dielectric constant κ in a uniform electric field:

(1) Relative permittivity (κ): The ratio of the strength of the applied electric field $\vec{E}_o$ to the strength of the reduced electric field $\vec{E}$ on introducing the dielectric between the plates of the capacitor is called relative permittivity (κ) of the dielectric medium.

$$\kappa = \frac{E_o}{E}$$

(2) Relation between the induced charge density σ_b on the dielectric and the free charge density σ_f on the plates:

$$\because E_o = \frac{\sigma_f}{\varepsilon_o}, \quad E_i = \frac{\sigma_b}{\varepsilon_o} \quad \text{and} \quad E = \frac{E_o}{\kappa} = \frac{\sigma_f}{\varepsilon_o \kappa}$$

Also $E = E_o - E_i$

$$\therefore \frac{\sigma_f}{\varepsilon_o \kappa} = \frac{\sigma_f}{\varepsilon_o} - \frac{\sigma_b}{\varepsilon_o}$$

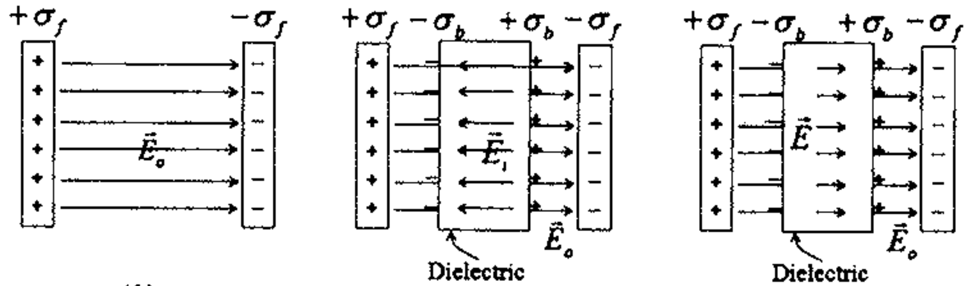
Or $\dfrac{\sigma_b}{\varepsilon_o} = \dfrac{\sigma_f}{\varepsilon_o} - \dfrac{\sigma_f}{\varepsilon_o \kappa}$

Or $\sigma_b = \sigma_f - \dfrac{\sigma_f}{\kappa} = \sigma_f\left(1 - \dfrac{1}{\kappa}\right)$

Or $\sigma_b = \sigma_f\left(\dfrac{\kappa - 1}{\kappa}\right)$

Since κ is always greater than 1, so the induced charge density σ_b on the dielectric is less than the free charge density σ_f on the plates.

(3) Electric field due to induced charges on the dielectric: As shown in the figure, let $\vec{E}_o$ be the external electric field, $\vec{E}_i$ be the electric field due to induced charges on the dielectric and $\vec{E}$ is the net electric field. σ_f and σ_b are the free and bound charge density on the dielectric and on the plates respectively.



(1) (2) (3)

$$\because \vec{E} = \vec{E}_o - \vec{E}_i$$

Or $\vec{E}_i = \vec{E}_o - \vec{E}$

Or $E_i = E_o - E$

Or $E_i = E_o - \dfrac{E_o}{\kappa}$

Or $E_i = E_o\left(1 - \dfrac{1}{\kappa}\right)$

Since κ is always greater than 1, so the induced electric field $\vec{E}_i$ is less than the electric field $\vec{E}_o$.

(4) Polarisation density or Polarisation vector ($\vec{P}$): Polarisation vector is defined as the electric dipole moment induced per unit volume in the dielectric slab when place in an electric field. Thus

$$\vec{P} = n\,\vec{p}$$

where n is the number of molecules per unit volume and $\vec{p}$ is the average electric dipole moment per molecule.

Note: $\vec{P} = n\,\vec{p}$ where $\vec{p}$ is the average dipole moment per molecule, n is the number of molecules per unit volume. Any molecule develops a dipole moment which is proportional to the applied field i.e., $\vec{p} = \varepsilon_0 \alpha\,\vec{E}$ where α is the atomic (electronic) polarizability, ε_0 is the permittivity and $\vec{E}$ is the net electric field inside the dielectric.

(5) To show that $P = \sigma_b$:

Consider a dielectric slab of length l and area A. Then volume of the slab is Al. If σ_b is the induced charge density, then dipole moment of the slab is $(\sigma_b \mathrm{A})\,l$.

By definition, Polarisation vector $P = \dfrac{(\sigma_b A)l}{Al} = \sigma_b$

Thus the induced surface charge density is equal in magnitude to the polarisation vector P.

Note: The net electric field $\vec{E}$ can also be expressed in terms of polarisation vector P as

$$E = E_o - E_i = E_o - \frac{\sigma_b}{\varepsilon_o} = E_o - \frac{P}{\varepsilon_o}$$

(6) Electric susceptibility (χ): Electric susceptibility is the different between the relative permittivity of the medium (κ) and that of a vacuum. i.e.,

$$\chi = \kappa - 1$$

$$\because P = \varepsilon_o \chi E$$

$$E = E_o - \frac{P}{\varepsilon_o} = E_o - \frac{\varepsilon_o \chi E}{\varepsilon_o} = E_o - \chi E$$

$$E_o = E + \chi E = E(1+\chi)$$

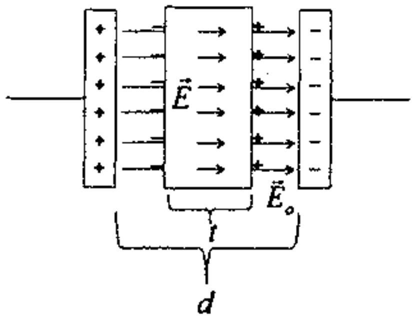
$$\frac{E_o}{E} = 1 + \chi$$

$$\kappa = 1 + \chi$$

$$\chi = \kappa - 1$$

Note: The susceptibility is the quantity that depends upon the nature of the dielectric. χ is unitless and dimensionless. $\vec{P} = n\,\vec{p} = n\varepsilon_0 \alpha\,\vec{E} = \varepsilon_0 \chi\,\vec{E}$ where $\chi = n\alpha$

Capacitance of a parallel plate capacitor with a dielectric slab: The capacitance of a parallel plate capacitor of area A and plate separation d is



$$C_o = \frac{\varepsilon_o A}{d} \text{ -------- (1)}$$

Magnitude of electric field between the plates is

$$E_o = \frac{\sigma}{\varepsilon_o} = \frac{q}{A\varepsilon_o}$$

where $\sigma = \frac{q}{A}$ is the charge density.

Potential difference between the capacitor plates is $V_o = E_o d$.

When a dielectric slab of thickness t ($t < d$) is place between the plates of the capacitor, then the magnitude of electric field between the plates is $E = \frac{E_o}{\kappa}$ and the potential difference between the capacitor plates is

$$V = E_o(d-t) + Et = E_o(d-t) + \frac{E_o}{\kappa}t$$

$$V = E_o\left[(d-t) + \frac{t}{\kappa}\right] = \frac{q}{A\varepsilon_o}\left[(d-t) + \frac{t}{\kappa}\right]$$

$$\frac{q}{V} = \frac{A\varepsilon_o}{(d-t) + \frac{t}{\kappa}}$$

But $C = \frac{q}{V}$ is the capacitance of the capacitor with dielectric slab between the plates.

$$\therefore C = \frac{A\varepsilon_o}{(d-t) + \frac{t}{\kappa}} = \frac{A\varepsilon_o}{d - t + \frac{t}{\kappa}} = \frac{A\varepsilon_o}{d - t\left(1 - \frac{1}{\kappa}\right)} \text{ -------- (2)}$$

From equation (1) and equation (2) we see that $C > C_o$

Special case:

If $t = d$

$$\therefore C = \frac{A\varepsilon_o}{(d-d)+\frac{d}{\kappa}} = \frac{A\varepsilon_o}{d}\kappa = \kappa C_o$$

Discharging action of sharp points (Corona discharge): If the conductor is given a charge q, then the charge density σ at the pointed end will be very high. Consequently, the electric field near the pointed end will be very high which may cause the ionisation or electrical breakdown of the surrounding air. The oppositely charged ions neutralise the pointed end while the similarly charged ions are repelled away.

Van de graaff generator: It is an electrostatic generator capable of building up high potential differences of the order of 10^7 volt.

Principle:

(1) Discharging action of sharp points (corona discharge) i.e.,electric discharge takes place in air or gases readily at a pointed ends of conductors.

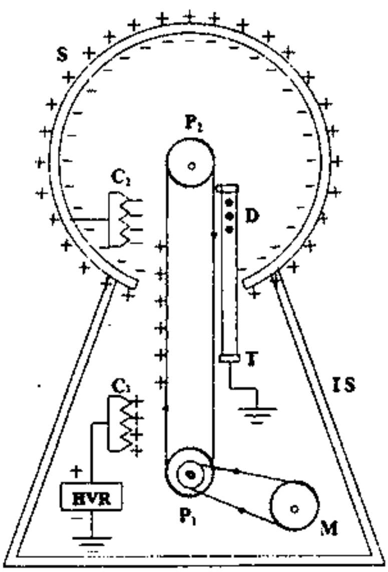
(2) If a charged conductor is brought into internal contact with a hollow conductor, all of its charge transfers to the hollow conductor, howsoever high the potential of the latter may be.

Construction:

Van de Graaff Generator consists of a large (about a few metres in radius) copper spherical shell (S) supported on an insulating stand (IS) which is of several metres high above the ground.

A belt made of insulating fabric (silk, rubber, etc.) is made to run over the pulleys (P_1, P_2) operated by an electric motor (M) such that it ascends on the side of the combs.

Comb (C_1) near the lower pulley is connected to High Voltage Rectifier (HVR) whose other end is earthed. Comb (C_2) near the upper pulley is connected to the sphere S through a conducting rod.



A tube (T) with the charged particles to be accelerated at its top and the target at the bottom is placed as shown in the figure. The bottom end of the tube is earthed for maintaining lower potential.

To avoid the leakage of charges from the sphere, the generator is enclosed in the steel tank filled with air or nitrogen at very high pressure (15 atmospheres).

Working:

Let the positive terminal of the High Voltage Rectifier (HVR) is connected to the comb (C_1). Due to action of points, electric wind is caused and the positive charges are sprayed on to the belt (silk or rubber). The belt made ascending by electric motor (EM) and pulley (P_1) carries these charges in the upward direction.

The comb (C_2) is induced with the negative charges which are carried by conduction to inner surface of the collecting sphere (dome) S through a metallic wire which in turn induces positive charges on the outer surface of the dome.

The comb (C_2) being negatively charged causes electric wind by spraying negative charges due to action of points which neutralize the positive charges on the belt. Therefore the belt does not carry any charge back while descending. (Thus the principle of conservation of charge is obeyed.)

The process continues for a longer time to store more and more charges on the sphere and the potential of the sphere increases considerably. When the charge on the sphere is very high, the leakage of charges due to ionization of surrounding air also increases.

Maximum potential occurs when the rate of charge carried in by the belt is equal to the rate at which charge leaks from the shell due to ionization of air.

Now, if the positively charged particles which are to be accelerated are kept at the top of the tube T, they get accelerated due to difference in potential (the lower end of the tube is connected to the earth and hence at the lower potential) and are made to hit the target for causing nuclear reactions, etc.

Use: The high potential difference set up in the van de Graaff generator is used to accelerate charged particles like protons, deutrons, α-particles, etc to a high energy of about 10 MeV.

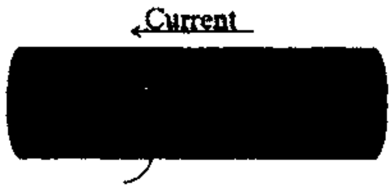
Note:

(1) At C_1, the positive charges ionised the air in its surrounding, the electrons moves to C_1, while the positive ions move to the right and stick to the belt (insulator). The electrons neutralise the positive charges in C_1, but the density of the positive charges is maintained by HVR.

(2) The positive charges in the belt induced negative charges at C_2, while the positive charges are collected at the outer surface of the sphere (dome). In the space between the belt and C_2, the air is ionised, the electrons move to the belt to neutralise the positive charges, while the positive ions move to the right i.e., to C_2 to neutralise the negative charges. So in the sphere there is a deficiency of negative charges which make it a positively charged sphere.

Current electricity: The study of electric charge in motion is called current electricity

Electric current: The flow of electric charges through a conductor constitutes an electric current.



Area perpendicular to the direction of flow of charge

Quantitatively, electric current in a conductor across an area held perpendicular to the direction of flow of charge is defined as the amount of charge flowing across that area per unit time.
If a charge Δq passes through an area in time t to $t + \Delta t$, then the current I at time t is given by

$$I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

If the current is steady i.e., the rate of flow of charge does not change with time, then

$$I = \frac{q}{t}$$

The unit of electric current is ampere (A). It is a scalar quantity.

One ampere of current is said to flow through a wire if one coulomb of charge flows in one second.

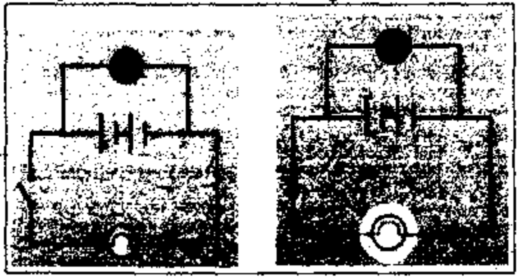
$$1A = \frac{1C}{1s}$$

Note:
(1) Current flows from high potential energy to low potential energy.
(2) Positive charge moves from high potential energy to low potential energy.
(3) Electron moves from low potential energy to high potential energy.

Electronic current: The direction of electronic current is along the direction of the electrons.

Conventional current: The direction of conventional current is along the direction of the positive charge or opposite to the direction of motion of the electron.

Electromotive force (EMF): The EMF of a source is equal to the maximum potential difference between its terminals when it is in the open circuit i.e., when it is not sending any current in the circuit.



The EMF of a source is the difference in potential i.e., $\varepsilon = V_B - V_A = \frac{W}{q}$

So EMF of a battery represents the strength of the battery required to move the positive charges from low potential to high potential inside it.

Note: The potential drop across the terminals of a cell when a current is being drawn from it is called its terminal potential difference (V). ε is always greater than V.

EMF Vrs Potential difference

Electromotive force	Potential difference
It is the work done by a source in taking a unit charge once round the complete circuit	It is the amount of work done in taking a unit charge from one point of a circuit to another
It exists even when the circuit is opened	It exists only when the circuit is closed
It is larger than the p.d. across any circuit element	It is always less than the EMF
It is independent of the external resistance in the circuit	It is dependent on the external resistance
It is equal to the maximum potential difference between the two terminals of a source when it is in the open circuit	Potential difference may exist between any two points of a closed circuit

Ohm's law: The current flowing through a conductor is directly proportional to the potential difference applied across its ends, provided the temperature and other physical conditions remain unchanged i.e.,

$$V \propto I$$
$$V = IR$$

R is called the resistance of the conductor.

The SI unit of resistance is ohm (Ω). 1 Ω = 1 volt/ 1 ampere

The dimensions of resistance is $[R]=\frac{[V]}{[I]}=\frac{[W/q]}{[I]}=\frac{[ML^2T^{-2}]}{[A][AT]}=[ML^2T^{-3}A^{-2}]$

International ohm: It is defined as the resistance of 106.3 cm long mercury column of 1 mm^2 cross-sectional area and mass 14.4521 g at $0^0\,C$

Resistance: The resistance of a conductor is the property by virtue of which it opposes the flow of charges through it.

Conductance: The conductance of a conductor is the ease with which electric charges flow through it. It is equal to the reciprocal of its resistance and is denoted by G. Thus

$$G=\frac{1}{R}$$

The SI unit of conductance is ohm^{-1} or mho or siemens (S)

Factors affecting the resistance (Resistivity): At constant temperature, the resistance R of a conductor depends on the following factors.

(1) Length (l): $R \propto l$

(2) Area of cross-section (A): $R \propto \frac{1}{A}$

(3) Nature of the material:

$$\therefore R \propto \frac{l}{A} \qquad \text{Or} \qquad R=\rho\frac{l}{A}$$

ρ is called the resistivity or the specific resistance of the material of the conductor. It depends on the nature of the material, temperature etc but not on the size and shape.

The SI unit of ρ is Ωm.

Resistivity (ρ): Specific resistance or resistivity of a material is the resistance offered by $1m$ length of wire of the material having area of cross-section of $1m^2$ i.e.,

$$R = \rho\frac{l}{A} = \rho\frac{1}{1} \qquad \Rightarrow R = \rho$$

Conductivity: The reciprocal of the resistivity of a material is called its conductivity and is denoted by σ. Thus

$$\sigma = \frac{1}{\rho}$$

The SI unit of conductivity is $ohm^{-1}\ m^{-1}$ or mho m^{-1} or S m^{-1}

Current density: The current density at any point inside a conductor is defined as the amount of charge flowing per second through a unit area held normal to the direction of the flow of charge at that point.

Area (A) perpendicular to the direction of flow of charge
(1)

Area (A) inclined at an angle θ to the direction of current (I)
(2)

In figure (1) Current density is $j = \frac{q/t}{A} = \frac{I}{A}$

In figure (2) Current density is $j = \frac{I}{A\cos\theta}$

$$\text{Or } I = jA\cos\theta = \vec{j}\cdot\vec{A}$$

Current density is a vector quantity. Its SI unit is Am^{-2}

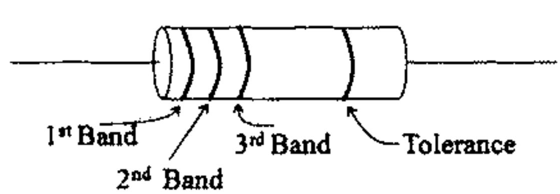
Classification of materials in terms of resistivity:

(1) Conductors: The materials which conduct electric current fairly well are called conductors.
$\rho = 10^{-8}\ \Omega m$ to $10^{-6}\ \Omega m$.

(2) Insulators: The materials which do not conduct electric current are called insulators.
$\rho > 10^{4}\ \Omega m$.

(3) Semiconductors: These are the materials whose resistivities lie between those of conductors and insulators.
ρ lies between $10^{-6}\ \Omega m$ and $10^{4}\ \Omega m$.

Carbon resistor and colour code: A carbon resistor is made from powdered carbon mixed with a binding material and baked into a small tube with the wire attached to each end.



Colour code

Aids	Colour	1^{st} Band	2^{nd} Band	3^{rd} Band (×)	Tolerance
B	Black	0	0	$\times 10^0$	
B	Brown	1	1	$\times 10^1$	
R	Red	2	2	$\times 10^2$	Gold ± 5%
O	Orange	3	3	$\times 10^3$	
Y	Yellow	4	4	$\times 10^4$	Silver ± 10%
Great	Green	5	5	$\times 10^5$	
Britain	Blue	6	6	$\times 10^6$	No colour ± 20%
Very	Violet	7	7	$\times 10^7$	
Good	Grey	8	8	$\times 10^8$	
Wife	White	9	9	$\times 10^9$	

Mechanism of current flow in a conductor – Drift velocity and relaxation time: Just after collision, let $\vec{u}_1, \vec{u}_2, \vec{u}_3, \ldots \vec{u}_n$ be the velocities of the 1^{st}, 2^{nd}, 3^{rd}, ...n^{th} electron, whose average velocity is $\vec{u} = \frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \ldots + \vec{u}_n}{n} = 0$.

Force experience by an electron is $\vec{F} = -e\vec{E}$

The acceleration of the electron is $\vec{a} = -\frac{e\vec{E}}{m}$

Before the 1^{st} electron will suffer the next collision, the velocity will be

$$\vec{v}_1 = \vec{u}_1 + \vec{a}\,\tau_1$$

The velocities of the other electrons are

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\,\tau_2, \qquad \vec{v}_3 = \vec{u}_3 + \vec{a}\,\tau_3, \ldots \qquad \vec{v}_n = \vec{u}_n + \vec{a}\,\tau_n.$$

The average velocity of all the n electrons is $\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \ldots + \vec{v}_n}{n}$

$$\vec{v}_d = \frac{(\vec{u}_1 + \vec{a}\,\tau_1) + (\vec{u}_2 + \vec{a}\,\tau_2) + (\vec{u}_3 + \vec{a}\,\tau_3) + \ldots + (\vec{u}_n + \vec{a}\,\tau_n)}{n}$$

$$\vec{v}_d = \frac{\vec{u}_1 + \vec{a}\,\tau_1 + \vec{u}_2 + \vec{a}\,\tau_2 + \vec{u}_3 + \vec{a}\,\tau_3 + \ldots + \vec{u}_n + \vec{a}\,\tau_n}{n}$$

$$\vec{v}_d = \frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \ldots + \vec{u}_n + \vec{a}\,\tau_1 + \vec{a}\,\tau_2 + \vec{a}\,\tau_3 + \ldots + \vec{a}\,\tau_n}{n}$$

$$\vec{v}_d = \frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \ldots + \vec{u}_n}{n} + \frac{\vec{a}\,\tau_1 + \vec{a}\,\tau_2 + \vec{a}\,\tau_3 + \ldots + \vec{a}\,\tau_n}{n}$$

$$\vec{v}_d = \frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \ldots + \vec{u}_n}{n} + \vec{a}\left(\frac{\tau_1 + \tau_2 + \tau_3 + \ldots + \tau_n}{n}\right)$$

$$\vec{v}_d = 0 + \vec{a}\,\tau$$

$$\vec{v}_d = \vec{a}\,\tau$$

$$\vec{v}_d = -\frac{e\vec{E}\tau}{m}$$

Consider the magnitude only we have

$$v_d = \frac{eE\tau}{m}$$

Or $\quad v_d = \dfrac{eV\tau}{m\,l}$

τ is the average time between two successive collisión called the relaxation time. $\vec{v}_d$ is called the drift velocity.

Drift velocity: The drift velocity of the electrons is defined as the average velocity gained by the free electrons of a conductor moving in the opposite direction of the external applied electric field.

The drift velocity of the electrons in a conductor is given by

$$\vec{v}_d = \vec{a}\tau$$

Where $\vec{a}$ is the acceleration and τ is the rèlaxation time.

$$\because \vec{a} = \frac{\vec{F}}{m} = -\frac{e\vec{E}}{m}$$

Where $\vec{F}$ is the electric force, $\vec{E}$ is the electric field, m is the mass of the electron and e is the charge on the electron.

$$\therefore \vec{v}_d = -\frac{e\vec{E}\tau}{m}$$

In magnitude, the drift velocity is $v_d = \dfrac{eE\tau}{m}$

Again $\because E = \dfrac{V}{l}$

Where V is the potential difference across the conductor and l is the length of the conductor.

$$\therefore v_d = \frac{eV\tau}{m\,l}$$

Note:

(1) The average time that the electron spends between two successive collisions is called the relaxation time (τ). Its value is of the order of $10^{-14}\,s$

(2) The average velocity with which free electrons get drifted in a metallic conductor under the influence of electric field is called drift velocity (v_d). The drift velocity of free electrons is of the order of $10^{-5}\,ms^{-1}$ or $0.01\,mm\,s^{-1}$

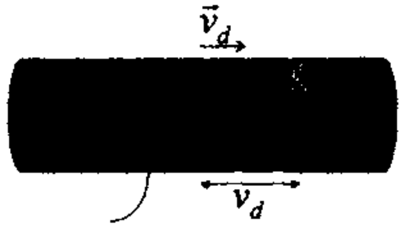
Relation between electric current and drift velocity:

Let n be the number of electrons per unit volume (electron density) moving with drift velocity $\vec{v}_d$ through an area A in a conductor in a unit time.

The total number of electrons per unit volume (electron density) crossing the area A is

$$q = neAv_d$$

$$\because I = \frac{q}{t} \qquad \text{Or} \qquad I = \frac{q}{1} = q$$



Area A perpendicular to the direction of flow of electrons

$\therefore$ The electric current is $I = neAv_d$

Hence current flowing through a conductor is directly proportional to the drift velocity i.e.,

$$I \propto v_d$$

Again $\frac{I}{A} = nev_d$

$\therefore j = nev_d$

Where j is the current density. In vector form, $\vec{j} = -ne\vec{v}_d$

Note: Since the drift velocity of the electrons is $v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$

(1) The current through the conductor $I = neAv_d = neA\frac{eE\tau}{m} = \frac{ne^2 A\tau E}{m}$

(2) Also the current through the conductor $I = neAv_d = neA\frac{eV\tau}{ml} = \frac{ne^2 A\tau V}{ml}$

Validity of Ohm's law or Derivation of ohm's law:

The drift velocity of the electrons is $v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$

The current through the conductor is $I = neAv_d = neA\frac{eV\tau}{ml}$

$$\text{So } \frac{V}{I} = \frac{ml}{ne^2 A\tau}$$

At a fixed temperature, the quantities m, l, n, e, A and τ all have constant values for a given conductor.

$$\text{So } \frac{V}{I} = \frac{ml}{ne^2 A\tau} = \text{constant} = R$$

where $R = \frac{ml}{ne^2 A\tau}$

Hence $V \propto I$ which is Ohm's law.

Resistivity in terms of electron density and relaxation time: The resistance R of a conductor of length l, area of cross-section A and resistivity ρ is given by

$$R = \rho\frac{l}{A}$$

$$\text{But } R = \frac{ml}{ne^2 A\tau}$$

$$\therefore \rho = \frac{m}{ne^2\tau}$$

ρ depends on the electron density of the conductor, and the relaxation time. It is independent of the dimensions of the conductor.

Mobility of charge carriers (μ): The mobility of a charge carrier is defined as the drift velocity acquired by it in a unit electric field.

$$\mu = \frac{v_d}{E} = \frac{eE\tau}{mE} = \frac{e\tau}{m}$$

For electron $\mu_e = \frac{e\tau_e}{m_e}$

For hole $\mu_h = \frac{e\tau_h}{m_h}$ (in semi conductor)

μ is positive for both electrons and holes, although their drift velocities are opposite to each other.

Or

$$\because I = \frac{ne^2 A\tau E}{m}$$

$$\text{Or } \frac{I}{neAE} = \frac{e\tau}{m} \text{ -------- (1)}$$

$$\text{By definition } \mu = \frac{v_d}{E} = \frac{eE\tau}{mE} = \frac{e\tau}{m} \text{ -------- (2)}$$

Equating equation (1) and equation (2) we have

$$\mu = \frac{I}{neAE}$$

Note: How easy the electrons accelerate in a conductor, in the presence of the electric field is the mobility. Or how often the electrons hit something in a conductor to get scattered.

Temperature dependence of resistance: It has been found that in the normal range of temperatures, the resistance of a metallic conductor increases linearly with the rise in temperature. Therefore resistance-temperature graph is a straight line as shown in the figure. Consider a metallic conductor having resistance R_o at $t_o{}^0C$ and R_1 at $t_1{}^0C$. Then in the normal range of temperatures, the increase in resistance i.e., $(R_1 - R_o)$ is

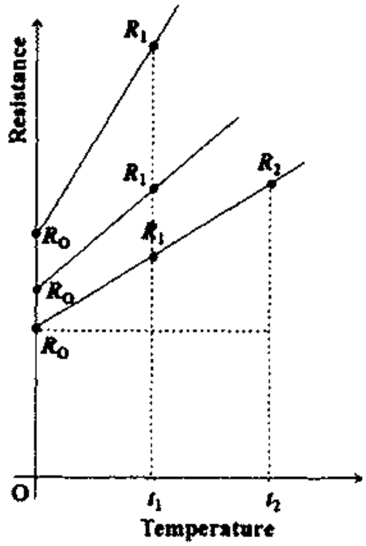
(i) Directly proportional to the initial resistance i.e.,

$R_1 - R_o \propto R_o$

(ii) Directly proportional to the rise in temperature i.e.,

$R_1 - R_o \propto t_1$

(iii) Depends upon the nature of the material.



Combining the first two, we get,

$R_1 - R_o \propto R_o t_1$

$R_1 - R_o = \alpha R_o t_1$ ------ (1)

Where α is called temperature co-efficient of resistance. Its value depends upon the nature of the material and temperature

Rearranging equation (1), we get

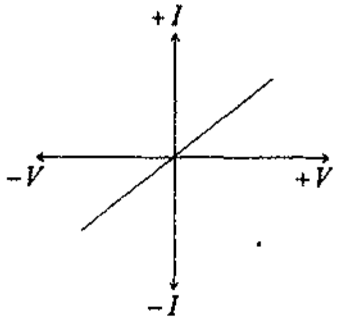
$R_1 = R_o(1 + \alpha t_1)$

And $\alpha = \frac{R_1 - R_o}{R_o t_1}$

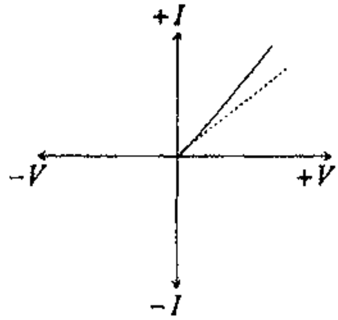
Temperature co-efficient of resistance of a conductor is the increase in resistance per ohm original resistance per ${}^0C$ rise in temperature.

Limitations of ohm's law – ohmic and non-ohmic conductors: A conductor that obeys Ohm's Law is called an ohmic conductor and the one that does not obey Ohm's law is called a non-ohmic conductor.

Ohmic conductors: A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field (figure 1)

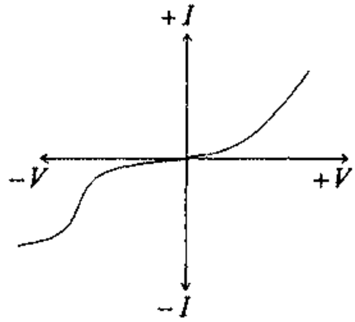


(Figure 1)

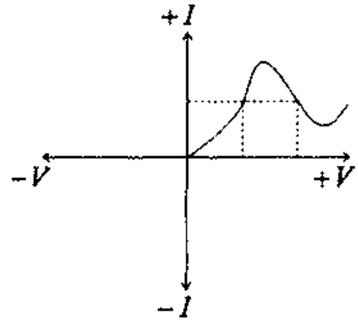


(Figure 2)

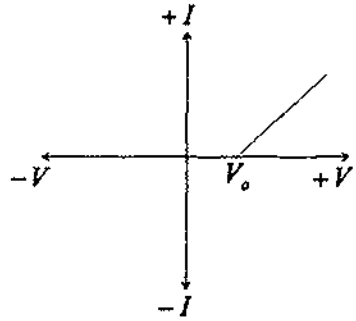
Non-ohmic conductors: Non-ohmic substance may have one or more of the following properties:
(1) V ceases to be proportional to I (figure 2)
(2) The relation between V and I depends on the sign of V. In other words, if I is the current for a certain V, then reversing the direction of V keeping its magnitude fixed, does not produce a current of the same magnitude as I in the opposite direction (figure 3).
(3) The relation between V and I is not unique, i.e., there is more than one value of V for the same current I (figure 4).
(4) The straight line V-I graph does not pass through the origin (figure 5).



(Figure 3-diode)



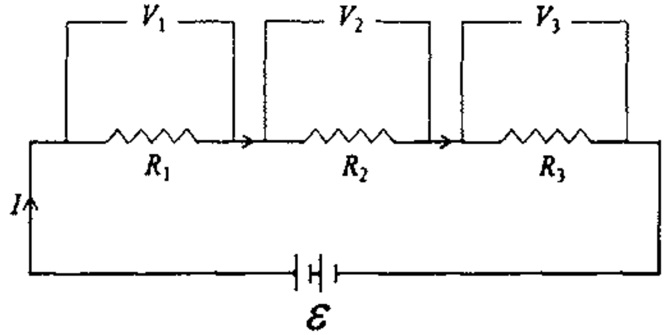
(Figure 4-GaAs)



(Figure 5-Water voltmeter)

Resistances in series: A number of resistors are said to be connected in series if the same current flows through each resistor and there is only one path for the current to flow through all of them.

Consider three resistors of resistances R_1, R_2 and R_3 connected in series across a battery of EMF ε volt as shown in the figure. Let I be the circuit current. By Ohm's law



$$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$$

If R_S is the equivalent resistance of the series combination, then we must have

$$\varepsilon = IR_S$$

$$\text{But } \varepsilon = V_1 + V_2 + V_3$$

$$IR_S = IR_1 + IR_2 + IR_3$$

$$R_S = R_1 + R_2 + R_3$$

The equivalent resistance of n resistances connected in series will be

$$R_S = R_1 + R_2 + R_3 + ... + R_n$$

Note:

(1) The current in each resistor is the same

(2) The total resistance of the circuit is equal to the sum of individual resistances plus the internal resistance of the cell if any.

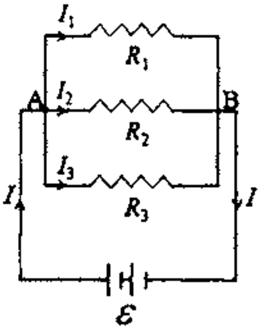
(3) The voltage drop across any resistor is directly proportional to the resistance of that resistor.

(4) The current in the circuit is independent to the relative positions of the resistors in the circuit

(5) The total resistance in the series circuit is more than the largest resistance in the circuit.

(6) The main disadvantage of a series circuit is that if one device (or resistor) fails, the current in the whole circuit ceases.

Resistances in parallel: A number of resistors are said to be connected in parallel if the voltage across each resistor is the same.



Consider three resistors of resistances R_1, R_2 and R_3 connected in parallel between points A and B across a battery of EMF ε volt as shown in the figure. Let I_1, I_2, I_3 be the currents through the resistances R_1, R_2 and R_3 respectively. By Ohm's law

$$I_1 = \frac{\varepsilon}{R_1}, \qquad I_2 = \frac{\varepsilon}{R_2}, \qquad I_3 = \frac{\varepsilon}{R_3}$$

If R_P is the equivalent resistance of the series combination, then we must have

$$I = \frac{\varepsilon}{R_P}$$

But $I = I_1 + I_2 + I_3$ resistance of the series

$$\frac{\varepsilon}{R_P} = \frac{\varepsilon}{R_1} + \frac{\varepsilon}{R_2} + \frac{\varepsilon}{R_3}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The equivalent resistance of n resistances connected in parallel will be

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

Note:

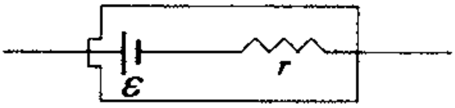
(1) The voltage across each resistor is the same

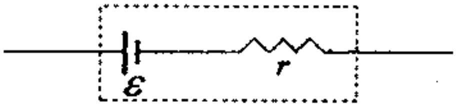
(2) The current through any resistor is inversely proportional to its resistance.

(3) The total current in the circuit is equal to the sum of currents in its parallel branches.

(4) The reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.
(5) As the number of parallel branches is increased, the total resistance of the circuit is decreased.
(6) The total resistance of the circuit is always less than the smallest of the resistances.

Internal resistance of a cell: The resistance offered by the electrolyte of a cell to the flow of current between its electrodes is called internal resistance of the cell



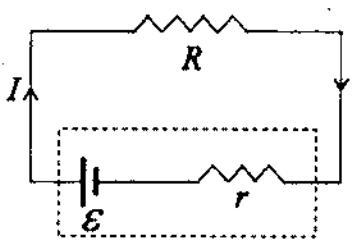


It depends on:
(1) Nature of the electrolyte (NaC*l* or $CuSo_4$) etc
(2) Nature of the electrodes
(3) It is directly proportional to the concentration of the electrolyte
(4) It varies inversely as the common area of the electrodes immersed in the electrolyte

Relation between internal resistance (r), EMF (ε) and terminal potential difference (V) of a cell: Consider a cell of EMF ε and internal resistance r connected to an external resistance R as shown in the figure. Suppose a constant current I flows through this circuit. By definition of EMF,

$$\varepsilon = V + V'$$

By Ohm's law $V = IR$ and $V' = Ir$



Hence the EMF of a cell is $\varepsilon = IR + Ir = I(R + r)$

The current in the circuit is $I = \dfrac{\varepsilon}{R + r}$

Potential difference across R is $V = IR = \dfrac{\varepsilon R}{R + r}$

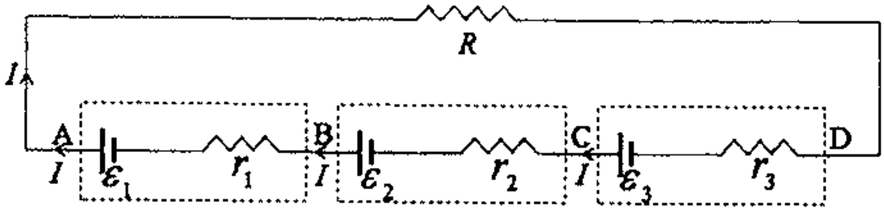
$$\because \varepsilon = V + Ir$$
$$Ir = \varepsilon - V$$

The internal resistance of a cell is $r = \dfrac{\varepsilon - V}{I} = \dfrac{\varepsilon - V}{\frac{V}{R}} = \left(\dfrac{\varepsilon - V}{V}\right)R$

Note: The terminal voltage V of a cell is the voltage across its load (R) when the cell is delivering current i.e., the cell circuit is closed.

Combinations of cells in series: When the negative terminal of one cell is connected to the positive terminal of the other cell and so on, the cells are said to be connected in series.

As shown in the figure, suppose three cells of EMF $\varepsilon_1, \varepsilon_2$ and ε_3 and internal resistances r_1, r_2 and r_3 are connected in series between points A and D. Let I be the current flowing through the series combination.



The potential difference across A and B is $V_{AB} = \varepsilon_1 - Ir_1$
The potential difference across B and C is $V_{BC} = \varepsilon_2 - Ir_2$
The potential difference across C and D is $V_{CD} = \varepsilon_3 - Ir_3$

The potential difference across A and D is $V_{AD} = V_{AB} + V_{BC} + V_{CD}$

$$V_{AD} = (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2) + (\varepsilon_3 - Ir_3)$$
$$V_{AD} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 - I(r_1 + r_2 + r_3)$$
$$V_{AD} = \varepsilon_{eq} - Ir_{eq}$$

But $V_{AD} = IR$

$$\therefore IR = \varepsilon_{eq} - Ir_{eq}$$
$$IR + Ir_{eq} = \varepsilon_{eq}$$
$$I(R + r_{eq}) = \varepsilon_{eq}$$
$$I = \frac{\varepsilon_{eq}}{R + r_{eq}}$$

If $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon$ (say)
and $r_1 = r_2 = r_3 = r$ (say)
Then $\varepsilon_{eq} = \varepsilon + \varepsilon + \varepsilon = 3\varepsilon$
and $r_{eq} = r + r + r = 3r$

$$\therefore I = \frac{3\varepsilon}{R + 3r}$$

For n number of cells in series, the current I will be $I = \frac{n\varepsilon}{R + nr}$

Special Cases:
(1) If $R >> nr$,
Then $I = \frac{n\varepsilon}{R}$ (n times the current $\frac{\varepsilon}{R}$ that can be drawn from one cell)
(2) If $R << nr$
Then $I = \frac{n\varepsilon}{nr} = \frac{\varepsilon}{r}$ (the current given by a single cell)

Combinations of cells in parallel: When the positive terminals of all the cells are connected together, and in the like manner all the negative terminals are connected together, the cells are said to be connected in parallel.
As shown in the figure, suppose two cells of EMF ε_1 and ε_2, and internal resistances r_1 and r_2 respectively are connected in parallel. Suppose the current I_1 and I_2 from the positive terminals of the two cells flow towards the junction A, and current I flows out.

As the two cells are connected in parallel between the same two points A and B, the potential difference V across both cells must be the same.

The potential difference between the terminals of the first cell is

$$V = \varepsilon_1 - I_1 r_1 \qquad \Rightarrow I_1 = \frac{\varepsilon_1 - V}{r_1}$$

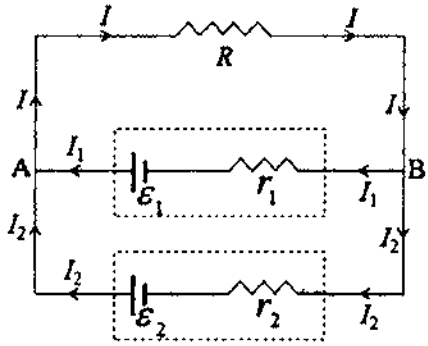
The potential difference between the terminals of the second cell is

$$V = \varepsilon_2 - I_2 r_2 \qquad \Rightarrow I_2 = \frac{\varepsilon_2 - V}{r_2}$$

Hence $I = I_1 + I_2 = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} = \frac{\varepsilon_1}{r_1} - \frac{V}{r_1} + \frac{\varepsilon_2}{r_2} - \frac{V}{r_2}$

$$I = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$I = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2}\right) - V\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$



$$V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2}\right) - I$$

$$V = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2}\right) \times \left(\frac{r_1 r_2}{r_1 + r_2}\right) - I \times \left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

$$V = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}\right) - I \times \left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

If we replace the parallel combination by a single cell of EMF ε_{eq} and internal resistance r_{eq} then

$$V = \varepsilon_{eq} - I r_{eq}$$

Comparing the above two equations we have

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

If $r_1 = r_2 = r$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon$

Then $\varepsilon_{eq} = \frac{\varepsilon r + \varepsilon r}{r + r} = \frac{2\varepsilon r}{2r} = \varepsilon$ and $r_{eq} = \frac{r r}{r + r} = \frac{r^2}{2r} = \frac{r}{2}$

$$I = \frac{\varepsilon_{eq}}{R + r_{eq}} = \frac{\varepsilon}{R + \frac{r}{2}} = \frac{\varepsilon}{\frac{2R + r}{2}}$$

$$I = \frac{2\varepsilon}{2R + r}$$

For m number of cells in parallel, the current I will be $I = \frac{m\varepsilon}{mR + r}$

Special Cases:

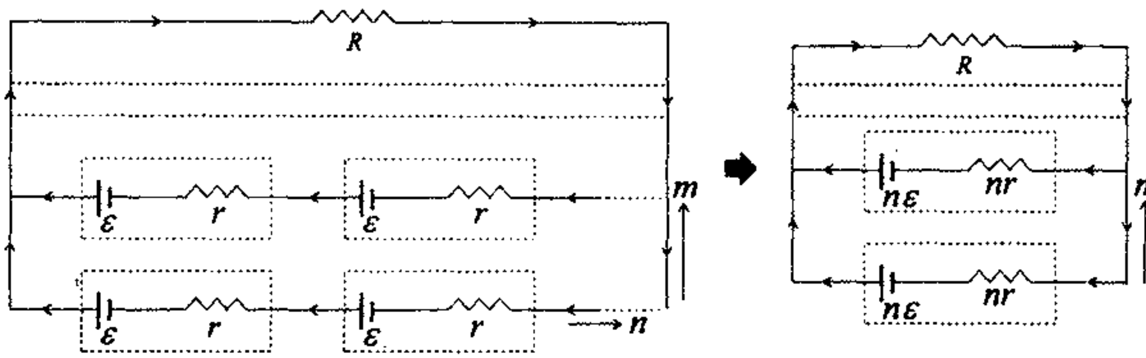
(1) If $mR << r$

Then $I = \frac{m\varepsilon}{r}$ (*m* times the current $\frac{\varepsilon}{r}$ given by a single cell)

(2) If $mR >> r$

Then $I = \frac{\varepsilon}{R}$ (the current drawn from one cell)

Mixed grouping of cells: In this combination, a certain number of identical cells are joined in series, and all such rows are then connected in parallel with each other.



As shown in the figure, suppose *n* cells, each of EMF ε and internal resistance *r*, are connected in series in each row and *m* such rows are connected in parallel across the external resistance *R*.

Net EMF of each row of *n* cells in series = $n\varepsilon$

Net internal resistance of each row of *n* cells in series = nr

Total EMF of *m* cells in parallel = $n\varepsilon$

Total internal resistance of *m* cells in parallel $= \frac{nr}{m}$

The current through the external resistance *R* is

$$I = \frac{\text{Total EMF}}{\text{Total resistance}} = \frac{n\varepsilon}{R + \frac{nr}{m}} = \frac{mnr}{mR + nr}$$

The current *I* will be maximum if $mR + nr$ is minimum.

$$mR + nr = \left(\sqrt{mR}\right)^2 + \left(\sqrt{nr}\right)^2$$

$$mR + nr = \left(\sqrt{mR}\right)^2 + \left(\sqrt{nr}\right)^2 - 2\sqrt{mR}\sqrt{nr} + 2\sqrt{mR}\sqrt{nr}$$

$$mR + nr = \left[\left(\sqrt{mR}\right) - \left(\sqrt{nr}\right)\right]^2 + 2\sqrt{mR}\sqrt{nr}$$

$mR + nr$ will be minimum if $\sqrt{mR} - \sqrt{nr}$ is equal to zero i.e.,

$$\sqrt{mR} - \sqrt{nr} = 0$$

$$\sqrt{mR} = \sqrt{nr}$$

$$mR = nr$$

$$\therefore R = \frac{nr}{m}$$

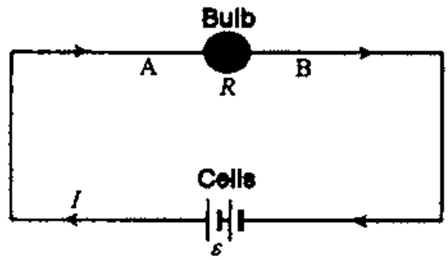
External resistance = Total internal resistance of the cells

Note: In a mixed grouping of cells, the current through the external resistance will be maximum if the external resistance is equal to the total internal resistance of the cells.

Heating effect of current: The phenomenon of the production of heat in a resistor by the flow of an electric current through it is called heating effect of current or joule heating

Note:
(1) The speed of the electrons does not increase beyond a constant drift speed.
(2) The metal ions begin to vibrate about their mean positions more and more violently. The average kinetic energy of the ions increases. This increases the temperature of the conductor.

Heat produced by electric current - Joule's law: Consider a conductor AB of resistance R as shown in the figure. A source of EMF maintains a potential difference V between its ends A and B and sends a steady current I from A to B.



Potential at point A is V_A
Potential at point B is V_B
Clearly $V_A > V_B$
The decrease in potential across AB is

$$V = V_A - V_B$$

The amount of charge that flows across AB in time t is

$$q = It$$

The loss of potential energy U is

$$U = qV = (It)V = VIt \text{ ---------------- (1)}$$

$$U = qV = (It)(IR) = I^2Rt \text{ ---------------- (2)}$$

Or

$$U = VIt = (IR)(It) = I^2Rt$$

$$U = I^2Rt = \frac{I^2R^2t}{R} = \frac{V^2t}{R} \text{ ---------------- (3)}$$

Or

$$U = VIt = V\left(\frac{V}{R}\right)t = \frac{V^2t}{R}$$

The loss of potential energy U is equal to the heat H produced in the conductor.

$$H = VIt \text{ joule} = \frac{VIt}{4.18}\text{cal ---------------- (4)}$$

$$H = I^2Rt \text{ joule} = \frac{I^2Rt}{4.18}\text{cal ---------------- (5)}$$

$$H = \frac{V^2t}{R} \text{ joule} = \frac{V^2t}{4.18 \times R}\text{ cal ---------------- (6)}$$

These equations are known as Joule's law of heating.

According to this law, the heat produced in a resistor is
(1) directly proportional to the square of current for a given R
(2) directly proportional to the resistance R for a given I
(3) inversely proportional to the resistance R for a given V
(4) directly proportional to the time for which the current flows through the resistor.

Note:
1 cal in physics = 4.18 J
1 Cal in food = 1000 cal = 4180 J

Electric power: The power of an electrical appliance is the rate at which electrical energy is converted into other forms of energy like heat, light etc. For example a 60W bulb converts 60J of electrical energy into heat and light each second.
Thus

$$P = \frac{\text{Electrical energy}}{\text{time}} = \frac{VIt}{t} = VI \text{ watt}$$

$$P = \frac{\text{Electrical energy}}{\text{time}} = \frac{I^2Rt}{t} = I^2R \text{ watt}$$

$$P = \frac{\text{Electrical energy}}{\text{time}} = \frac{V^2t}{Rt} = \frac{V^2}{R} \text{ watt}$$

The SI unit of power is watt (W) i.e., $1\, watt = \frac{1\, joule}{1\, s}$

Hence electric power of a circuit or a device is one *watt* if a current of $1A$ flows through it, when a potential difference of $1V$ is maintained across it. (From $P = VI$)

Note:
$1kW = 1000\ W$
$1MW = 10^3\ kW = 10^6\ W$
The commercial unit of power is horse power (HP) where $1HP = 746\ W$

Electrical energy: The loss of electrical potential energy in maintaining current in a circuit is called electrical energy consumed in the circuit.

Electric energy, $W = Pt = VIt\ joule = I^2Rt\ joule$
$1\ joule = 1\ watt \times 1\ second$

1 *kilowatt-hour* is defined as the electric energy consumed by an appliance of 1 *kilowatt* in 1 *hour*.

1 *kilowatt-hour* = 1 *kilowatt* × 1 *hour*
1 *kilowatt-hour* = 1000 *watt* × 3600 *second*
1 *kilowatt-hour* = 3.6×10^6 *joule*

Note:
(1) $1kW$-h is also called Board of trade (B.O.T) unit or unit of electricity.
(2) The resistance of the wires supplying electric current is very small, therefore these wires do not heat up when current passes through them.

(3) The resistance of the filament of a lamp is very high, therefore it shows more heating effect when electric current passes through it.
(4) Heater wire must have a high resistivity and high melting point.

Power rating: The power rating of an electrical appliance is the electrical energy consumed per second by the appliance when connected across the marked voltage of the mains.
If a voltage V applied across a circuit element of resistance R sends current I through it, then power rating of the element will be

$$P = VI = I^2R = \frac{V^2}{R}$$

Note:
(1) Suppose the voltmeter reads V *volt* and the ammeter reads I *amp*, then the power rating of the electric lamp will be

$P = VI$ *watt*

(2) When a 60W and a 100W bulbs are connected in series, the 40W bulb will glow brighter than the 100W bulb.
(3) When a 60W and a 100W bulbs are connected in parallel, the 100W bulb will glow brighter than the 60W bulb.
(4) In series connection, if any bulb get fused, then others will not glow.
(5) In parallel connection, if any bulb get fused, then others will continue to glow.

Kirchhoff's first law or Kirchhoff's current law (KCL) or junction rule: In an electric circuit, the algebraic sum of currents at any junction is zero.

Or

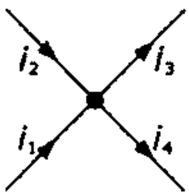
The sum of currents entering a junction is equal to the sum of currents leaving that junction.

Mathematically, this law may be expressed as $\sum I = 0$

Example $i_1 + i_2 + (-i_3) + (-i_4) = 0$

$i_1 + i_2 - i_3 - i_4 = 0$

$i_1 + i_2 = i_3 + i_4$



Sign convention for applying junction rule
(1) The currents flowing towards the junction are taken as positive.
(2) The currents flowing away from the junction are taken as negative.

Note: Kirchhoff's current law is based on the law of conservation of charge.

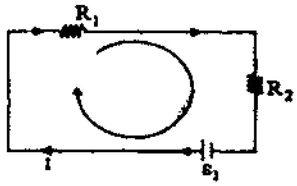
Kirchhoff's second law or Kirchhoff's voltage law (KVL) or loop rule: Around any closed loop of a network, the algebraic sum of changes in potential must be zero.

Or

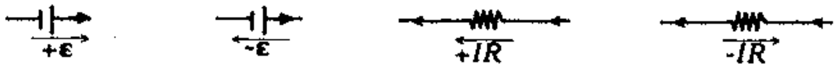
The algebraic sum of EMFs in any loop of a circuit is equal to the sum of the products of currents and resistances in it.
Mathematically, the loop rule may be expressed as

$\sum \Delta V = 0$ or $\sum \varepsilon = \sum IR$

Example



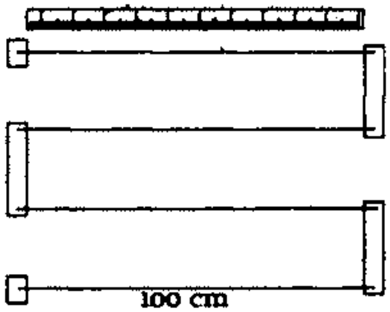
$\varepsilon_1 = IR_1 + IR_2$

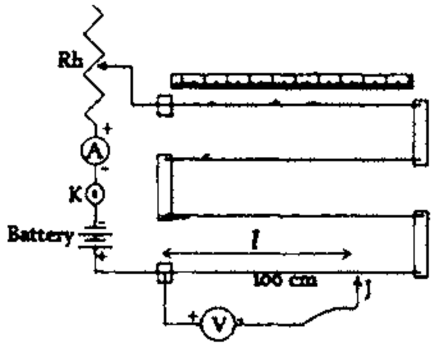


Sign convention for applying loop rule
(1) We can take any direction (clockwise or anti-clockwise) as the direction of traversal.
(2) The EMF of cell is taken as positive if the direction of traversal is from its negative to the positive terminal (through the electrolyte)

Note: Kirchhoff's voltage law is based on the law of conservation of energy.

Potentiometer: A potentiometer is a device used to measure an unknown EMF or potential difference accurately.

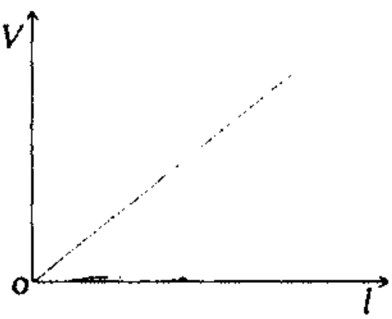




Principle: The basic principle of a potentiometer is that when a constant current flows through a wire of uniform cross-sectional area and composition, the potential drop across any length of the wire is directly proportional to that length.

$$V = IR = I\left(\rho\frac{l}{A}\right) = \left(\frac{I\rho}{A}\right)l$$

$$\therefore V \propto l$$



Potential drop $V \propto$ length l

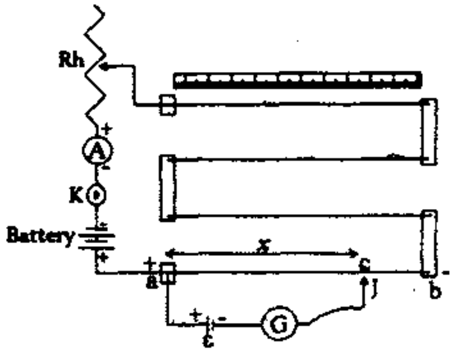
Sensitiveness of a potentiometer: A potentiometer is sensitive if
(1) It is capable of measuring very small potential differences
(2) It shows a significant change in balancing length for a small change in the potential difference being measured.
The sensitivity of a potentiometer depends on the potential gradient along its wire. Smaller the potential gradient, greater will be the sensitivity of the potentiometer.

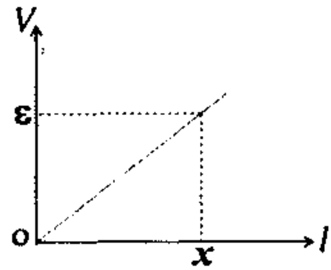
Potential gradient: The potential drop per unit length of the potentiometer wire is known as potential gradient. It is given by $k = \frac{V}{l}$

The SI unit of potential gradient is Vm^{-1} and its practical unit is Vcm^{-1}.

Uses of a Potentiometer:

(1) Measurement of EMF of a cell:





When the Galvanometer shows no deflection at point c, the EMF of the cell is ε, where ε = potential difference across 'ac'

$\because V \propto l$, and let ac = x,

$\therefore$ from the graph ε is known

(2) Determination of internal Resistance of a cell:

When key k' is opened,

$\varepsilon \propto l_1$ or $\varepsilon = k\,l_1$

When key k' is closed,

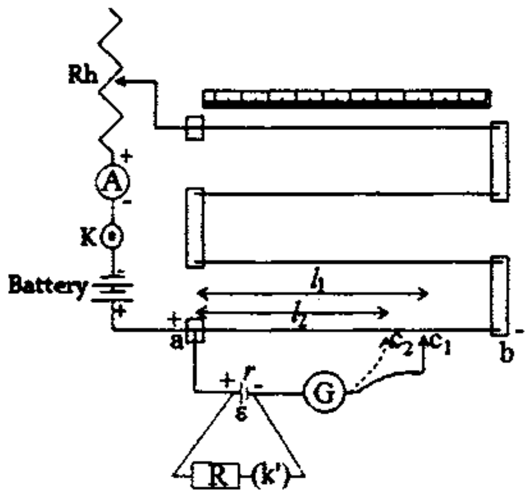
$V \propto l_2$ or $V = k\,l_2$

We know that $r = \dfrac{\varepsilon - V}{V} \times R$

$$r = \frac{kl_1 - kl_2}{kl_2} \times R$$

$\therefore$ The internal resistance of the cell is

$$r = \left(\frac{l_1 - l_2}{l_2}\right) \times R$$

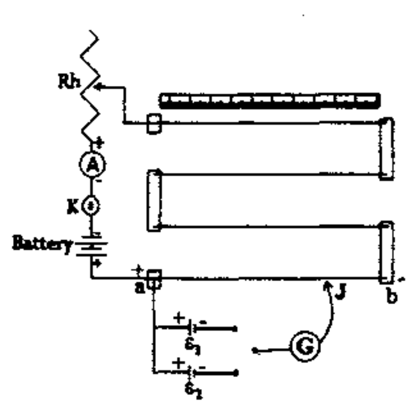


Comparison the EMF of two cells: Let ε_1 be the EMF of the first cell when the Galvanometer shows no deflection at l_1 and ε_2 be the EMF of the second cell when the Galvanometer shows no deflection at l_2. Thus

$\varepsilon_1 \propto l_1 \quad \Rightarrow \varepsilon_1 = k\,l_1$

And $\quad \varepsilon_2 \propto l_2 \quad \Rightarrow \varepsilon_2 = k\,l_2$

Or $\quad \dfrac{\varepsilon_2}{\varepsilon_1} = \dfrac{k\,l_2}{k\,l_1} = \dfrac{l_2}{l_1}$



Advantages of a potentiometer:

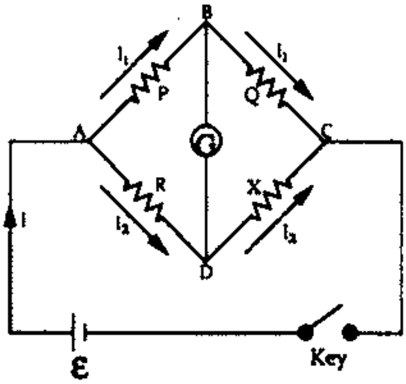
(1) It is a null deflection method, and therefore the balance condition can be found with a high degree of sensitivity.

(2) No current is drawn from the circuit under test. Therefore, it can measure the EMF of a cell accurately.

(3) The scale can be made long for maximum accuracy.
(4) Results are dependent only on measurements of length and the values of standard resistances.

Disadvantages of a potentiometer:
(1) It is slow operation.
(2) The potentiometer wire must be of uniform cross-sectional area.
(3) The temperature of the potentiometer wire must remain constant.

Wheatstone bridge: It is an arrangement of four resistances used to determine one of these resistances quickly and accurately in terms of the remaining three resistances.



Principle: Wheatstone bridge principle states that when the bridge is balanced, the products of the resistances of the opposite arms are equal i.e., $PX = QR$ or $X = (Q/P) \times R$

The values of resistances P, Q, R, and X are so adjusted that the galvanometer shows no deflection. It means that no current is flowing in arm BD and hence potential at B is equal to the potential at D.

Let in a balanced bridge, same current I_1 flow through P and Q and same current I_2 flow through R and X.
Then

$V_A - V_B = I_1 P$ ------- (1) $\qquad$ $V_B - V_C = I_1 Q$ ------- (2)

$V_A - V_D = I_2 R$ ------- (3) $\qquad$ $V_D - V_C = I_2 X$ ------- (4)

$$\because V_B = V_D$$

$\therefore$ Equation (1) equal equation (3) i.e., $I_1 P = I_2 R$ ------- (5)
Also equation (2) equal equation (4) i.e., $I_1 Q = I_2 X$ ------- (6)
On dividing equation (5) by equation (6) we get

$$\frac{P}{Q} = \frac{R}{X} \qquad \text{Or } X = \frac{Q}{P} \times R$$

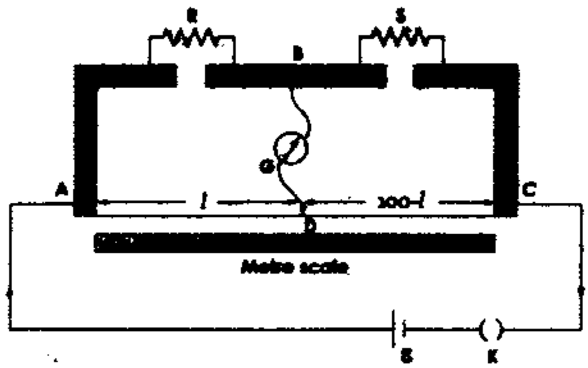
Sensitivity of a wheatstone bridge: A wheatstone bridge is said to be sensitive if it shows a large deflection in the Galvanometer for a small change of resistance in the resistance arm.

The sensitivity of a wheatstone bridge depends on two factors:

(1) Relative magnitudes of the resistances in the four arms of the bridge. The bridge is most sensitive when all the four resistances are of the same order.
(2) Relative positions of battery and Galvanometer.

Metre bridge or slide wire bridge: It is the simplest practical application of the wheatstone bridge that is used to measure an unknown resistance.

Principle: Its working is based on the principle of wheatstone bridge. When the bridge is balanced, $\frac{P}{Q}=\frac{R}{X}$



At the balanced condition of the metre bridge, we have

$$\frac{P}{Q}=\frac{R}{X}$$

Let AD = l cm and DC = $(100 - l)$ cm

P = $\sigma l\ \Omega$ and Q = $\sigma\ (100 - l)\ \Omega$

σ is the resistance per unit length of the wire.

$$\therefore \frac{R}{S}=\frac{\sigma l}{\sigma(100-l)}=\frac{l}{100-l}$$

Or $S=\frac{100-l}{l}\times R$

Knowing l and R, the unknown resistance S can be determined.

Determination of resistivity: If r is the radius of the wire, L be its length, A is the area of cross-section S be the resistance and ρ be the resistivity. Then we have

$$S=\rho\frac{L}{A}$$

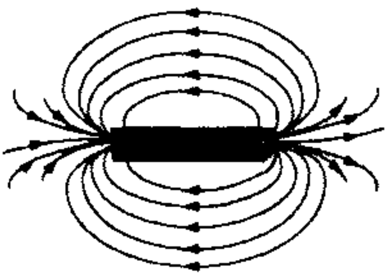
$$\rho=\frac{SA}{L}$$

$$\rho=\frac{S\pi r^2}{L}$$

If S, r and L are known then ρ is known.

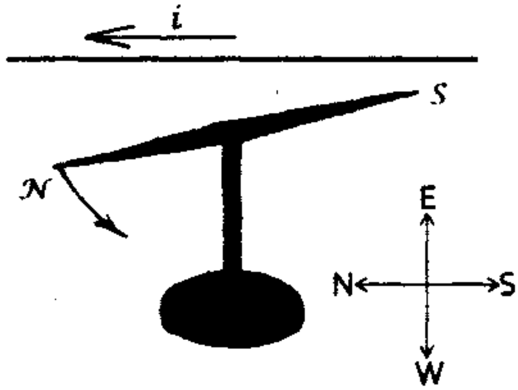
MAGNETIC EFFECTS OF CURRENT AND MAGNETISM

Concept of magnetic field: The space around a magnet within which its influence can be experienced is called its magnetic field. Like electric field, magnetic field is a vector field. We used the symbol $\vec{B}$ for magnetic field. The SI unit of magnetic field is *tesla* (T)



Note:
(1) A moving charge or a current creates a magnetic field in the space surrounding it.
(2) The magnetic field exerts a force on a moving charge or a current in the field.

Oersted's experiment: Oersted found that when current was allowed to flow through a wire placed parallel to the axis of a magnetic needle kept directly below the wire, the needle deflected from its normal position. When current was reversed through the wire, the needle was found to deflect in the opposite direction.

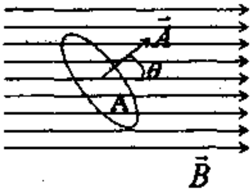


Note: The direction can be remembered with the help of the word SNOW. It indicates that if the current flows from South to North and the wire is held Over the needle, the north pole is deflected towards the West.

Direction of the current OVER the needle	Deflection of the NORTH pole of the needle	Direction of the current BELOW the needle	Deflection of the NORTH pole of the needle
S to N	WEST	S to N	EAST
N to S	EAST	N to S	WEST

Conclusions From Oersted's Experiment:
(1) A current-carrying conductor produces a magnetic field.
(2) The larger is the magnitude of the current in the conductor, the stronger is the magnetic field.
(3) The magnetic field produced by the current-carrying conductor is at right angles to it. (i.e., plane of the concentric circles is perpendicular to the current-carrying conductor)
(4) A current-carrying conductor produces a magnetic field consisting of circular lines of force concentric with the conductor.
(5) If the charge is in motion, in addition to the electric field, it produces a magnetic field.

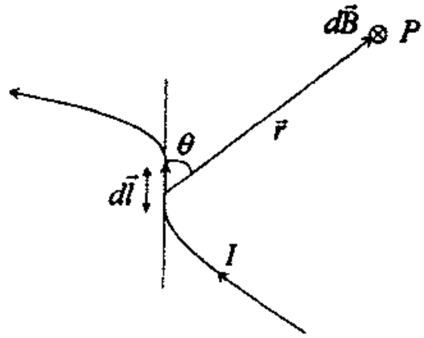
Magnetic flux: The magnetic flux ϕ_B linked with a surface held in a magnetic field $\vec{B}$ is defined as the number of magnetic lines of force crossing a closed area A. If θ is the angle between the direction of the field and normal to the area, then



$$\phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$$

The SI unit of magnetic flux ϕ_B is *weber* (Wb) i.e., $Wb = T\cdot m^2$

Biot-Savart law: According to Biot-Savart law, the magnitude of the field $d\vec{B}$ is
(1) directly proportional to the current I through the conductor i.e., $dB \propto I$
(2) directly proportional to the length dl of the current element i.e., $dB \propto dl$
(3) directly proportional to sin θ i.e., $dB \propto \sin\theta$
(4) Inversely proportional to the square of the distance r of the point P from the current element i.e., $dB \propto \frac{1}{r^2}$



Combining all these four factors we get

$$dB \propto \frac{I\, dl \sin\theta}{r^2}$$

Or $$dB = k\frac{I\, dl \sin\theta}{r^2}$$

The proportionality constant k depends on the medium between the observation point P and the current element $I\, dl$ and the system of unit chosen. For free space and in SI unit,

$$k = \frac{\mu_o}{4\pi} = 10^{-7}\ TmA^{-1} \text{ or } Wbm^{-1}A^{-1}$$

Or $$\mu_o = 4\pi \times 10^{-7}\ TmA^{-1}$$

Where μ_o is the permeability of free space.

$$\therefore dB = \frac{\mu_o}{4\pi}\frac{I\, dl \sin\theta}{r^2}$$

Note: Biot-Savart law holds strictly for steady currents.

In vector form:

$$dB = \frac{\mu_o}{4\pi}\frac{I\, dl\, r \sin\theta}{r^3}$$

Or $$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I\,(d\vec{l} \times \vec{r})}{r^3}$$

The direction of $d\vec{B}$ is perpendicular to the plane of $d\vec{l}$ and $\vec{r}$. It is given by right-hand screw rule.

Note: Just as the charge q is the source of electrostatic field, the source of magnetic field is the current element $I\, d\vec{l}$.

SI unit of magnetic field from Biot-Savart law: According to Biot-Savart law

$$dB = \frac{\mu_o}{4\pi}\frac{I\, dl \sin\theta}{r^2}$$

If $I = 1A$, $dl = 1m$, $r = 1m$ and $\theta = 90^0$ so that $\sin 90^0 = 1$, then

$$dB = \frac{\mu_o}{4\pi}\frac{1\times1\times1}{1^2} = \frac{\mu_o}{4\pi}$$

Or $$dB = \frac{4\pi\times10^{-7}}{4\pi} = 10^{-7}\, tesla$$

Thus one *tesla* is 10^7 times the magnetic field produced by a conducting wire of length one metre and carrying current of one ampere at a distance of one metre from it and perpendicular to it.

Note: $dB = 10^{-7} tesla = 10^{-7} \times 10^{7} tesla = 1 tesla$

Special cases:

(1) If $\theta = 0^0$, $\sin\theta = 0$ so that $dB = 0$ i.e., the magnetic field is zero at points on the axis of the current element.

(2) If $\theta = 90^0$, $\sin\theta = 1$ so that dB is maximum i.e., the magnetic field due to a current element is maximum in a plane passing through the element and perpendicular to its axis.

Biot-Savart law vrs Coulomb's law:

Points of difference

Biot-Savart law	Coulomb's law
The magnetic field is produced by the vector source, the current element $I\,d\vec{l}$	The electrostatic field is produced by a scalar source, the electric charge q
The direction of the magnetic field is perpendicular to the plane containing the displacement vector $\vec{r}$ and the current element $I\,d\vec{l}$	The direction of the electrostatic field is along the displacement vector joining the source and the field point.

Points of similarity

Both fields depend inversely on the square of the distance from the source to the point of observation.
Both are long range fields.

Note:

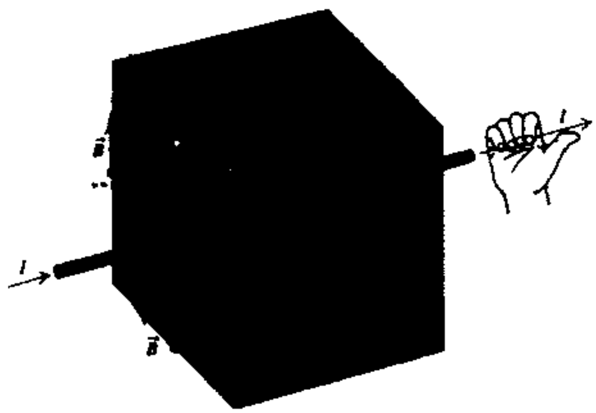
(1) Biot-Savart law is also called Laplace's law and inverse square law.

(2) This law cannot be tested directly because it is not possible to have a current carrying conductor of length dl.

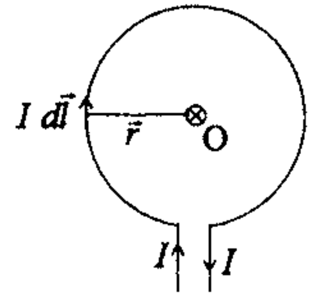
(3) Current element is the product of current I and length of very small segment $d\vec{l}$ of current carrying conductor. The current element $I\,d\vec{l}$ is a vector. Its direction is tangent to the element and acts in the direction of current flow in the conductor.

Right hand grip rule: Grip the wire with your right hand with thumb pointing in the direction of the conventional current. Then curled fingers point in the direction of the magnetic lines of force.

Note: ⊙ shows that the field (magnetic field or electric field) comes out of the plane of the paper and ⊗ shows that the field (magnetic field or electric field) entering the plane of the paper.



Magnetic field at the centre of circular current loop: Consider a circular loop of wire of radius $\vec{r}$ carrying current I. We wish to calculate its magnetic field at the centre O. Consider a current element $I\,d\vec{l}$ of the loop. According to Biot-Savart law, the magnetic field at the centre O due to this element is



$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I\,(d\vec{l}\times\vec{r})}{r^3}$$

$$dB = \frac{\mu_o}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_o}{4\pi}\frac{I\,dl\sin 90^0}{r^2} = \frac{\mu_o}{4\pi}\frac{I\,dl}{r^2} \qquad [\because I\,d\vec{l}\perp\vec{r}]$$

Hence the total magnetic field at the centre O is

$$B = \int dB = \int\frac{\mu_o}{4\pi}\frac{I\,dl}{r^2} = \frac{\mu_o}{4\pi}\frac{I}{r^2}\int dl$$

$$B = \frac{\mu_o}{4\pi}\frac{I}{r^2}\cdot l = \frac{\mu_o}{4\pi}\frac{I}{r^2}\cdot 2\pi r$$

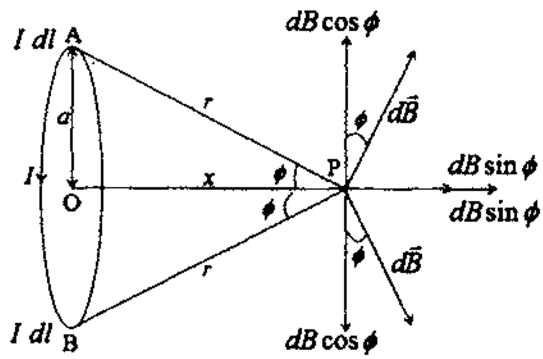
$$\therefore B = \frac{\mu_o I}{2r}$$

If instead of a single loop, there is a coil of n turns, all wound over one another, then

$$B = \frac{\mu_o nI}{2r}$$

Note: The direction of the magnetic field at O is inward i.e., into the plane of the paper and perpendicular to it.

Magnetic field on the axis of a circular current loop: Consider a circular loop of wire of radius a and carrying current I, as shown in the figure. Let the plane of the loop be perpendicular to the plane of the paper. We wish to find field B at an axial point P at a distance x from the centre O.



From Biot-Savart law, the magnetic field at the centre O due to the current element $I\,dl$ at A is

$$dB = \frac{\mu_o}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$\because I\,dl \perp r$

$$\therefore dB = \frac{\mu_o}{4\pi}\frac{I\,dl\sin 90^0}{r^2} = \frac{\mu_o}{4\pi}\frac{I\,dl}{r^2}$$

The field dB lies in the plane of the paper and is perpendicular to r. Let ϕ be the angle between AP and OP. Then dB can be resolved into two rectangular components.

(1) $dB\sin\phi$ along the axis,

(2) $dB\cos\phi$ perpendicular to the axis.

The cosine components for the loop will be equal and opposite and will cancel out. Their axial components will be in the same direction, i.e., along OP and get added up.

$\therefore$ Total magnetic field at the point P in the direction OP is

$$B = \int dB\sin\phi$$

But $\sin\phi = \frac{a}{r}$ and $dB = \frac{\mu_o}{4\pi}\frac{I\,dl}{r^2}$

$$B=\frac{\mu_o Ia}{4\pi r^3}\int dl=\frac{\mu_o Ia}{4\pi r^3}\cdot 2\pi a$$

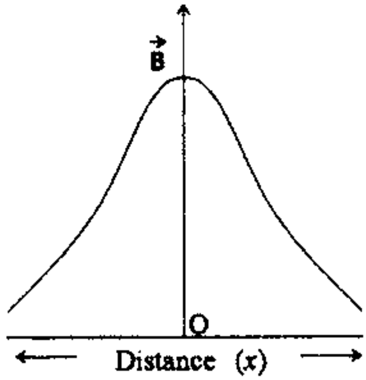
$$B=\frac{\mu_o Ia^2}{2r^3}$$

Now $r=\sqrt{a^2+x^2}$

$$\therefore B=\frac{\mu_o Ia^2}{2(a^2+x^2)^{\frac{3}{2}}}$$

If the coil consists of n turns, then

$$B=\frac{\mu_o nIa^2}{2(a^2+x^2)^{\frac{3}{2}}}$$



Special cases:

(1) At the centre of the current loop, $x = 0$

$$B=\frac{\mu_o nIa^2}{2(a^2)^{\frac{3}{2}}}=\frac{\mu_o nIa^2}{2a^3}=\frac{\mu_o nI}{2a}$$

Or $B=\frac{\mu_o nIa^2}{2a^3}\cdot\frac{\pi}{\pi}=\frac{\mu_o nIA}{2\pi a^3}$ where A is the area of the loop

(2) At the axial point lying far away from the coil, $x >> a$

$$B=\frac{\mu_o nIa^2}{2(x^2)^{\frac{3}{2}}}=\frac{\mu_o nIa^2}{2x^3}$$

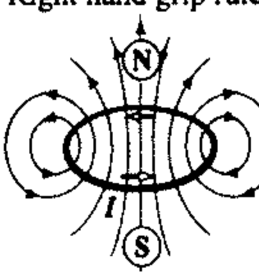
$B=\frac{\mu_o nIa^2}{2x^3}\cdot\frac{\pi}{\pi}=\frac{\mu_o nIA}{2\pi x^3}$ where A is the area of the loop

(3) At the axial point where $x = a$

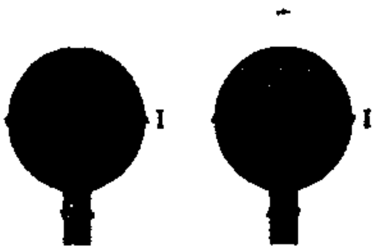
$$B=\frac{\mu_o nIa^2}{2(a^2+a^2)^{\frac{3}{2}}}=\frac{\mu_o nIa^2}{2(2a^2)^{\frac{3}{2}}}=\frac{\mu_o nIa^2}{2^{\frac{5}{2}}a^3}=\frac{\mu_o nI}{2^{\frac{5}{2}}a}$$

Direction of magnetic field:

(1) Right hand palm rule
(2) Clock rule
(3) Right hand grip rule



(1)



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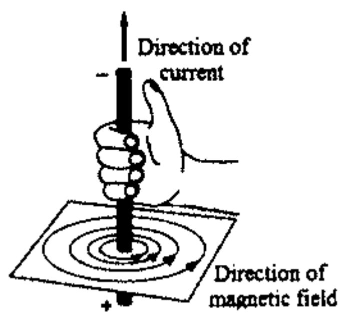
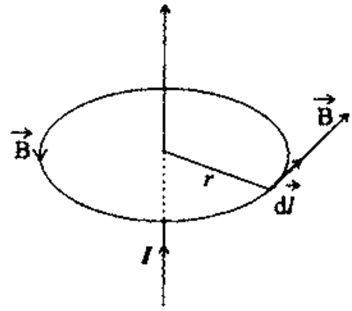


(3)

Ampere's circuital law: Ampere's circuital law states that the line integral of the magnetic field $\vec{B}$ around any closed circuit is equal to μ_o times the total current I threading or passing through this closed circuit. Mathematically,

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I$$



Proof:

The magnetic field $\vec{B}$ at a distance r from the straight current carrying conductor is given by

$$B = \frac{\mu_o I}{2\pi r}$$

Let $d\vec{l}$ be a small element of the closed path. Thus

$$\oint \vec{B}\cdot d\vec{l} = \oint B dl \cos 0^0 = \oint B dl = B\oint dl$$

$$\oint \vec{B}\cdot d\vec{l} = \frac{\mu_o I}{2\pi r}\cdot 2\pi r$$

$$\therefore \oint \vec{B}\cdot d\vec{l} = \mu_o I$$

This prove Ampere's law. This law is valid for any assembly of current and for any arbitrary closed loop.

Note:

(1) Ampere's circuital law is not independent of the Biot-Savart law. It can be derived from the Biot-Savart law.

(2) Ampere's circuital law holds for steady currents which do not change with time.

Magnetic field due to a straight conductor carrying current: Consider a long straight conductor carrying current I in the direction shown in the figure. It is desired to find the magnetic field $\vec{B}$ at a point P at a perpendicular distance r from the conductor.

Applying Ampere's circuital law to this closed path, we have

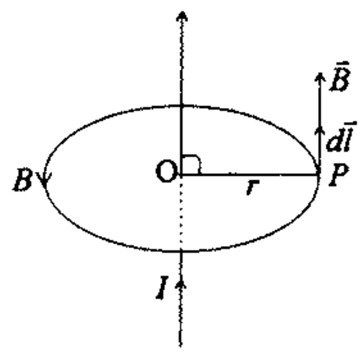
$$\oint \vec{B}\cdot d\vec{l} = \mu_o I$$

$$\oint B\, dl \cos 0^0 = \mu_o I$$

$$B\oint dl = \mu_o I$$

$$B \times 2\pi r = \mu_o I$$

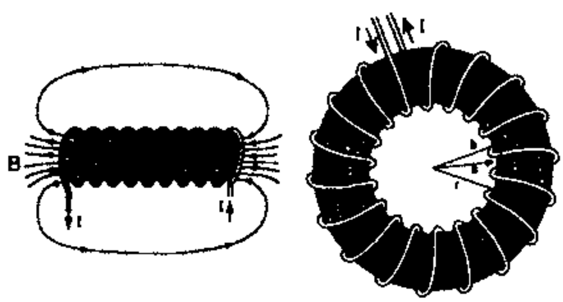
$$\therefore B = \frac{\mu_o I}{2\pi r}$$



A straight solenoid: A long coil of wire consisting of closely packed loops is called a solenoid.

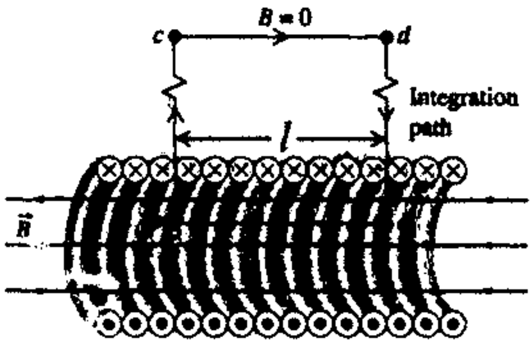
Or

A solenoid means an insulated copper wire wound closely in the form of a helix.



A toroidal solenoid or a toroid: A solenoid bend in the form of a closed ring is called a toroidal solenoid or a toroid.

Magnetic field inside a straight solenoid: The magnetic field inside a closely wound long solenoid is uniform everywhere and zero outside it. The figure shows the sectional view of a long solenoid. At various turns of the solenoid, current comes out of the plane of paper at points marked ⊙ and enters the plane of the paper at points marked ⊗.



Central part of solenoid

Consider a rectangular closed path *abcd* as the Amperean loop.

According to Ampere's circuital law, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I$$

Now $\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} + \int_d^a \vec{B}\cdot d\vec{l}$

But $\int_b^c \vec{B}\cdot d\vec{l} = \int_b^c B\,dl\cos 90^0 = 0$

$\int_c^d \vec{B}\cdot d\vec{l} = 0$ outside the solenoid, and

$\int_d^a \vec{B}\cdot d\vec{l} = \int_d^a B\,dl\cos 90^0 = 0$

$\therefore \oint \vec{B}\cdot d\vec{l} = \int_a^b B\,dl\cos 0^0 = \int_a^b B\,dl = B\int_a^b dl = Bl$ ------ (1)

where *l* is the length of side *ab* of the rectangular loop *abcd*, *n* is the number of turns per unit length of the solenoid, then *nl* is the number of turns in length *l* of the solenoid. Therefore the total current threading the loop *abcd* is *nlI*

Again from Ampere's circuital law, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I = \mu_o (nlI) \text{ ------ (2)}$$

Equating equation (1) and equation (2) gives

$$Bl = \mu_o nlI$$

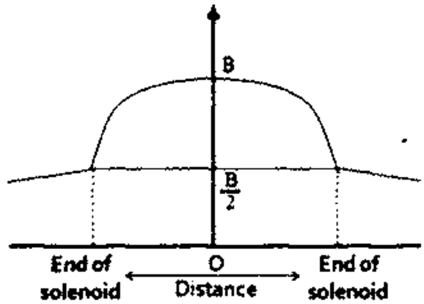
$$\text{Or } B = \mu_o nI$$

Note:

(1) *B* depends upon *n* and *I*. It does not depend upon the position within the solenoid. Therefore magnetic field inside the solenoid is uniform.

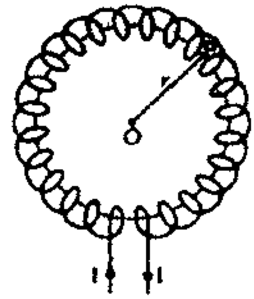
(2) If the solenoid is iron-cored of relative permeability μ_r, the magnitude of the magnetic field inside the solenoid is $B = \mu_o \mu_r nI = \mu\, nI$

(3) At points near the end of the air-cored solenoid, the magnitude of the magnetic field is



$$B = \frac{1}{2}\mu_o nI$$

Magnetic field due to a toroidal solenoid or a toroid: Consider a thin air-cored toroid having a large radius. Let I is the current through the toroid, r is the mean radius of the toroid, n is the number of turns per unit length and B is the magnitude of the magnetic field inside the toroid.



According to Ampere's circuital law, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I$$

Now $\oint \vec{B}\cdot d\vec{l} = \oint B\,dl\cos 0^0 = \oint B\,dl = B\oint dl = B(2\pi\, r)$ ------- (1)

Again from Ampere's circuital law, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I = \mu_o[n(2\pi\, r)I] \text{ ------ (2)}$$

Equating equation (1) and equation (2) gives

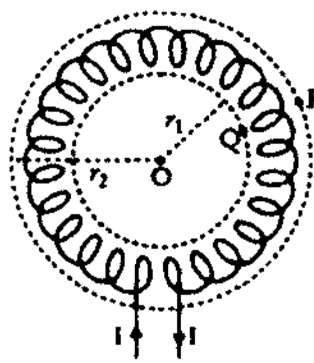
$$B(2\pi\, r) = \mu_o[n(2\pi\, r)I]$$

$$\text{Or } B = \mu_o nI$$

If N is the total number of turns of a toroid, then $N = n(2\pi r)$ or $n = \dfrac{N}{2\pi r}$.

The magnetic field $B = \dfrac{\mu_o NI}{2\pi\, r}$

Note:



(1) The maximum and minimum magnetic field strengths within the toroid is $B_{max} = \dfrac{\mu_o NI}{2\pi\, r_1}$ and $B_{min} = \dfrac{\mu_o NI}{2\pi\, r_2}$ respectively.

(2) If the toroid is iron-cored of relative permeability μ_r, then the magnitude of the magnetic field inside the toroid is $B = \dfrac{\mu_o \mu_r NI}{2\pi\, r}$ or $B = \dfrac{\mu NI}{2\pi\, r}$.

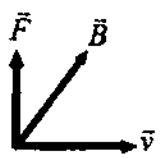
Force on a moving charge in a magnetic field: The electric charges moving in a magnetic field experience a force, while there is no such force on static charges.

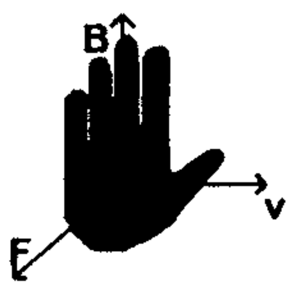
Suppose a positive charge q moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$, It is found from experiments that the charge q moving in the magnetic field $\vec{B}$ experiences a force $\vec{F}$ such that

(1) The force is proportional to the magnitude of the magnetic field i.e., $F_m \propto B$

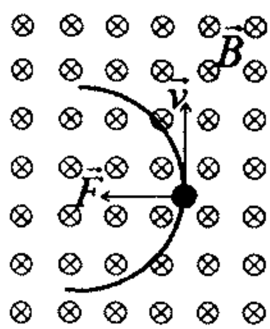
(2) The force is proportional to the charge q i.e., $F_m \propto q$

(3) The force is proportional to the component of the velocity v in the perpendicular direction of the field B i.e., $F_m \propto v\sin\theta$





Right hand palm rule



Combining the above factors, we get

$$F_m \propto Bqv\sin\theta$$

$$F_m = k\,Bqv\sin\theta$$

$$F_m = Bqv\sin\theta \qquad [\because k = 1]$$

F is called magnetic Lorentz force.

In vector form $\vec{F}_m = q\,(\vec{v} \times \vec{B})$

Special cases:

(1) If $v = 0$, then $F_m = 0$

(2) If $\theta = 0^0$ or $\theta = 180^0$, then $F_m = 0$

(3) If $\theta = 90^0$, then $F_m = qvB$. Here F_m is maximum

Definition of magnetic field: The magnetic field at a point may be defined as the force acting on a unit charge moving with a unit velocity at right angle to the direction of the field.

$$\because F_m = Bqv\sin\theta$$

$$\text{Or} \quad B = \frac{F_m}{qv\sin\theta}$$

If $F = 1N$, $q = 1C$, $v = 1ms^{-1}$, $\theta = 90^0$, then the SI unit of B is

$$B = \frac{1N}{1C \cdot 1ms^{-1} \cdot \sin 90^0}$$

$$B = \frac{1N}{1A \cdot 1m} = 1NA^{-1}m^{-1} = 1\,tesla\,(T)$$

Note: $1\,gauss = 10^{-4}\,tesla$

Lorentz force : The total force experienced by a charged particle moving in a region where both electric and magnetic fields are present, is called Lorentz force.

A charge q in an electric field $\vec{E}$ experiences the electric force $\vec{F}_e = q\vec{E}$

A charge q in a magnetic field $\vec{B}$ experiences the magnetic force $\vec{F}_m = q\,(\vec{v} \times \vec{B})$

The total force or Lorentz force is

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\,(\vec{E} + \vec{v} \times \vec{B})$$

Work done by a magnetic force on a charged particle is zero:

The magnetic force $\vec{F} = q\,(\vec{v} \times \vec{B})$ always acts perpendicular to the velocity $\vec{v}$ or the direction of motion of the charge q.

$$\vec{F} \cdot \vec{v} = q\,(\vec{v} \times \vec{B}) \cdot \vec{v} = 0$$

According to Newton's second law, we have

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$m\frac{d\vec{v}}{dt} \cdot \vec{v} = 0$$

$$\frac{m}{2}\left[\frac{d\vec{v}}{dt} \cdot \vec{v} + \vec{v} \cdot \frac{d\vec{v}}{dt}\right] = 0$$

$$\frac{m}{2}\frac{d}{dt}(\vec{v}\cdot\vec{v})=0$$

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right)=0$$

$$\frac{dK}{dt}=0 \qquad \Rightarrow K=\text{constant}$$

Thus the magnetic force does not change the kinetic energy of the charge particle. This indicates that speed of the particle does not change.

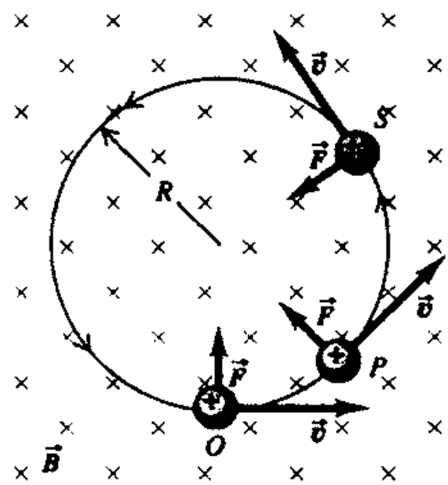
Note: According to work- energy theorem, the change in kinetic energy is equal to the work done on the particle by the net force.

Motion of a charged particle in a uniform magnetic field: When a charged particle having charge q and velocity $\vec{v}$ enters a magnetic field $\vec{B}$, it experiences a force $\vec{F}=q\,(\vec{v}\times\vec{B})$. The direction of this force is perpendicular to both $\vec{v}$ and $\vec{B}$. The magnitude of this force is

$$F=q\,vB\sin\theta$$

(1) When the initial velocity is parallel to the magnetic field:
Here $\theta=0^0$, so the force $F=q\,vB\sin 0^0=0$

(2) When the initial velocity is perpendicular to the magnetic field:
Here $\theta=90^0$, so the force $F=q\,vB\sin 90^0=q\,vB$



This force continuously deflects the particle sideways without changing its speed and the particle will move along a circle perpendicular to the field. Thus the magnetic force provides the necessary centripetal force. Let R be the radius of the circular path, then

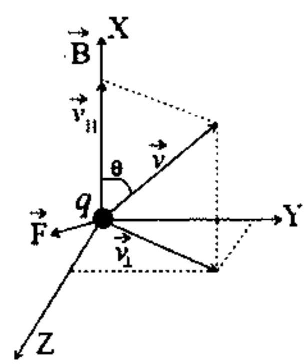
$$\frac{mv^2}{R}=q\,vB$$

$$\text{Or}\quad R=\frac{mv}{qB}$$

Period of revolution $T=\frac{2\pi R}{v}$

$$\text{Or } T=\frac{2\pi}{v}\frac{mv}{qB}=\frac{2\pi\,m}{qB}$$

Frequency of revolution $f_c=\frac{1}{T}=\frac{qB}{2\pi\,m}$. This frequency is called cyclotron frequency.



(3) When the initial velocity makes an arbitrary angle with the magnetic field direction:
Consider a charged particle q entering a uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ inclined at an angle θ with the direction $\vec{B}$ as shown in the figure.

The velocity $\vec{v}$ can be resolved into two rectangular components,

$$v_{\parallel} = v\cos\theta \qquad \text{and} \qquad v_{\perp} = v\sin\theta.$$

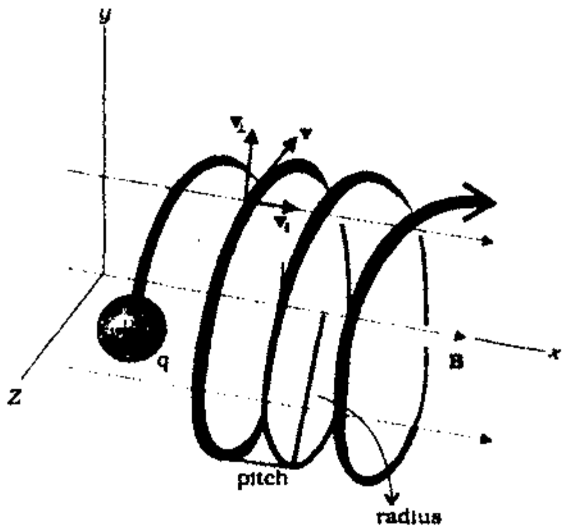
The charged particle experiences a force

$$F_m = qv_{\perp}B \qquad \text{and} \qquad F_c = \frac{mv_{\perp}^2}{r}$$

$$\therefore F_c = F_m \qquad \text{Or} \qquad \frac{mv_{\perp}^2}{r} = qv_{\perp}B$$

$$\text{Or} \quad r = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB}$$

The period of revolution is $T = \frac{2\pi r}{v_{\perp}} = \frac{2\pi}{v\sin\theta}\frac{mv\sin\theta}{qB} = \frac{2\pi m}{qB}$



The linear distance travelled by the charged particle in the direction of the magnetic field during its period of revolution is called pitch of the helical path.

$$\text{Pitch} = v_{\parallel} \times T = v\cos\theta \times \frac{2\pi m}{qB} = \frac{2\pi m v\cos\theta}{qB}$$

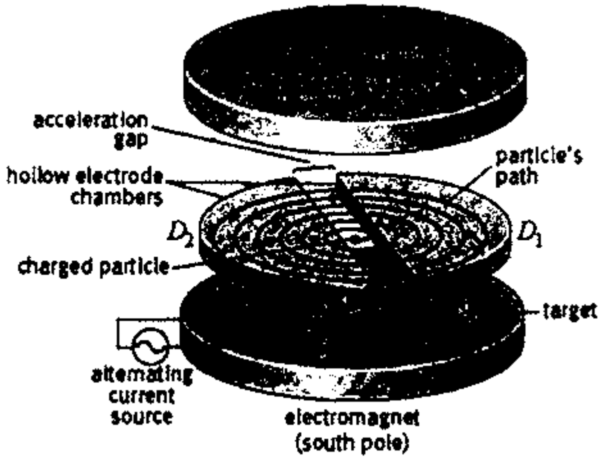
Cyclotron or Magnetic resonance accelerator: It is a device used to accelerate charged particles like protons, deutrons, α-particle, etc., to very high energy.

Principle: A charged particle can be accelerated to very high energy by making it pass through a moderate electric field a number of times. This can be done with the help of a perpendicular magnetic field which throws the charged particle into a circular motion, the frequency of which does not depend on the speed of the particle and the radius of the circular orbit.

Construction: The cyclotron consists of two D shape metallic chambers marked D_1 and D_2. They are separated by a very small gap. An alternating high voltage is applied across the gap. The voltage is of the order of 10^4 V and its frequency is of the order of 10^7 Hz. The dees are closed in a steel box which is placed between the pole pieces of a very strong magnet. The magnetic field is applied perpendicular to the plane of the dees.

Working: An alternating voltage is applied across the dees. The positive ion to be accelerated is introduced at the centre of the dees. Suppose at that instant D_1 is negative.

The particle will be accelerated towards D_1 and will describe a semicircle. By the time it comes to the edge of D_1 the polarity of the dees is reverse. D_2 becomes negative and D_1 becomes positive.
The particle is further accelerated and it describes a circle of larger radius. As it emerges from D_2, the polarity again changes and this will go on. Every time the particle crosses the gap, it accelerates. By applying a deflecting electric field the highly accelerated ion is removed.



Theory: Let a particle of charge q and mass m enter a region of magnetic field $\vec{B}$ with a velocity $\vec{v}$ normal to the field $\vec{B}$. The particle will follow a circular path provided by the magnetic field.

The centripetal force on charge q is $F_c = \frac{mv^2}{r}$

The magnetic force $F_m = qvB\sin 90^0 = qvB$

$$\text{Or } \frac{mv^2}{r} = qvB \qquad \Rightarrow r = \frac{mv}{qB}$$

Period of revolution of charged particle is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

Frequency of revolution of charged particle is given by

$$f_c = \frac{1}{T} = \frac{qB}{2\pi m}$$

This frequency is called cyclotron frequency or magnetic resonance frequency.

Angular frequency of the charged particle is given by

$$\omega = 2\pi f_c = 2\pi \frac{qB}{2\pi m} = \frac{qB}{m}$$

Maximum K.E of the accelerated ions: The ions will attain maximum velocity near the periphery of the dees. If v_o is the maximum velocity acquired by the ions and r_o is the radius of the dees, then

$$\frac{mv_o^2}{r_o} = qv_oB \qquad \Rightarrow v_o = \frac{qBr_o}{m}$$

The maximum K.E of the ion will be

$$K_{max} = \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(\frac{qBr_o}{m}\right)^2 = \frac{q^2B^2r_o^2}{2m}$$

Limitations of Cyclotron:

(1) According to Einstein's special theory of relativity, the mass of the particle increases with the increase in its velocity as $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$ where m_o is the rest mass of the particle. At high velocities, the cyclotron frequency $f = \frac{qB}{2\pi m}$ will decrease due to increase in mass. This will throw the particles out of resonance with the oscillating field. That is, as the ions reach the gap between the dees, the polarity of the dees is not reverse at that instant. Consequently the ions are not accelerated further.

The above drawback is overcome either by increasing magnetic field as in a synchrotron or by decreasing the frequency of the alternating electric field as in a synchro-cyclotron.

(2) Electrons cannot be accelerated in a cyclotron. A large increase in their energy increases their velocity to a very large extent. This throw the electron out of step with the oscillating field.

(3) Neutrons, being electrically neutral, cannot be accelerated in a cyclotron.

Uses of cyclotron:

(1) The high energy particles produced in a cyclotron are used to bombard nuclei and study the resulting nuclear reactions and hence investigate nuclear structure.

(2) It is used to implant ions into solids and modify their properties or even synthesise new materials.

(3) It is used to produce radioactive isotopes which are used in hospitals for diagnosis and treatment.

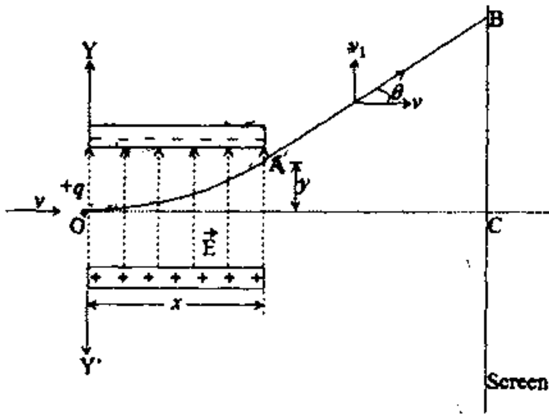
Note:

(1) The frequency depends on the mass of the charged particles in motion.

(2) The time spent by the positive ion inside the dee is T/2 i.e., independent of the values of the velocity of the ion and radius of its path.

(3) Another name of cyclotron is magnetic resonance accelerator.

Force on a charged particle in a uniform electric field: Consider a charge $+q$ of mass m moves with constant velocity v along the X axis, entering a uniform electric field $\vec{E}$ as shown in the figure.



Force on the charged particle is $F = qE$ along OY

Acceleration on the charged particle is $a = \frac{F}{m} = \frac{qE}{m}$ along OY

Time taken to traverse the field is $t = \frac{x}{v}$

If y is the transverse deflection during time t, then

$$y = 0 \times t + \frac{1}{2}at^2 = \frac{1}{2}at^2$$

$$y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{v}\right)^2 = \frac{qE}{2mv^2}x^2$$

$$\text{Or} \quad y = k\,x^2$$

This is the equation of the parabola. Therefore inside the electric field, the charged particle follow the parabolic path OA.

Note:

(1) As the charged particle leaves the electric field, it follow a straight path. (AB in the figure)

(2) When the charged particle just leaves the electric field, vertical deflection produced is

$$y = \frac{qE}{2mv^2}x^2$$

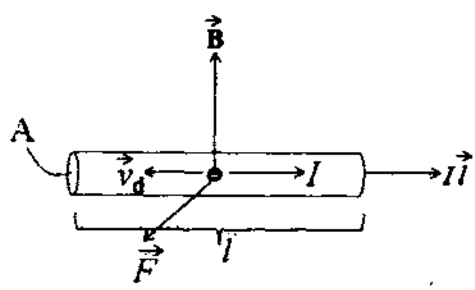
(3) As the charged particle leaves the electric field, its vertical velocity is given by

$$v_1 = v_o + at = 0 + \left(\frac{qE}{m}\right)\left(\frac{x}{v}\right) = \frac{qE\,x}{mv}$$

(4) Angle with the X axis at which the charged particle emerges from the electric field is

$$\tan\theta = \frac{v_1}{v} = \frac{qE\,x}{mv^2} \qquad \Rightarrow \theta = \tan^{-1}\left(\frac{qE\,x}{mv^2}\right)$$

Force on a current carrying conductor placed in a uniform magnetic field: Consider a conductor of length l and area of cross-section A, carrying current I as shown in the figure. The electron drift towards left with drift velocity $\vec{v}_d$.



Each electron will experience a magnetic Lorentz force which is given by

$$\vec{f} = -e(\vec{v}_d \times \vec{B})$$

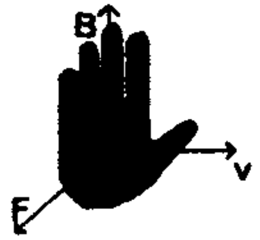
If n is the number of free electrons per unit volume, then the total number of free electrons is

$$N = nlA$$

$\therefore$ The total force on all the electrons in a conductor is

$$\vec{F} = -nlA[e(\vec{v}_d \times \vec{B})]$$

$$\vec{F} = nA[e(-l\,\vec{v}_d \times \vec{B})]$$



If $I\,\vec{l}$ represent a current element in the direction of current, then vector $\vec{l}$ and $\vec{v}_d$ will have opposite direction. Hence

$$\vec{F} = nA[e(v_d\vec{l} \times \vec{B})]$$

$$\vec{F} = neAv_d(\vec{l} \times \vec{B})$$

But $neAv_d = I$

$$\therefore \vec{F} = I(\vec{l} \times \vec{B})$$

The magnitude of the force on the current carrying conductor is given by

$$F = IlB\sin\theta$$

Where θ is the angle between the direction of the magnetic field and the direction of flow of current.

Special cases:

(1) If $\theta = 0^0$ or 180^0

$F = IlB(0) = 0$

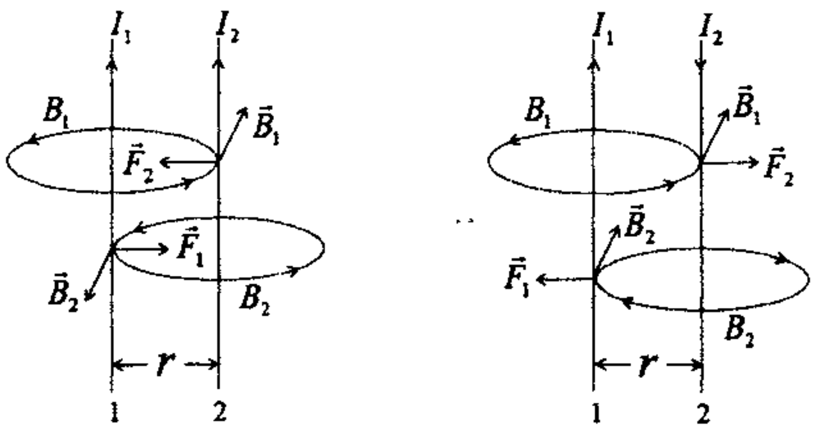
Thus a current carrying conductor placed parallel to the direction of the magnetic field does not experience any force.

(2) If $\theta = 90^0$

$F = IlB\sin 90^0 = IlB$

Thus a current carrying conductor placed perpendicular to the direction of a magnetic field experience a maximum force.

Force between two parallel current-carrying conductors: If currents are in the same direction, the conductors attract each other, if current are in opposite direction, the conductors repel each other. Thus like currents attract while unlike currents repel.



Consider two infinitely long parallel conductors 1 and 2 carrying currents I_1 and I_2 respectively. Suppose the conductors are separated by a distance r in the plane of the paper. The magnetic field produced by the current I_2 at any point on the conductor 1 is

$$B_2 = \frac{\mu_o I_2}{2\pi r}$$

The force acting on the conductor 1 of unit length will be

$$F = IlB\sin\theta$$

Or $$F_1 = I_1(1)\,B_2\sin 90^0 = I_1B_2 = \frac{\mu_o I_1 I_2}{2\pi r}$$

Similarly the force acting on the conductor 2 of unit length will be

$$F_2 = I_2B_1 = \frac{\mu_o I_1 I_2}{2\pi r}$$

If the two currents I_1 and I_2 are in the same direction, the forces between them are attractive. But if the two currents I_1 and I_2 are in opposite direction, the forces between them are repulsive.

Note:

(1) If two straight current-carrying conductors of unequal length are held parallel to each other, then the force on the long conductor is due to the magnetic field of short conductor.
If I_1 = current through the short conductor of length l,
I_2 = current through the long conductor of length L
And if r is the separation between these parallel conductors, then

$$\text{Force on long conductor} = \text{Force on short conductor} = \frac{\mu_o I_1 I_2}{2\pi r}\cdot l$$

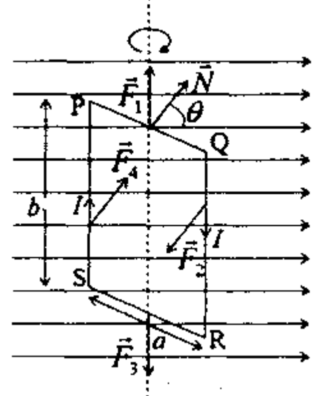
(2) Currents in the same direction (parallel).
(3) Currents in opposite direction (series).

Definition of ampere: One ampere is defined as that current flowing in each of the two infinitely long parallel conductors $1m$ apart, which result of exactly $2 \times 10^{-7}\,N$ per metre length of each conductor.

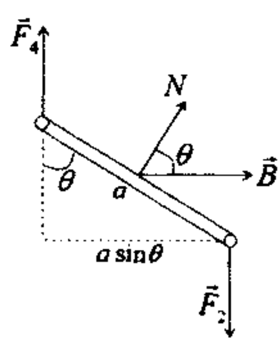
If $I_1 = I_2 = 1A$, $r = 1m$, The force per unit length on each conductor is

$$f = \frac{F}{l} = \frac{\mu_o I_1 I_2}{2\pi r} = \frac{4\pi\times10^{-7}(1)(1)}{2\pi(1)} = 2\times10^{-7}\,Nm^{-1}$$

Torque experienced by a current loop in a uniform magnetic field: As shown in the figure (1), consider a rectangular coil PQRS suspended in a uniform magnetic field $\vec{B}$, with its axis perpendicular to the field.



(1)



(2)

Let I = current flowing through the coil PQRS

a, b = sides of the coil PQRS

$A = ab$ = area of the coil

θ = angle between the direction of $\vec{B}$ and that of the vector $\vec{N}$ drawn normal to the plane of the coil.

The force on the side PQ is $\vec{F}_1 = I(\vec{a} \times \vec{B})$

Its magnitude is, $F_1 = IaB\sin(90^0 - \theta) = IaB\cos\theta$

The force on the side QR is $\vec{F}_2 = I(\vec{b} \times \vec{B})$

Its magnitude is, $F_2 = IbB\sin 90^0 = IbB$

The force on the side RS is $\vec{F}_3 = I(\vec{a} \times \vec{B})$

Its magnitude is, $F_3 = IaB\sin(90^0 + \theta) = IaB\cos\theta$

The force on the side QR is $\vec{F}_4 = I(\vec{b} \times \vec{B})$

Its magnitude is, $F_4 = IbB\sin 90^0 = IbB$

The forces $\vec{F}_1$ and $\vec{F}_3$ act along the axis of the loop, as shown in the figure (1). These forces are equal, opposite and collinear. So they give rise to no net forces or torques.

The forces $\vec{F}_2$ and $\vec{F}_4$ are equal and opposite but not collinear. So they form a couple.

The perpendicular distance between the two forces is $a\sin\theta$, as shown in the figure (2).

The magnitude of the torque $\vec{\tau}$ on the loop is given by

τ = Force × perpendicular distance

$\tau = IbB \times a\sin\theta = IBA\sin\theta$

Or $\tau = mB\sin\theta$

Where $m = IA$ is the magnitude of the magnetic dipole moment. (not yet done)

In vector notation, the torque $\vec{\tau}$ is given by

$$\vec{\tau} = \vec{m} \times \vec{B}$$

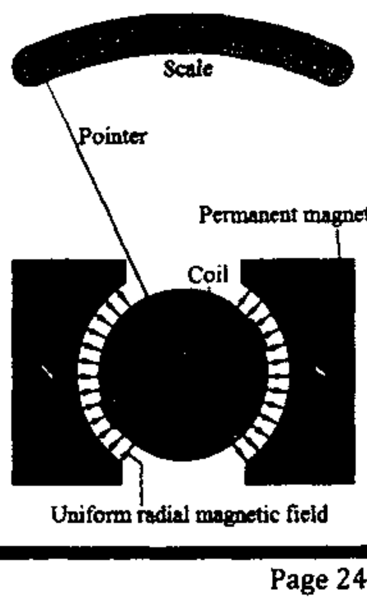
If the rectangular loop has N turns, the torque is

$$\vec{\tau} = N(\vec{m} \times \vec{B})$$

Special cases:

(1) When $\theta = 0^0$, $\tau = 0$ i.e., the torque is minimum when the plane of the loop is perpendicular to the magnetic field.

(2) When $\theta = 90^0$, $\tau = NIBA$ i.e., the torque is maximum when the plane of the loop is parallel to the magnetic field. Thus $\tau_{max} = NIBA$.



Moving coil galvanometer: A galvanometer is a device to detect current in a circuit.

Principle: The operating principle of the galvanometer is that a current carrying coil placed in a magnetic field experiences a torque, the magnitude of which depends on the strength of current.

Construction: A moving coil galvanometer consists of a rectangular coil wound on a non metallic frame and suspended from a torsion head with a phosphor bronze wire in a magnetic filed B produced by a permanent magnet NS. The magnetic lines of force remain nearly parallel to the plane of the coil, as it rotates on passing the current through it. This is achieved by making the pole pieces of the field magnet concave and by having a soft ion core at the centre of the coil. The lower end of the coil is attached to a spring which wind up as the coil rotates.

Working: The deflecting torque acting on the coil due to the magnetic field, when a current is passed through it is

$$\tau_{def} = NIBA$$

Where N is the number of turns in the coil, A is the area of the plane of the coil, I is the current flowing through the coil and B is the magnetic field produced by the field magnet.

As the coil rotates, a restoring torque sets up in the suspension fibre and the spring. If the deflection is α then the restoring torque is

$$\tau_{rest} = k\alpha$$

Where k is the restoring torque per unit twist, called the torsion constant.
In equilibrium,

Deflecting torque = Restoring torque

$$NIBA = k\alpha$$

$$I = \frac{k}{NBA}\alpha$$

$$I = G\alpha$$

Where G is called galvanometer constant or current reduction factor of the galvanometer.

$$I \propto \alpha$$

Thus the deflection produced is directly proportional to the current passed through the galvanometer.

Figure of merit of a galvanometer: It is defined as the current which produces a deflection of one scale division in the galvanometer and is given by

$$G = \frac{I}{\alpha} = \frac{k}{NBA}$$

Sensitivity of a galvanometer: A galvanometer is said to be sensitive if it shows large scale deflection even when a small current is passed through it or a small voltage is applied across it.

Current sensitivity: It is defined as the deflection produced by the galvanometer when a unit current flows through it.

$$\text{Current sensitivity, } I_S = \frac{\alpha}{I} = \frac{NBA}{k} \text{ ------- (1)}$$

Note: If $N \to 2N$, then $I_S \to 2I_S$

That is, when the number of turns N is doubled, current sensitivity is also doubled.

Voltage sensitivity: It is defined as the deflection produced by the galvanometer when a unit potential difference is applied across its ends.

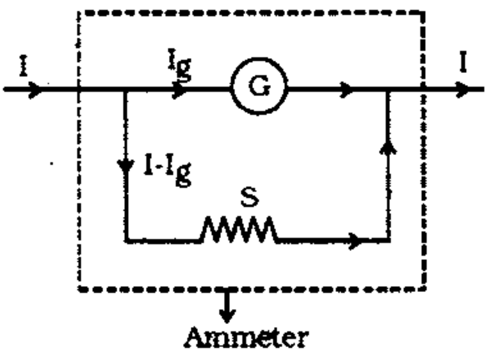
$$\text{Voltage sensitivity, } V_S = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{kR} \text{ ------- (2)}$$

Note: If $N \to 2N$, then $R \to 2R$
Hence voltage sensitivity remains unchanged.
An interesting point to note is that, Increasing the current sensitivity does not necessarily, increase the voltage sensitivity.

Factors on which the sensitivity of a moving coil galvanometer depends:
(1) Number of turns N in its coil
(2) Magnetic field B
(3) Area A of the coil
(4) Torsion constant k of the spring and suspension wire

Factors by which the sensitivity of a moving coil galvanometer can be increase:
(1) By increasing the number of turns N of the coil
(2) By increasing the magnetic field B
(3) By increasing the area A of the coil
(4) By decreasing the value of torsion constant k.

Conversion of a galvanometer into an ammeter: As shown in the figure, let G be the resistance of the galvanometer, I_g is the current with which galvanometer gives full scale deflection, S is the shunt resistance and $I - I_g$ is the current through the shunt.



P.D across the galvanometer = P.D across the shunt

$$I_g G = (I - I_g)S$$

$$S = \frac{I_g}{I - I_g} \times G$$

So by connecting a shunt of resistance S across the given galvanometer, we get an ammeter of desired range. Moreover

$$I_g = \frac{S}{G+S} \times I$$

The deflection in the galvanometer is proportional to I_g and hence to I.

Note:
(1) An ammeter is a shunted or low resistance galvanometer. Its effective resistance is

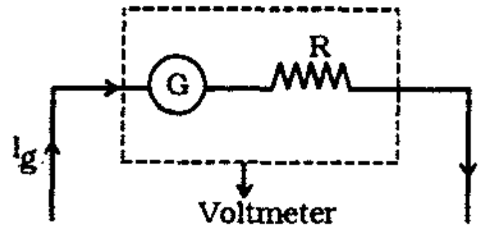
$$R_A = \frac{GS}{G+S} < S$$

(2) The effective resistance $R_A \ll G$
(3) Higher the range of ammeter to be prepared from a given galvanometer, lower is the value of the shunt resistance required for the purpose.
(4) The ammeter of lower range has a higher resistance than the ammeter of higher range.
(5) The range of the ammeter can be increased but it cannot be decreased.

Shunt: A shunt is a low resistance which is connected in parallel with a galvanometer (or ammeter) to protect it from strong current.

Uses of shunt:
(1) To prevent a galvanometer from being damaged due to large current.
(2) To convert a galvanometer into ammeter.
(3) To increase the range of an ammeter.

Conversion of a galvanometer into a voltmeter: As shown in the figure, let G be the resistance of the galvanometer, I_g is the current with which galvanometer gives full scale deflection, R is the high series resistance which restricts the current to safe limit I_g.



$\therefore$ Total resistance in the circuit is $R + G$

$$I_g = \frac{V}{R+G}$$

$$R+G = \frac{V}{I_g}$$

$$\text{Or} \quad R = \frac{V}{I_g} - G$$

So by connecting a high resistance R in series with the galvanometer, we get a voltmeter of desired range. Moreover, the deflection in the galvanometer is proportional to current I_g and hence to V.

Note: A voltmeter is a high resistance galvanometer. Its effective resistance is

$$R_V = R + G >> G$$

Magnets and magnetism: A magnet is a material that has both attractive and directive properties. It attracts small pieces of iron, nickel, cobalt, etc. This property of attraction is called magnetism.

Artificial magnets: The pieces of iron and other magnetic materials can be made to acquire the properties of natural magnets. Such magnets are called artificial magnets.

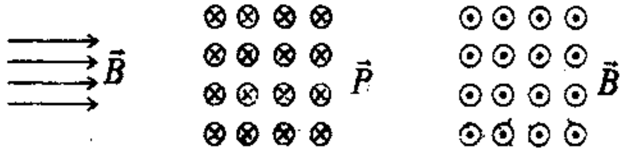
(1) Bar magnet: It is a bar of circular or rectangular cross-section.
(2) Magnetic needle: It is a thin magnetised steel needle having pointed ends and is pivoted at its centre so that it is free to rotate in a horizontal plane.
(3) Horse shoe magnet: It has the shape of a horse shoe.
(4) Ball ended magnet: It is a thin bar of circular cross-section ending in two spherical balls.

Basic properties of magnets:
(1) Attractive property
(2) Directive property
(3) Like poles repel and unlike poles attract
(4) Magnetic poles always exist in pairs
(5) Magnetic induction

Some important definitions connected with magnetism:

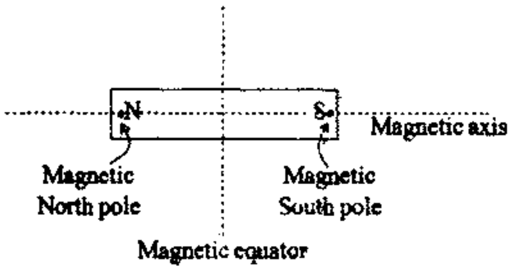
(1) Magnetic field: The space around a magnet within which its influence can be experienced is called its magnetic field.

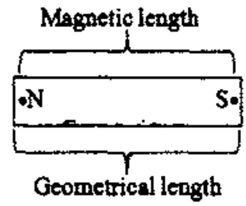
(2) Uniform magnetic field: The magnetic field in a region is said to be uniform if it has the same magnitude and direction at all points of that region.



(3) Magnetic poles: These are the regions of apparently concentrated magnetic strength in a magnet where the magnetic attraction is maximum.

(4) Magnetic axis: The line passing through the poles of a magnet is called the magnetic axis of the magnet.





(5) Magnetic equator: The line passing through the centre of the magnet and at right angles to the magnetic axis is called the magnetic equator of the magnet.

(6) Magnetic length: The distance between two poles of the magnet is called the magnetic length of the magnet. It is slightly less than the geometrical length of the magnet. It is found that

$$\frac{\text{Magnetic length}}{\text{Geometrical length}} = 0.84$$

Coulomb's law of magnetic force: This law states that the force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.

If q_{m1} and q_{m2} are the pole strengths of the two magnetic poles, separated by a distance r apart, then the force between them is given by

$$F \propto \frac{q_{m1}q_{m2}}{r^2}$$

Or $F = k\frac{q_{m1}q_{m2}}{r^2}$

Or $F = \frac{\mu_o}{4\pi}\frac{q_{m1}q_{m2}}{r^2}$

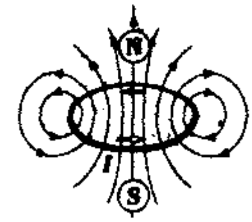
where μ_o is the permeability of free space and is equal to $4\pi \times 10^{-7}\, Hm^{-1}$

Note: If $q_{m1} = q_{m2} = 1 unit$, $r = 1m$ then

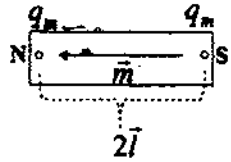
$$F = \frac{\mu_o}{4\pi}\frac{(1)(1)}{(1)^2} = 10^{-7} N$$

Hence a unit magnetic pole may be defined as that pole which when placed in vacuum at a distance of one metre from an identical pole repels it with a force of 10^{-7} newton.

Magnetic dipole: An arrangement of two equal and opposite magnetic poles separated by a small distance is called a magnetic dipole. Every bar magnet is a magnetic dipole. A current carrying loop behaves as a magnetic dipole. An atom acts as a magnetic dipole due to the circulatory motion of the electrons around its nucleus.



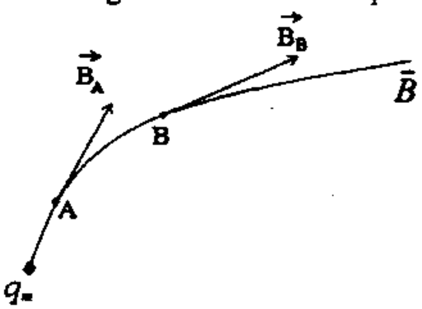
Magnetic dipole moment: The magnetic dipole moment of a magnetic dipole is defined as the product of its pole strength and magnetic length. It is a vector quantity, directed from S-pole to N-pole.

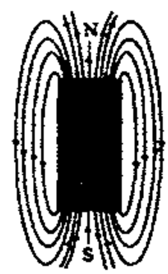


$$\vec{m} = q_m \times 2\vec{l}$$

Where q_m is the pole strength and $2\vec{l}$ is the magnetic length of the dipole. The SI unit of pole strength is ampere-metre (Am), and the SI unit of magnetic dipole moment is ampere-metre^2 (Am^2) or joule per tesla (JT^{-1})

A magnetic line of force: A magnetic line of force may be defined as the curve the tangent to which at any point gives the direction of the magnetic field at that point. It may also be defined as the path along which a unit north pole would tend to move if free to do so.





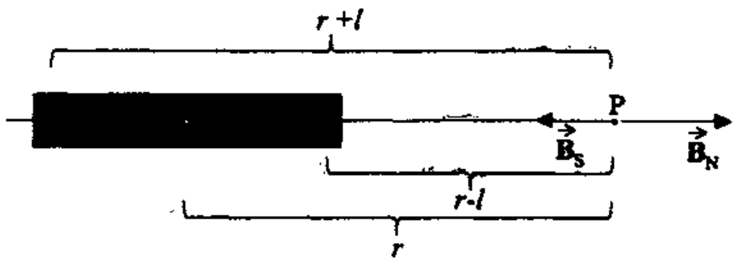
Properties of line of force:

(1) Magnetic lines of force are closed curves which start in air from N-pole and end at the S-pole and then return to the N-pole through the interior of the magnet.

(2) The lines of force never cross each other

(3) They start from the end on the surface of the magnet normally

Magnetic field of a bar magnet at an axial point: Let NS be a bar magnet of length $2l$ and of pole strength q_m. Suppose the magnetic field is to be determined at a point P which lies on the axis of the magnet at a distance r from its centre, as shown in the figure.



The magnetic field at P due to the S-pole is

$$B_S = \frac{\mu_o}{4\pi}\frac{q_m}{(r+l)^2} \text{ along PS}$$

The magnetic field at P due to the N-pole is

$$B_N = \frac{\mu_o}{4\pi}\frac{q_m}{(r-l)^2} \text{ along NP}$$

The net field at P is

$$B_{axial} = B_N - B_S$$

$$B_{axial} = \frac{\mu_o}{4\pi}\frac{q_m}{(r-l)^2} - \frac{\mu_o}{4\pi}\frac{q_m}{(r+l)^2} = \frac{\mu_o q_m}{4\pi}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$B_{axial} = \frac{\mu_o q_m}{4\pi}\left[\frac{4rl}{(r^2-l^2)^2}\right] = \frac{\mu_o}{4\pi}\frac{(q_m \times 2l)2r}{(r^2-l^2)^2}$$

$$B_{axial} = \frac{\mu_o}{4\pi}\frac{2mr}{(r^2-l^2)^2} \qquad [\because m = q_m \times 2l]$$

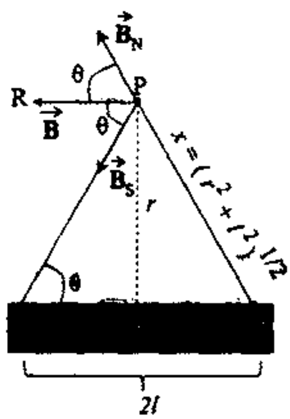
For short bar magnet, $l << r$, therefore we have

$$B_{axial} = \frac{\mu_o}{4\pi}\frac{2mr}{r^4} = \frac{\mu_o}{4\pi}\frac{2m}{r^3} \text{ along NP}$$

Clearly, the magnetic field at any axial point of magnetic dipole is in the same direction as that of its magnetic dipole moment i.e., from S-pole to N-pole, so we can write

$$\vec{B}_{axial} = \frac{\mu_o}{4\pi}\frac{2\vec{m}}{r^3}$$

Magnetic field of a bar magnet at an equatorial point: Let NS be a bar magnet of length $2l$ and of pole strength q_m. Suppose the magnetic field is to be determined at a point P which lies on the equatorial line of the magnet at a distance r from its centre, as shown in the figure.



The magnetic field at P due to the S-pole is

$$B_S = \frac{\mu_o}{4\pi}\frac{q_m}{x^2} \text{ along PS}$$

The magnetic field at P due to the N-pole is

$$B_N = \frac{\mu_o}{4\pi}\frac{q_m}{x^2} \text{ along NP}$$

As the magnitude of B_S and B_N are equal, so their vertical components get cancelled while the horizontal components add up along PR.
Hence the net field at P is

$$B_{equa} = B_N \cos\theta + B_S \cos\theta = 2\, B_N \cos\theta$$

$$B_{equa} = 2 \cdot \frac{\mu_o}{4\pi}\frac{q_m}{x^2} \cdot \frac{l}{x}$$

$$B_{equa} = 2 \cdot \frac{\mu_o}{4\pi}\frac{q_m l}{x^3} = 2 \cdot \frac{\mu_o}{4\pi}\frac{q_m l}{(r^2+l^2)^{\frac{3}{2}}} = \frac{\mu_o}{4\pi}\frac{(q_m \times 2l)}{(r^2+l^2)^{\frac{3}{2}}}$$

$$B_{equa} = \frac{\mu_o}{4\pi}\frac{m}{(r^2+l^2)^{\frac{3}{2}}}$$

For short bar magnet, $l << r$, therefore we have

$$B_{equa} = \frac{\mu_o}{4\pi}\frac{m}{r^3} \text{ along PR}$$

Clearly, the magnetic field at any equatorial point of a magnetic dipole is in the direction opposite to that of its magnetic dipole moment i.e., from N-pole to S-pole. So we can write

$$\vec{B}_{equa} = -\frac{\mu_o}{4\pi}\frac{\vec{m}}{r^3}$$

Note:

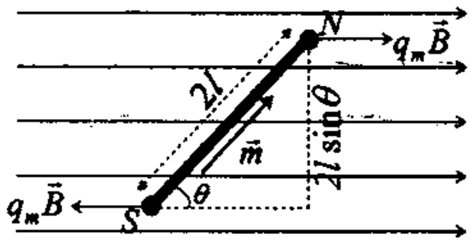
(1) For $l << r$, we have

$$B_{axial} = \frac{\mu_o}{4\pi}\frac{2m}{r^3} \quad \text{and} \quad B_{equa} = \frac{\mu_o}{4\pi}\frac{m}{r^3}$$

$$\therefore B_{axial} = 2B_{equa}$$

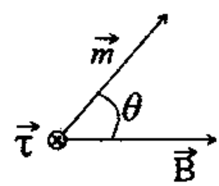
(2) When the point P lies on the axial line of the magnetic dipole, it is called end-on position or tangent A (*tan* A) position. However when point P lies on the equatorial line of the magnetic dipole, it is known as broad-side-on position or tangent B (*tan* B) position.

Torque on a magnetic dipole (bar magnet) in a uniform magnetic field: Let *NS* be a bar magnet of length $2l$ placed in a uniform magnetic field $\vec{B}$. Let q_m be the pole strength of its each pole. Let the magnetic axis of the bar magnet make an angle θ with the field $\vec{B}$ as shown in the figure.



Force on N-pole = $q_m B$ along $\vec{B}$

Force on S-pole = $q_m B$ opposite to $\vec{B}$



The moment of a couple or torque is given by

$$\tau = q_m B \times 2l\sin\theta = (q_m \times 2l)B\sin\theta$$

Or $\tau = mB\sin\theta$

where $m = q_m \times 2l$ is the magnetic dipole moment of the bar magnet.

In vector notation,

$$\vec{\tau} = \vec{m} \times \vec{B}$$

The SI unit of τ is *Nm* and that of *B* is *tesla* (*T*). Therefore, the SI unit of *m* is NmT^{-1} or JT^{-1} or Am^2.

Special cases:

(1) When the magnet lies along the direction of the magnetic field,

$\theta = 0^0$, $\sin\theta = 0$, $\tau = 0$.

Thus the torque is minimum. $\tau_{min} = 0$

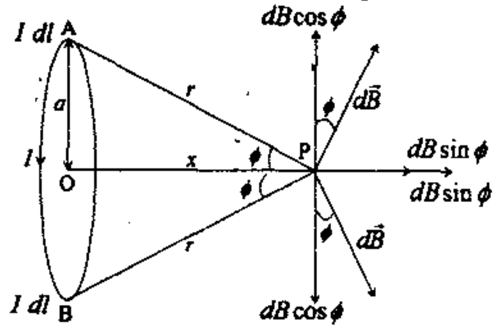
(2) When the magnet lies perpendicular to the direction of the magnetic field,

$\theta = 90^0$, $\sin\theta = 1$, $\tau = mB$.

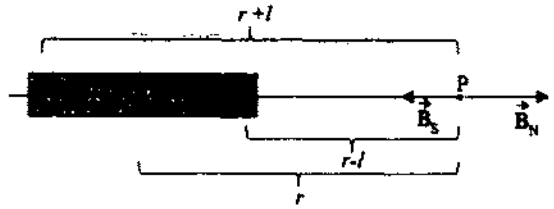
Thus the torque is maximum. $\tau_{max} = mB$

Definition of magnetic dipole moment: If $B = 1$, $\theta = 90^0$, then $\tau = m$.
Hence the magnetic dipole moment may be defined as the torque acting on a magnetic dipole placed perpendicular to a uniform magnetic field of unit strength.

Current loop as a magnetic dipole:



(1)



(2)

In figure (1) we have $B = \frac{\mu_o}{4\pi}\frac{2IA}{x^3}$ for $x >> a$

In figure (2) we have $B_{axial} = \frac{\mu_o}{4\pi}\frac{2m}{r^3}$ for $r >> l$

On comparing the two equations, we have

$$m = IA$$

In vector notation

$$\vec{m} = I\vec{A}$$

Hence the magnetic dipole moment is equal to the product of the current in the loop and the area of the loop.

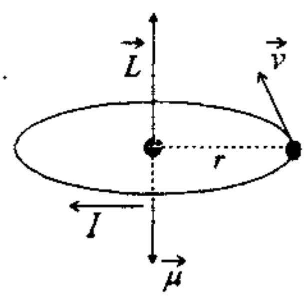
Magnetic dipole moment of a revolving electron: Let an electron of charge e and mass m_e revolve along a circular path of radius r. The corresponding current I is given by

$$I = \frac{e}{T} = \frac{e}{\frac{2\pi r}{v}} = \frac{e\,v}{2\pi r}$$

$$IA = \frac{e\,v}{2\pi r}A = \frac{e\,v}{2\pi r}(\pi r^2) = \frac{e\,v\,r}{2}$$

Where A is the area of the loop, and the above equation can also be written as

$$\mu = IA = \frac{e\,v\,r}{2}\cdot\frac{m_e}{m_e} = \frac{e}{2\,m_e}m_e v\,r$$



Where μ is the magnetic dipole moment.

The angular momentum of the electron is $L = m_e v\,r$.

Hence $\mu = \frac{e}{2\,m_e}L$

In vector form

$$\vec{\mu} = -\frac{e}{2\,m_e}\vec{L}$$

Note:

(1) According to Bohr's quantisation condition, the angular momentum of the electron in any permissible orbit is integral multiple of $h/2\pi$ where h is a planck's constant.

$$L = \frac{nh}{2\pi} \qquad \text{where } n = 1, 2, 3, \ldots$$

$$\mu = \frac{e}{2\,m_e} L = \frac{e}{2\,m_e}\frac{nh}{2\pi} = n\left(\frac{e\,h}{4\pi\,m_e}\right)$$

(2) Bohr magneton μ_B is defined as the magnetic moment associated with an electron due to its orbital motion in the first orbit of hydrogen atom.

$$\mu_B = \frac{e\,h}{4\pi\,m_e}$$

Gauss's law in magnetism: The surface integral of a magnetic field over a closed surface is always zero. Or the net magnetic flux through a closed surface is always zero i.e.,

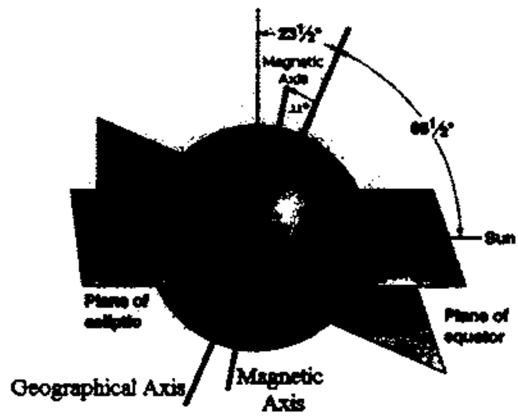
$$\oint \vec{B}\cdot d\vec{S} = 0$$

Some definitions in connection with earth's magnetism:

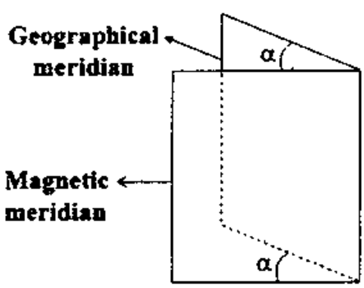
(1) Geographic axis or Geographical axis: The straight line passing through the geographical north and south poles of the earth is called geographic axis

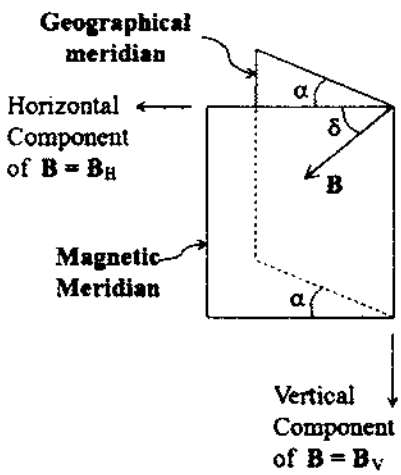
(2) Magnetic axis: The straight line passing through the magnetic north and south poles of the earth is called magnetic axis

(3) Magnetic & Geographical meridian: The vertical plane passing through the magnetic axis is called magnetic meridian and through the geographic axis is called geographical meridian



Elements of earth's magnetic field: The earth's magnetic field at a place can be completely described by three parameters which are called elements of earth's magnetic field





(1) Magnetic declination: The angle between the geographical meridian and the magnetic meridian at a place is called the magnetic declination (α) at that place.

(2) Angle of dip or magnetic inclination: The angle made by the earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called angle of dip (δ) at any place.

Note: The angle of dip δ is 0^0 at the magnetic equator, and 90^0 at the magnetic pole.

(3) Horizontal component of the earth's magnetic field: It is the component of earth's total magnetic field $\vec{B}$ in the horizontal direction in the magnetic meridian.

Note:

The horizontal component of the earth's magnetic field is $B_H = B\cos\delta$.

The vertical component of the earth's magnetic field is $B_V = B\sin\delta$.

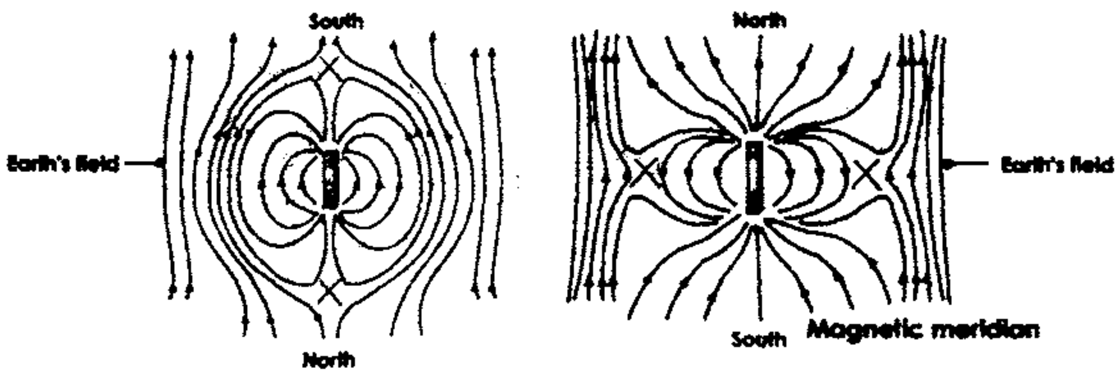
$$\frac{B_V}{B_H} = \frac{B\sin\delta}{B\cos\delta} = \tan\delta$$

At the magnetic equator, $\delta = 0^0$, $\therefore B_H = B\cos 0^0 = B$

At the magnetic pole, $\delta = 90^0$, $\therefore B_H = B\cos 90^0 = 0$

Thus the value of B_H is different at different places on the surface of the earth.

Neutral point: It is the point where the magnetic field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field. The resultant magnetic field at the neutral point is zero.



Classification of magnetic materials:

(1) Diamagnetic substances: Diamagnetic substances are those which develop feeble magnetisation in the opposite direction of the magnetising field. Such substances are feeble repelled by magnets and tend to move from stronger to weaker parts of a magnetic field. Examples are water, copper, lead, tin, gold, silicon etc

(2) Paramagnetic substances: Paramagnetic substances are those which develop feeble magnetisation in the direction of the magnetising field. Such substances are feeble attracted by magnets and tend to move from weaker to stronger parts of a magnetic field. Examples are Manganese, aluminium, sodium etc

Curie Law: According to Curie law, the intensity of magnetisation B of the paramagnetic material is

(1) Directly proportional to the external magnetic field H in which the specimen is placed.

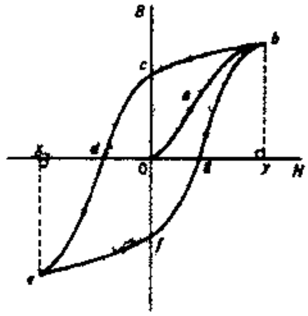
(2) Inversely proportional to the absolute temperature

$$B \propto \frac{H}{T}$$

$$B = C\frac{H}{T} \text{------- (1)}$$

Where C is a constant called Curie constant. Equation (1) is known as Curie law

(3) Ferromagnetic substances: Ferromagnetic substances are those which develop strong magnetisation in the direction of the magnetising field. They are strongly attracted by magnets and tend to move from weaker to stronger parts of a magnetic field. Examples are iron, cobalt, nickel, etc



Hysteresis: The lagging of the intensity of magnetization B (or magnetic induction) behind the magnetizing field H, when a magnetic specimen is taken through a cycle of magnetization, is called hysteresis.

Hysteresis loop: A hysteresis loop shows the relationship between the induced magnetic flux density and the magnetizing force.

Some important terms used to describe magnetic properties of materials:

(1) Magnetising field: The magnetic field that exists in vacuum and induces magnetism is called magnetising field. It is given by

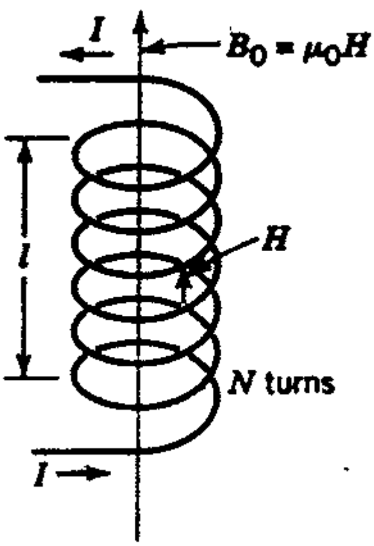


Figure (1)

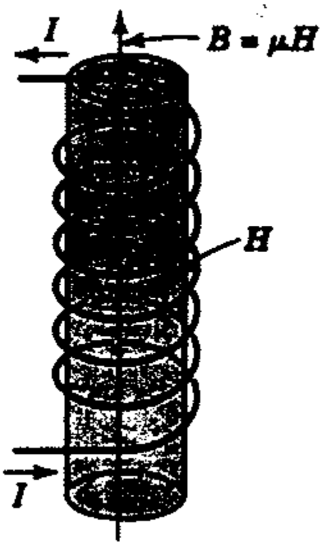


Figure (2)

$$\vec{B}_0 = \mu_0 \vec{H} \quad \text{Or} \quad \vec{H} = \frac{\vec{B}_0}{\mu_0}$$

Where $\vec{H}$ is the magnetising field, $\vec{B}_0$ is the magnetic induction in free space and μ_0 is the permeability of free space (figure 1).

$$\vec{B} = \mu \vec{H} \quad \text{Or} \quad \vec{H} = \frac{\vec{B}}{\mu}$$

Where $\vec{H}$ is the magnetising field, $\vec{B}$ is the magnetic induction and μ is the magnetic permeability of the magnetic material (figure 2).

(2) Magnetisation vector: The magnetic moment developed per unit volume of the material when placed in magnetising field is called magnetisation vector or intensity of magnetisation or simply magnetisation

$$\vec{M} = \lim_{\Delta v \to 0} \frac{\sum(\vec{p}_m)_n}{\Delta v} = \frac{magnetic\ dipole\ moment}{unit\ volume}$$

(3) Magnetic susceptibility: Magnetic susceptibility measures the ability of a substance to take up magnetisation when placed in a magnetic field. It is defined as the ratio of the intensity of magnetisation M to the magnetising field intensity H.

$$\vec{M} \propto \vec{H} \quad \text{Or} \quad \vec{M} = \chi_m \vec{H}$$

$$|\vec{M}| = \chi_m |\vec{H}| \quad \text{Or} \quad \chi_m = \frac{M}{H}$$

χ_m is the magnetic susceptibility

(4) Magnetic induction: The total magnetie field inside a magnetic material is the sum of the external magnetising field $\vec{H}$ and the additional magnetic field $\vec{M}$ produced due to magnetisiation of the material . This is called magnetic induction $\vec{B}$

$$\vec{B} = \vec{B}_0 + \mu_0\vec{M} = \mu_0\vec{H} + \mu_0\vec{M} = \mu_0\vec{H} + \mu_0\chi_m \vec{H} = \mu_0(1+\chi_m)\vec{H}$$

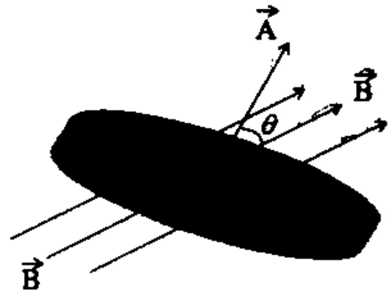
$$\because \vec{B} = \mu_0\mu_r \vec{H} \qquad \therefore \mu_r = 1 + \chi_m$$

(5) Magnetic permeability: Permeability is the measure of the extent to which a material can be penetrated or permeated by the magnetic field.

Magnetic flux: The magnetic flux through any surface placed in the magnetic field is the total number of magnetic lines of force crossing this surface normally.

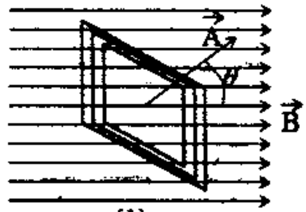
$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$

The unit of magnetic flux ϕ_B is weber (Wb) and the unit of magnetic field $\vec{B}$ is tesla (T). So $Wb = Tm^2$



Note:
1 Wb = 10^8 maxwell

Figure (1) $\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$

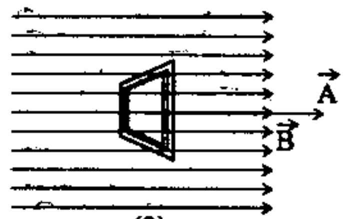


(1)

Figure (2) $\phi_B = \vec{B} \cdot \vec{A} = BA\cos 0°$

$\phi_B = BA$

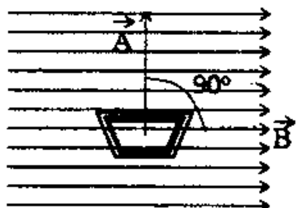
(i.e., the flux is maximum)



(2)

Figure (3) $\phi_B = \vec{B} \cdot \vec{A} = BA\cos 90°$

$\phi_B = 0$



(3)

Figure (4)

$\phi_B = \vec{B} \cdot \vec{A} = BA\cos 180°$

$\phi_B = -BA$

(i.e., the flux is negative and minimum)

(4)

Electromagnetic induction: Electromagnetic induction is the phenomenon of production of electric current or EMF in a closed coil, when magnetic flux linked with the coil is changed. The current and the EMF so produced are called induced current and induced EMF.

Laws of electromagnetic induction:

Faraday's first law: Whenever there is a change in magnetic flux linked with a closed circuit, an induced EMF (and hence current) is produced.

Note: For EMF to be induced in a coil, the magnetic flux linking the coil should change continuously.

Faraday's second law: The magnitude of the induced EMF is directly proportional to the rate of change of magnetic flux linked with the closed circuit.

$$\varepsilon = \frac{d\phi}{dt}$$

Lenz's law: The direction of induced current is such that it opposes the cause which produces it, i.e., it opposes the change in magnetic flux.

$$\varepsilon = -\frac{d\phi}{dt}$$

Lenz's law and Energy conservation: Work has to be done against the force of attraction or repulsion in order to move the magnet away from the coil or in bringing the magnet closer to the coil. It is this mechanical work which changes into electrical energy or EMF. Thus, Lenz's law is in accordance with the principle of conservation of energy.

Expression for the induced EMF:
If the flux changes from ϕ_1 to ϕ_2 in time t, then the induced EMF will be

$$\varepsilon \propto \frac{\phi_2 - \phi_1}{t}$$

Or $\varepsilon = k\frac{\phi_2 - \phi_1}{t}$ where the value of k is unity in SI unit

Or $\varepsilon = \frac{\phi_2 - \phi_1}{t}$

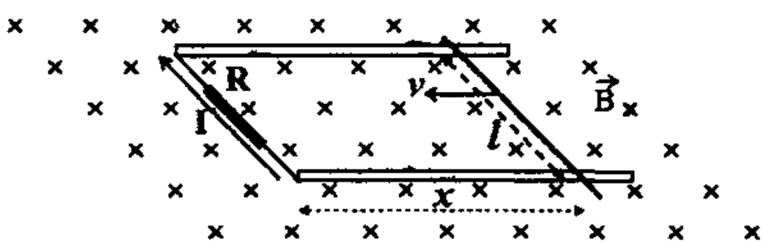
If the coil consists of N tightly wound turns, then $\varepsilon = N\frac{\phi_2 - \phi_1}{t}$

In differential form we have $\varepsilon = N\frac{d\phi}{dt}$

The magnitude and direction of induce EMF are given by $\varepsilon = -N\frac{d\phi}{dt}$

The minus sign on the RHS represents Lenz's law mathematically. In SI units, ε is measured in volts, ϕ in webers and t in seconds.

Motional EMF from Faraday's law: The EMF induced across the end of a conductor due to its motion in a magnetic field is called motional EMF.
Let a conductor of length l is moving with velocity v towards the left in a uniform magnetic field $\vec{B}$. The magnetic field $\vec{B}$ acts perpendicular to the plane of the rails vertically downward. During its motion, the area lx decreases.



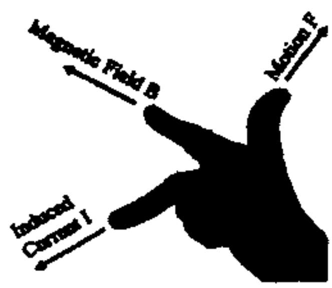
The magnetic flux $\phi = BA = B(lx)$

The induced EMF $\varepsilon = -\frac{d\phi}{dt}$

Or $\varepsilon = -\frac{d(Blx)}{dt}$

Or $\varepsilon = -Bl\frac{dx}{dt}$

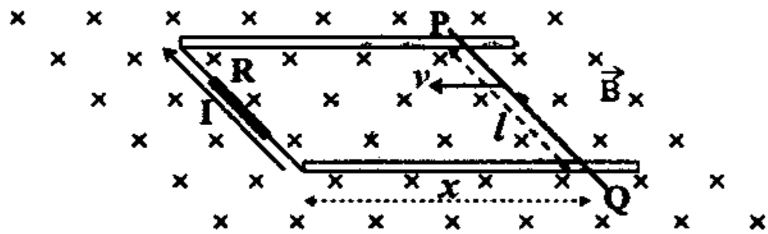
Or $\varepsilon = Blv$



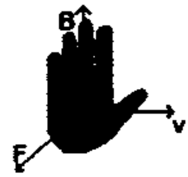
Where $\frac{dx}{dt} = -v$, the velocity v is in the decreasing direction of x

Note: To know the direction of current I, we have to use the Fleming's right hand rule.

Motional EMF from Lorentz force and energy consideration (for electrons):
As the arm PQ is move towards the left with a speed v, the free electrons on PQ also move with the same speed towards the left. The electrons experience a magnetic Lorentz force $F_m = qvB$



According to Flemings' left hand rule or Right hand palm rule, this force acts in the direction QP and hence the free electrons will move towards P. A negative charge accumulates at P and a positive charge at Q. An electric field E is set up in the conductor from Q to P. This field exerts a force $F_e = qE$ on the free electrons. The accumulation of charges at the two ends continues till these two forces balance each other, i.e.,



$$F_m = F_e$$

Or $qvB = qE$

$$vB = E$$

The potential difference between the ends Q and P is

$$V = El = vBl = Blv$$

Clearly, it is the magnetic force on the moving free electrons that maintains the potential difference and produces the EMF $\varepsilon = Blv$ called the motional EMF.

Note:

(1) Current induced in the loop $I_{induced} = \dfrac{\varepsilon}{R} = \dfrac{Blv}{R}$

(2) Force on the movable arm $F = IlB\sin 90^\circ = \left(\dfrac{Blv}{R}\right)lB = \dfrac{B^2l^2v}{R}$

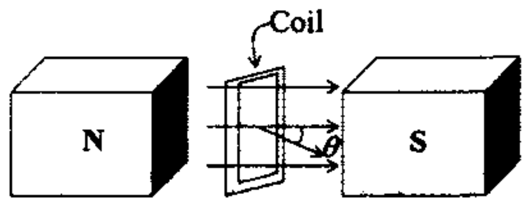
(3) Power delivered by the external force $P = Fv = \dfrac{B^2l^2v^2}{R}$

(4) Power dissipated as Joule loss $P_J = I^2R = \left(\dfrac{Blv}{R}\right)^2 R = \dfrac{B^2l^2v^2}{R}$

Induce EMF by changing relative orientation of the coil and the magnetic field – Theory of AC generator: Let a closed coil of area A rotates freely in a magnetic field B, and let θ be the angle between the perpendicular direction of the plane of the coil at any instant with the field B as shown in the figure.

The magnetic flux is $\phi = BA\cos\theta$

$$\phi = BA\cos\omega t$$



The induced EMF $\varepsilon = -\dfrac{d\phi}{dt}$

$$\varepsilon = -\frac{d(BA\cos\omega t)}{dt}$$

$$\varepsilon = -BA\frac{d}{dt}(\cos\omega t) = BA\omega\sin\omega t$$

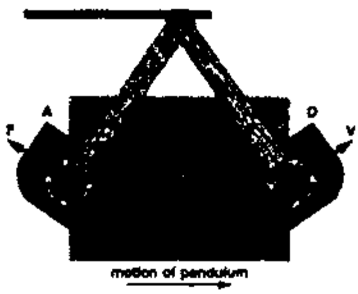
If the coil has N turns, then

$$\varepsilon = NBA\omega\sin\omega t = \varepsilon_0 \sin\omega t$$

$$\varepsilon = \varepsilon_0 \sin 2\pi f t$$

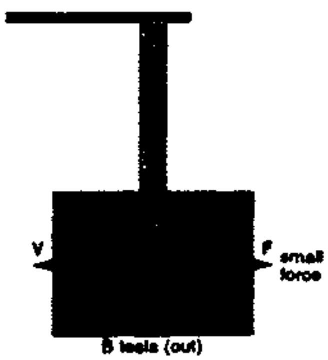
Such an EMF is called sinusoidal or alternating EMF.

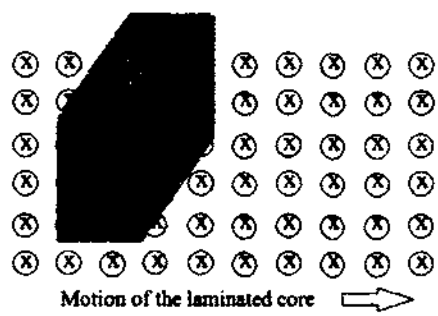
Eddy currents: Eddy currents are the current induced in a solid metallic masses when the magnetic flux threading through them changes.



In the first position AB, The metal sheet is swinging to the right and has just entered the magnetic field. A clockwise, circular eddy current is induced in the copper sheet to resist this motion.
In the second position CD, The metal sheet is still swinging to the right and is half way out of the magnetic field. An anti-clockwise, circular eddy current is induced in the copper sheet to resist this motion.

Minimisation:
(1) The effect of eddy currents can be minimized by reducing the area through which the eddy currents flows. Thus the pendulum plate with holes or slots reduces electromagnetic damping because the area decreases and the plate swings more freely.





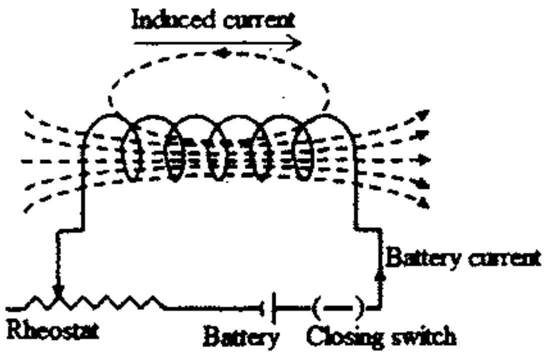
(2) These currents can be minimized by using thin laminated sheets in the core of the transformers rather than using solid thick metallic plate. The plane of the laminated thin sheets should be always arranged parallel to the magnetic field so that they cut across the eddy current paths. This type of parallel arrangement reduces the strength of eddy currents.

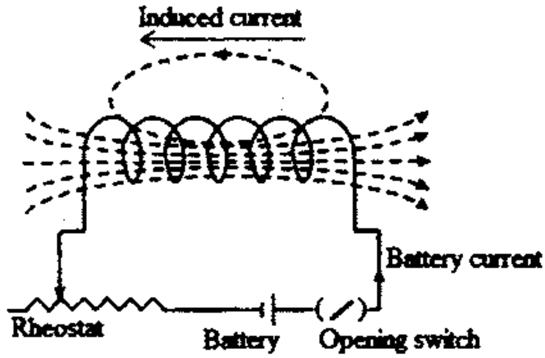
Practical Applications of Eddy Currents:
(i) Magnetic brake
(ii) Dead-beat galvanometer
(iii) Induction furnace
(iv) Energy meter

(v) Diathermy
(vi) Speedometer
(vii) Induction (i.e., ac) motor

Inductance: Inductance is the property in an electrical circuit where a change in the electric current through that circuit induces an electromotive force (EMF) that opposes the change in current.

Self-induction: Self-induction is the phenomenon of production of induced EMF in a coil when a changing current passes through it. This EMF is called self-induced EMF or back EMF.





The magnetic flux linked with a coil of N turns is proportional to the current i.e.,

$$N\phi \propto I \qquad \text{Or} \qquad N\phi = LI$$

L = self-inductance of the coil.

The induced EMF in the coil is $\varepsilon = -N\dfrac{d\phi}{dt} = -L\dfrac{dI}{dt}$

The SI unit of L is henry (H)

Coefficient of Self-Induction (Self-Inductance): Self-induction is the property of a coil (or a circuit) by virtue of which it opposes any change in the strength of current flowing through it by inducing an EMF in itself.
For this reason self-inductance is also called the inertia of electricity.

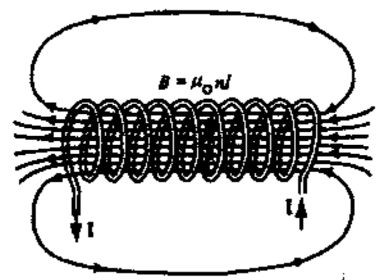
$$\varepsilon = -L\frac{dI}{dt}$$

A coil (or a circuit) has an inductance of 1 henry if current changing at the rate of 1 ampere per second through the coil induces an EMF of 1 volt.

Again $L = \dfrac{N\phi}{I}$

A coil (or a circuit) has an inductance of 1 henry if a current of 1 ampere in the coil set up a total flux of 1 weber (i.e., $N\phi = 1$ weber).

Self-inductance of a long solenoid: Consider a long air-cored solenoid of length l, area of cross-section A and having total number of turns N. For a long solenoid, the magnetic field inside is constant. If the solenoid is carrying a current I then magnetic field inside the solenoid is given by



$$B = \mu_0 nI = \frac{\mu_0 NI}{l} \qquad \text{where } n = \frac{N}{l} \text{ is the number of turns per unit length.}$$

Magnetic flux linked with each turn of solenoid is

$$\phi = BA = \frac{\mu_0 NI}{l} \cdot A$$

$\therefore$ Inductance of solenoid $L = \frac{N\phi}{I} = \frac{N}{I} \cdot \frac{\mu_0 NI}{l} \cdot A$

$$\text{Or } L = \frac{\mu_0 N^2 A}{l}$$

If the solenoid carries a core of relative permeability μ_r, then $L = \frac{\mu_0 \mu_r N^2 A}{l}$

Mutual-induction: Mutual-induction is the phenomenon of production of induced EMF in one coil due to a change of current in the neighbouring coil. The magnetic flux linked with a coil of N turns is proportional to the current that flows through the other i.e.,

$$N\phi \propto I \qquad \text{Or} \qquad N\phi = MI$$

M = mutual inductance between the two coils.

The mutual induced EMF set up in one coil when a current flows through the other is

$$\varepsilon = -N\frac{d\phi}{dt} = -M\frac{dI}{dt}$$

The SI unit of M is henry (H).

Theorem of Reciprocity:

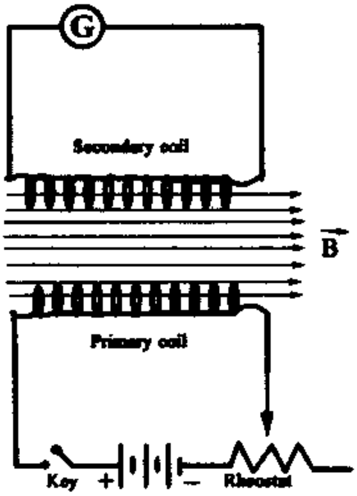
The magnetic flux linked with the secondary coil of N_s turns is proportional to the current in the primary coil i.e.,

$$N_s\phi_s \propto I_p \qquad \text{Or} \qquad N_s\phi_s = M_s I_p$$

M_s = mutual inductance of the secondary coil.

The mutual induced EMF in the secondary coil is

$$\varepsilon_s = -N_s\frac{d\phi_s}{dt} = -M_s\frac{dI_p}{dt}$$



Also the magnetic flux linked with the primary coil of N_p turns is proportional to the current in the secondary coil i.e.,

$$N_p\phi_p \propto I_s \qquad \text{Or} \qquad N_p\phi_p = M_p I_s$$

M_p = mutual inductance of the primary coil. The mutual induced EMF in the primary coil is

$$\varepsilon_p = -N_p\frac{d\phi_p}{dt} = -M_p\frac{dI_s}{dt}$$

It is found that if $\frac{dI_p}{dt} = \frac{dI_s}{dt}$

then $\varepsilon_s = \varepsilon_p$

And $M_s = M_p = M$ (say)

The above equation is called the theorem of reciprocity or reciprocity theorem of mutual inductance.

Coefficient of Mutual-Induction (Mutual-Inductance): Mutual induction is the property of two coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an induced EMF.

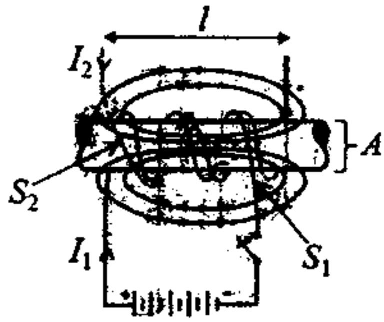
$$\varepsilon = -M\frac{dI}{dt}$$

Mutual inductance between two coils is 1 henry if current changing at the rate of 1 ampere per second in one coil induces an EMF of 1 volt in the other.

Again $M = \frac{N\phi}{I}$

Mutual inductance between two coils is 1 henry if a current of 1 ampere in one coil sets up a total flux of 1 weber in the other coil.

Mutual-inductance of two long solenoids: Consider two long air-cored solenoids S_1 and S_2 of the same length l. Solenoid S_2 surrounds solenoid S_1 completely as shown in the figure. The two solenoids are so closely wound that they have the same area of cross-section A. Let N_1 and N_2 be the total number of turns of solenoids S_1 and S_2 respectively.



Mutual inductance of S_2 with respect to S_1 is M_{21}: The magnetic field B_1 inside solenoid S_1 due to current I_1 through it is given by

$$B_1 = \mu_o n_1 I_1 = \frac{\mu_o N_1 I_1}{l} \qquad \text{where } n_1 = \frac{N_1}{l} \text{ is the number of turns per unit length.}$$

Since the solenoids are closely wound, the magnetic field inside solenoid S_2 is also B_1. Magnetic flux linked with each turn of solenoid S_2 is

$$\phi_2 = B_2 A = B_1 A = \frac{\mu_o N_1 I_1}{l}\cdot A$$

$\therefore$ Mutual inductance of solenoid S_2 is $M_{21} = \frac{N_2\phi_2}{I_1} = \frac{N_2}{I_1}\cdot\frac{\mu_o N_1 I_1}{l}\cdot A$

$$\text{Or } M_{21} = \frac{\mu_o N_1 N_2 A}{l}$$

Mutual inductance of S_1 with respect to S_2 is M_{12}: The magnetic field B_2 inside solenoid S_2 due to current I_2 through it is given by

$$B_2 = \mu_o n_2 I_2 = \frac{\mu_o N_2 I_2}{l} \qquad \text{where } n_2 = \frac{N_2}{l} \text{ is the number of turns per unit length.}$$

Since the solenoids are closely wound, the magnetic field inside solenoid S_1 is also B_2. Magnetic flux linked with each turn of solenoid S_1 is

$$\phi_1 = B_1 A = B_2 A = \frac{\mu_o N_2 I_2}{l}\cdot A$$

$\therefore$ Mutual inductance of solenoid S_1 is $M_{12} = \frac{N_1\phi_1}{I_2} = \frac{N_1}{I_2}\cdot\frac{\mu_o N_2 I_2}{l}\cdot A$

$$\text{Or } M_{12} = \frac{\mu_o N_1 N_2 A}{l}$$

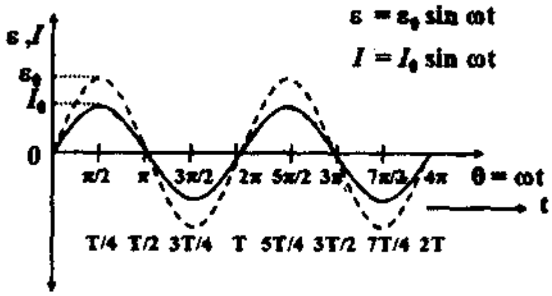
Thus we see that $M_{21} = M_{12} = M$

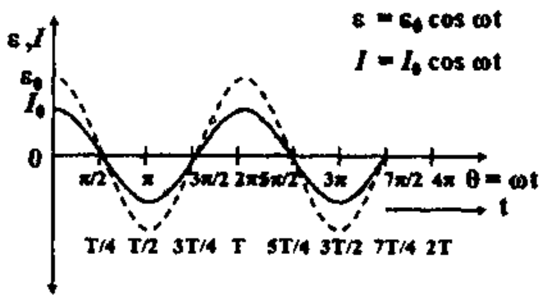
The mutual inductance between the two coils is the same no matter which of the two coils carries the current. Therefore no subscripts are needed.

$$M = \frac{\mu_o N_1 N_2 A}{l}$$

Note: The mutual induction between two coils depends upon
(a) the size and shape of the two coils
(b) their relative orientation
(c) separation between the coils
(d) material of the core on which they are wound.

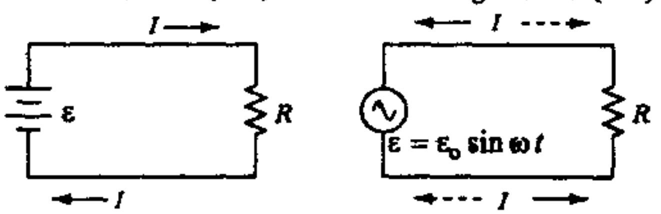
Alternating voltage: An alternating voltage (or EMF) is one whose magnitude changes continuously with time and direction reverses periodically.





Alternating current: An alternating current is that current whose magnitude changes continuously with time and direction reverses periodically.

Direct current (d.c.) Alternating current (a.c.)



The alternating EMF is given by $\varepsilon = \varepsilon_o \sin \omega t$

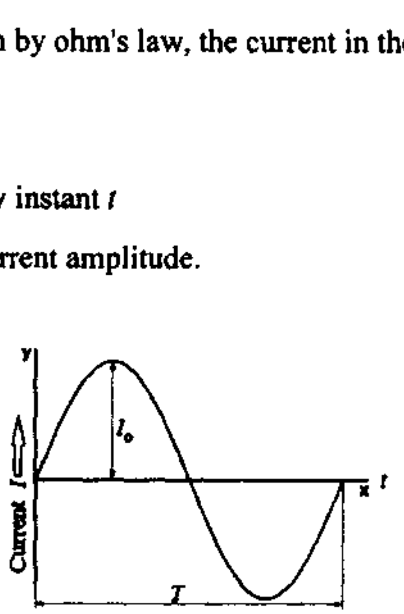
Suppose this EMF is applied to a circuit of resistance R. Then by ohm's law, the current in the circuit will be

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon_o}{R} \sin \omega t = I_o \sin \omega t$$

I is the instantaneous value of alternating current (a.c.) at any instant t

$I_o = \frac{\varepsilon_o}{R}$ is the peak or maximum value of a.c. and is called current amplitude.

Amplitude: The maximum value attained by an alternating current or voltage in either direction is called its amplitude or peak value and is denoted by I_o for current and ε_0 for voltage.



Time period: The time taken by an alternating current or voltage to complete one cycle of its variations is called its time period and is denoted by T

$$T = \frac{Angular\ displacement}{Angular\ velocity} = \frac{2\pi}{\omega}$$

Frequency: The number of cycles completed per second by an alternating current or voltage is called its frequency and is denoted by f

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \qquad \Rightarrow \omega = 2\pi f$$

Note:

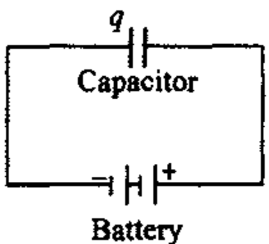
(1) An alternating current can be represented as

$$I = I_o \sin \omega t = I_o \sin 2\pi ft = I_o \sin \frac{2\pi}{T} t$$

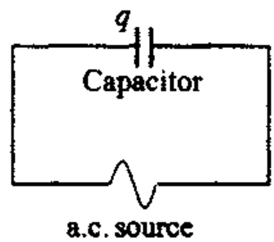
(2) A direct current is that current which flows with a constant magnitude in the same direction.

(3) The frequency of alternating current of voltage in India is $f = 50Hz$.

Mean or average value of alternating current and voltage: It is defined as that value of direct current which sends the same charge in a circuit in the same time as is sent by the given alternating current in its half time period. It is denoted by I_{av}, I_m or I_v



Time = T/2, Charge collected = q



Time = T/2, Charge collected = q

The alternating current at any instant t is given by

$$I = I_o \sin \omega t$$

The amount of charge that flows through the circuit in small time dt will be

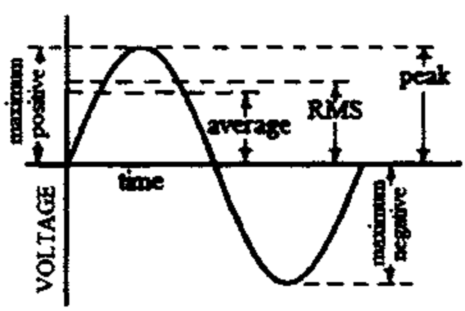
$$dq = Idt = I_o \sin \omega t dt$$

The total charge that flows through the circuit in the first half cycle is

$$q = \int_0^q dq = \int_0^{\frac{T}{2}} I_o \sin \omega t dt = I_o \left[-\frac{\cos \omega t}{\omega} \right]_0^{\frac{T}{2}} = -\frac{I_o}{\omega} [\cos \omega t]_0^{\frac{T}{2}}$$

$$q = -\frac{I_o}{2\pi / T} \left[\cos \frac{2\pi}{T} t \right]_0^{\frac{T}{2}} = -\frac{I_o}{2\pi / T} \left[\cos \pi - \cos 0^o \right] = -\frac{I_o}{2\pi / T} [-1 - 1]$$

$$q = \frac{I_o T}{\pi}$$



$\therefore$ The average value of a.c. over the first half cycle is

$$I_{av} = \frac{charge}{time} = \frac{q}{T/2} = \frac{2q}{T}$$

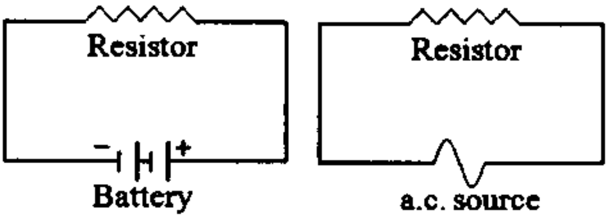
$$I_{av} = \frac{2}{T} \frac{I_o T}{\pi} = \frac{2}{\pi} I_o = 0.637 I_o$$

The similar relation can be proved for alternating

EMF, which is

$$\varepsilon_{av} = \frac{2}{\pi}\varepsilon_o = 0.637\varepsilon_o$$

Root mean square (RMS) or virtual or effective value of a.c.: It is defined as that value of direct current which produces the same heating effect in a given resistor as is produced by the given alternating current when passed for the same time. It is denoted by I_{rms}, I_v or I_{eff}



The amount of heat produced in small time dt will be

$$dH = I^2 R dt$$

If T is the time period of a.c., then heat produced in one complete cycle will be

$$H = \int_0^T I^2 R dt$$

$$H = R\int_0^T I_o^2 \sin^2 \omega t dt$$

$$H = RI_o^2 \int_0^T \sin^2 \omega t dt$$

$$H = RI_o^2 \int_0^T \frac{1-\cos 2\omega t}{2} dt$$

$$H = \frac{RI_o^2}{2}\int_0^T (1-\cos 2\omega t) dt$$

$$H = \frac{RI_o^2}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$H = \frac{RI_o^2}{2}\left[t - \frac{\sin 2\frac{2\pi}{T}t}{2\omega}\right]_0^T$$

$$H = \frac{RI_o^2}{2}\left[(T - \frac{\sin \pi}{2\omega}) - (0-0)\right]$$

$$H = \frac{RI_o^2}{2}T \text{ ---------------- } (1)$$

Let I_{eff} be the effective value of a.c. Then heat produced in time T must be

$$H = I_{eff}^2 RT \text{ ---------------- } (2)$$

Equating equation (1) and equation (2) we have

$$I_{eff}^2 RT = \frac{RI_o^2}{2}T$$

$$I_{eff}^2 = \frac{I_o^2}{2}$$

$$I_{eff} = \sqrt{\frac{I_o^2}{2}} = \frac{I_o}{\sqrt{2}} = 0.707 I_o$$

Root mean square (RMS) or virtual or effective value of alternating EMF: It is defined as that value of steady voltage that produces the same amount of heat in a given resistance as is produced by the given alternating EMF when applied to the same resistance for the same time. It is denoted by ε_{rms}, ε_v or ε_{eff}

The amount of heat produced in small time dt will be

$$dH = I^2 R dt$$

$$\because \varepsilon = IR \qquad \Rightarrow I = \frac{\varepsilon}{R} \qquad \text{or } I^2 = \frac{\varepsilon^2}{R^2}$$

$$\therefore dH = \frac{\varepsilon^2}{R^2} R dt \qquad \Rightarrow dH = \frac{\varepsilon^2}{R} dt$$

Let T be the time period of alternating EMF. Then heat produced in time T will be

$$H = \frac{1}{R}\int_0^T \varepsilon^2 dt = \frac{1}{R}\int_0^T \varepsilon_o^2 \sin^2 \omega t dt$$

$$H = \frac{\varepsilon_o^2}{R}\int_0^T \sin^2 \omega t dt$$

$$H = \frac{\varepsilon_o^2}{R}\int_0^T \frac{1-\cos 2\omega t}{2} dt$$

$$H = \frac{\varepsilon_o^2}{2R}\int_0^T (1-\cos 2\omega t) dt$$

$$H = \frac{\varepsilon_o^2}{2R}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$H = \frac{\varepsilon_o^2}{2R}\left[t - \frac{\sin 2\frac{2\pi}{T}t}{2\omega}\right]_0^T$$

$$H = \frac{\varepsilon_o^2}{2R}\left[(T - \frac{\sin \pi}{2\omega}) - (0-0)\right]$$

$$H = \frac{\varepsilon_o^2}{2R} T \text{ ---------------- (1)}$$

Let ε_{eff} be the effective value of alternating EMF. Then heat produced in time T must be

$$H = \frac{\varepsilon_{eff}^2}{R^2} RT \qquad \Rightarrow H = \frac{\varepsilon_{eff}^2}{R} T \text{ ---------------- (2)}$$

Equating equation (1) and equation (2) we have

$$\frac{\varepsilon_{eff}^2}{R} T = \frac{\varepsilon_o^2}{2R} T$$

$$\varepsilon_{eff}^2 = \frac{\varepsilon_o^2}{2}$$

$$\varepsilon_{eff} = \sqrt{\frac{\varepsilon_o^2}{2}} = \frac{\varepsilon_o}{\sqrt{2}} = 0.707\varepsilon_o$$

Importance of RMS values: An alternating voltage or current is always specified in terms of RMS values. Thus an alternating current of 10A is one which has the same heating effect as 10A d.c., under similar conditions. The domestic a.c. supply is 230V, 50Hz. It is the RMS or effective value. It means that the alternating voltage available has the same heating effect as 230V d.c., under similar conditions. The equation of this alternating voltage is

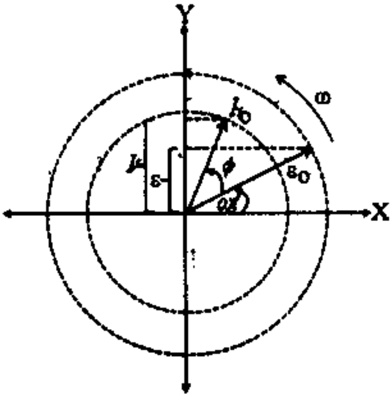
$$\varepsilon = \varepsilon_o \sin \omega t$$

$$\varepsilon = (\varepsilon_{eff} \times \sqrt{2}) \sin(2\pi \times f)t$$

$$\varepsilon = (230 \times \sqrt{2}) \sin(2\pi \times 50)t = 325.269 \sin 314.16\, t$$

Phasors and phasor diagrams: The rotating vector that represents a sinusoidally varying quantity is called a phasor.

Phasor diagram: A diagram that represents alternating current and voltage of the same frequency as rotating vectors (phasors) along with proper phase angle between them is called a phasor diagram or Argand diagram.



Suppose the alternating EMF and current in a circuit are given by

$$\varepsilon = \varepsilon_o \sin \omega t \quad \text{and } I = I_o \sin \omega t$$

If the current leads the EMF by ϕ, where ϕ is the phase angle between ε_0 and I_0, then we can write $I = I_0 \sin(\omega t + \phi)$ as shown in the figure.

If the current lags behind the EMF by ϕ then we can write $I = I_0 \sin(\omega t - \phi)$

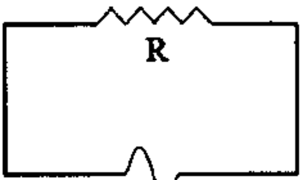
a.c. circuit containing only a resistor: As shown in the figure, a resistor of resistance R is connected to a source of alternating EMF ε given by

$$\varepsilon = \varepsilon_o \sin \omega t$$

Or $IR = \varepsilon_o \sin \omega t$

Or $I = \frac{\varepsilon_o}{R} \sin \omega t$

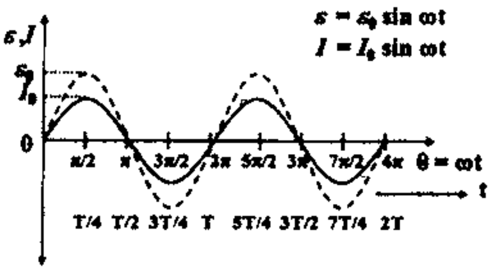
Or $I = I_o \sin \omega t$

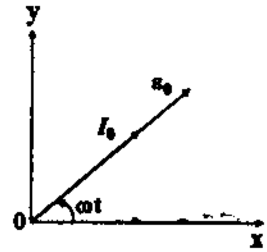


$\varepsilon_0 \sin \omega t$

Where $I_o = \frac{\varepsilon_o}{R}$ is the maximum or peak value of a.c.

The EMF ε and the current I are in the same phase in a purely resistive circuit. This means that both ε and I attain their zero, minimum and maximum values at the same time.





Note: $\because I_o = \frac{\varepsilon_o}{R}$ $\Rightarrow \varepsilon_o > I_o$

a.c. circuit containing only an inductor: As shown in the figure, an inductor L is connected to a source of alternating EMF ε given by

$$\varepsilon = \varepsilon_o \sin \omega t$$

The magnetic flux linked with a coil of N turns is proportional to the current i.e.,

$$N\phi \propto I \qquad \text{Or} \quad N\phi = LI$$

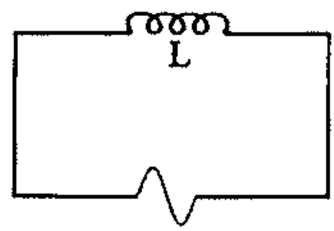
As the alternating current flows through the inductor, a back EMF is

$$\varepsilon = -N\frac{d\phi}{dt} = -L\frac{dI}{dt}$$

$\therefore$ net instantaneous EMF = Source EMF + back EMF

$$= \varepsilon + \left(-L\frac{dI}{dt}\right) = \varepsilon - L\frac{dI}{dt}$$

But this net EMF is zero because there is no resistance in the circuit.



$$\therefore 0 = \varepsilon - L\frac{dI}{dt}$$

Or $\varepsilon = L\frac{dI}{dt}$

Or $\varepsilon_o \sin \omega t = L\frac{dI}{dt}$

Or $dI = \frac{\varepsilon_o}{L}\sin \omega t \, dt$

Or $I = \int \frac{\varepsilon_o}{L}\sin \omega t \, dt = -\frac{\varepsilon_o}{\omega L}\cos \omega t + \text{constant}$

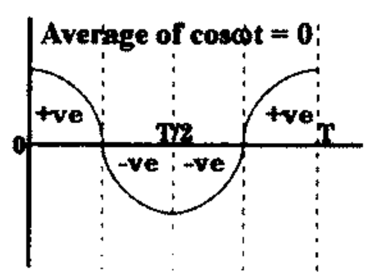
The average of current I over a time period T is zero. Now the average of cos ωt over a time period T is zero, hence the integration constant in the above equation must be zero i.e.,

$$0 = -\frac{\varepsilon_o}{\omega L}\cos \omega \times 0 + \text{constant} \qquad \Rightarrow \text{constant} = 0$$

$$\therefore I = -\frac{\varepsilon_o}{\omega L}\cos \omega t = I_o(-\cos \omega t)$$

Or $I = I_o \sin\left(\omega t - \frac{\pi}{2}\right)$

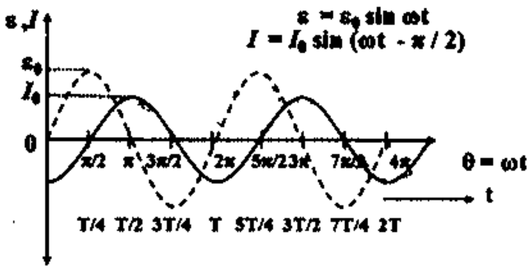
where $I_o = \frac{\varepsilon_o}{\omega L}$ is the peak value of a.c.

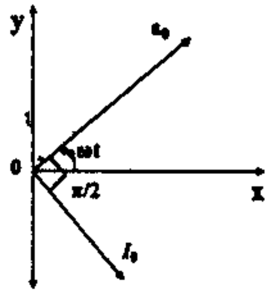


The current lags behind the EMF by $\frac{\pi}{2}$ or 90°

$$I_o = \frac{\varepsilon_o}{\omega L} = \frac{\varepsilon_o}{(2\pi f)L} = \frac{\varepsilon_o}{X_L}$$

$X_L = \omega L$ is called the inductive reactance





(a) For d.c., $f = 0$

$X_L = \omega L = (2\pi \times 0)L = 0$

$\therefore$ a pure inductance offers zero resistance to d.c.

(b) For a.c., $f = f$

$X_L = \omega L = (2\pi \times f)L \qquad \Rightarrow X_L \propto f$

$\therefore$ greater the frequency f, greater is the inductive reactance X_L and vice versa.

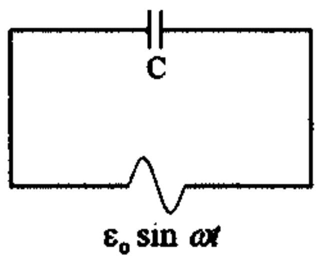
(c) Unit of X_L

$$X_L = \omega L = \frac{1}{s} \times H = \frac{1}{s} \times \frac{V}{A/s} = \frac{V}{A} = \Omega$$

Note: $\because I_o = \dfrac{\varepsilon_o}{\omega L} \qquad \Rightarrow \varepsilon_o > I_o$

a.c. circuit containing only a capacitor: As shown in the figure, a capacitor C is connected to a source of alternating EMF ε given by

$$\varepsilon = \varepsilon_o \sin \omega t$$



$$\because Q = C\varepsilon = C\varepsilon_o \sin \omega t$$

$$\therefore I = \frac{dQ}{dt} = \frac{d}{dt} C\varepsilon_o \sin \omega t$$

$$I = \omega C \varepsilon_o \cos \omega t$$

$$I = I_o \cos \omega t$$

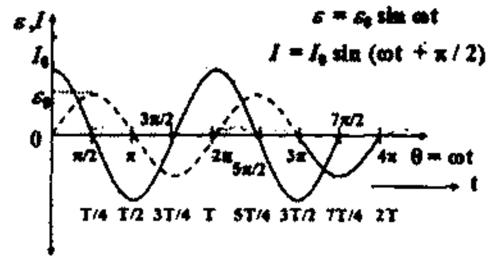
$$I = I_o \sin\left(\omega t + \frac{\pi}{2}\right)$$

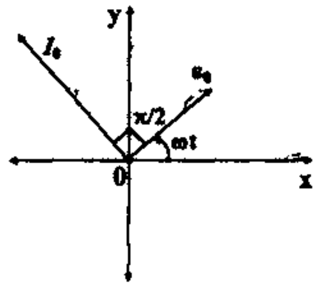
where $I_o = \omega C \varepsilon_o = \dfrac{\varepsilon_o}{\frac{1}{\omega C}}$ is the peak value of a.c.

The current leads the EMF by $\frac{\pi}{2}$ or 90°

$$I_o = \omega C \varepsilon_o = \frac{\varepsilon_o}{\frac{1}{\omega C}} = \frac{\varepsilon_o}{\frac{1}{(2\pi f)C}} = \frac{\varepsilon_o}{X_C}$$

$X_C = \frac{1}{\omega C}$ is called the capacitive reactance





(a) For d.c., $f = 0$

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi \times f)C} = \frac{1}{(2\pi \times 0)C} = \infty$$

$\therefore$ a pure capacitance offers infinite resistance to d.c. In other words, a capacitor blocks d.c.

(b) For a.c., $f = f$ i.e., $f \neq 0$

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi \times f)C} \qquad \Rightarrow X_C \propto \frac{1}{f}$$

$\therefore$ Greater the frequency f, smaller is the capacitive reactance X_C and vice versa.

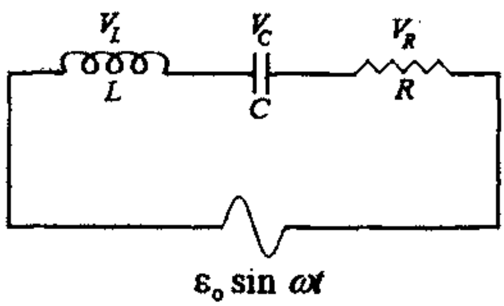
(c) Unit of X_C

$$X_C = \frac{1}{\omega C} = \frac{s}{F} = \frac{s}{C/V} = \frac{Vs}{As} = \frac{V}{A} = \Omega$$

Note: $\because I_o = \omega C \varepsilon_o \qquad \Rightarrow \varepsilon_o < I_o$

Series LCR circuit: As shown in the figure, an inductor L, a capacitor C and a resistor R are connected to a source of alternating EMF ε given by

$$\varepsilon = \varepsilon_o \sin \omega t$$



Let I be the current in the series circuit at any instant. The applied EMF appears as Voltage drops V_L, V_C and V_R across L, C and R respectively. Then

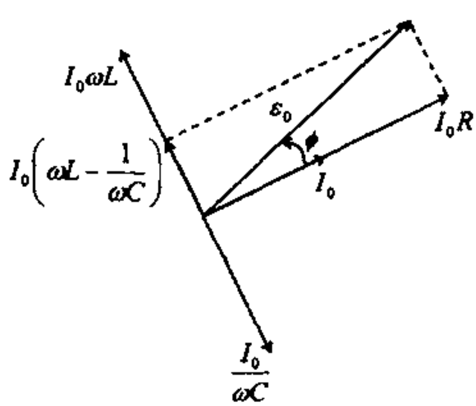
$V_L = IX_L = I\omega L$

$V_C = IX_C = \frac{I}{\omega C}$

$V_R = IR$

The length of the phasors for I, V_L, V_C and V_R are

$I_o, I_o\omega L, \frac{I_o}{\omega C}$ and $I_o R$.



As shown in the figure

(1) In L, current lags behind voltage by $\pi/2$

(2) In C, current leads the voltage by $\pi/2$

(3) In R, current and voltage are in phase.

The resultant of $I_o\omega L$ and $\frac{I_o}{\omega C}$

$$= I_o\omega L - \frac{I_o}{\omega C} = I_o\left(\omega L - \frac{1}{\omega C}\right)$$

The resultant phasor will be

$$\varepsilon_o = \sqrt{(I_oR)^2 + \left\{I_o\left(\omega L - \frac{1}{\omega C}\right)\right\}^2}$$

$$\text{Or } \varepsilon_o = I_o\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The impedance of the circuit is $Z = \frac{\varepsilon_o}{I_o} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$$\Rightarrow \varepsilon_o = Z\, I_o$$

$$\tan\phi = \frac{I_o\left(\omega L - \frac{1}{\omega C}\right)}{I_oR} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Series LCR circuit: Special cases

Case I: When $X_L > X_C$ i.e. $\omega L > 1/\omega C$,
$\tan\phi$ = positive or ϕ is positive
The current lags behind the EMF by phase angle ϕ and the LCR circuit is inductance - dominated circuit.

Case II: When $X_L < X_C$ i.e. $\omega L < 1/\omega C$,
$\tan\phi$ = negative or ϕ is negative
The current leads the EMF by phase angle ϕ and the LCR circuit is capacitance - dominated circuit.

Case III: When $X_L = X_C$ i.e. $\omega L = 1/\omega C$,
$\tan\phi = 0$ or ϕ is $0°$
The current and the EMF are in same phase. The impedance does not depend on the frequency of the applied EMF. LCR circuit behaves like a purely resistive circuit.

Resonance condition of a series LCR circuit: A series LCR circuit is said to be in the resonance condition when the current through it has its maximum value.

When $X_L = X_C$

$$\omega L = \frac{1}{\omega C}$$

$$\tan\phi = 0 \qquad \Rightarrow \phi = 0°$$

$$\because Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\therefore Z_{min} = R \qquad \text{and} \quad (I_o)_{max} = \frac{\varepsilon_o}{R}$$

The impedance (i.e., the resistance of the L-C-R circuit) offered by the circuit is minimum and the current is maximum. This condition is called resonant condition of LCR circuit and the frequency is called resonant frequency, denoted by f_r.

$$\omega_r L = \frac{1}{\omega_r C}$$

$$\text{Or } \omega_r = \frac{1}{\sqrt{LC}}$$

$$\text{Or } f_r = \frac{1}{2\pi\sqrt{LC}}$$

Characteristics of series resonant circuit:

(1) Resonance occurs in a series LCR circuit when $X_L = X_C$

(2) Resonance frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$

(3) The impedance is minimum and purely resistive.

(4) The current has a maximum value of $\frac{\varepsilon_o}{R}$ at resonant condition.

(5) The power dissipated in the circuit is maximum and is equal to $\frac{\varepsilon_{rms}^2}{R}$.

$$P = \frac{W}{t} = \frac{I_{rms}^2 Rt}{t} = I_{rms}^2 R = \frac{I_{rms}^2 R^2}{R} = \frac{\varepsilon_{rms}^2}{R}$$

Sharpness of resonance or Q-factor: Quality factor (Q–factor) is defined as the ratio of resonant frequency to band width.

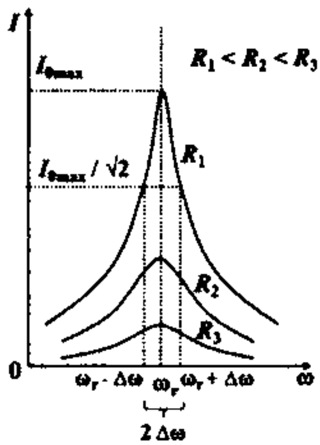
$$Q = \frac{\omega_r}{2\Delta\omega}$$

where $2\Delta\omega$ is the band width.

Or

Quality factor (Q–factor) is defined as the ratio of potential drop across either, the inductance or the capacitance to the potential drop across the resistance.

$$Q = \frac{V_L}{V_R} = \frac{\omega_r L}{R} \qquad \text{Or} \qquad Q = \frac{V_C}{V_R} = \frac{1}{\omega_r CR}$$



Note: The greater the Q-factor of resonance L-C-R circuit, the sharper is the resonance curve.

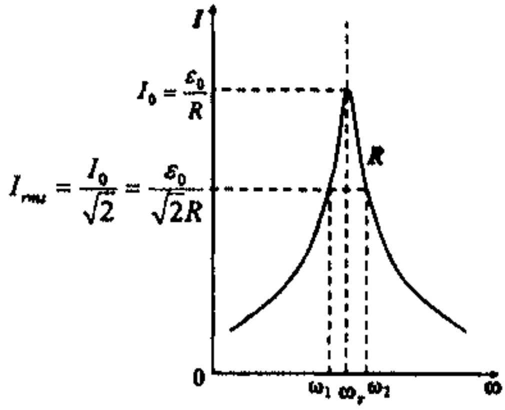
Q-factor can also be written as $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Example: Consider an L-C-R series circuit connected to 240V a.c. source. If Q-factor of the coil is 20, then voltage across L or C will be

$$V_L = V_C = Q \times V_R = 20 \times 240V = 4800V$$

Expression for Q-factor: Clearly at ω_r, the impedance (i.e., the resistance of the L-C-R circuit) is equal to R, while at ω_1 and ω_2 its value is $\sqrt{2}$ R.

$$\therefore Z=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}=\sqrt{2}R$$



Squaring both sides we have

$$R^2+\left(\omega L-\frac{1}{\omega C}\right)^2=2R^2$$

$$\left(\omega L-\frac{1}{\omega C}\right)^2=R^2$$

$$\omega L-\frac{1}{\omega C}=\pm R$$

We can write

$$\omega_1 L-\frac{1}{\omega_1 C}=-R \text{ ------------------ (1)}$$

$$\omega_2 L-\frac{1}{\omega_2 C}=+R \text{ ------------------ (2)}$$

Adding equation (1) and equation (2) we get

$$(\omega_1+\omega_2)L-\frac{1}{C}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)=0$$

$$(\omega_1+\omega_2)L-\frac{1}{C}\left(\frac{\omega_1+\omega_2}{\omega_1\omega_2}\right)=0$$

$$\frac{1}{C}\left(\frac{\omega_1+\omega_2}{\omega_1\omega_2}\right)=(\omega_1+\omega_2)L$$

$$\frac{1}{\omega_1\omega_2}=LC$$

$$\omega_1\omega_2=\frac{1}{LC}$$

Or $L=\frac{1}{C\omega_1\omega_2}$ ------------------ (3)

Subtracting equation (1) from equation (2), we get

$$(\omega_2-\omega_1)L+\frac{1}{C}\left(\frac{1}{\omega_2}-\frac{1}{\omega_1}\right)=2R$$

$$(\omega_2-\omega_1)L+\frac{1}{C}\left(\frac{\omega_2-\omega_1}{\omega_1\omega_2}\right)=2R$$

$$(\omega_2-\omega_1)\left[L+\frac{1}{C}\frac{1}{\omega_1\omega_2}\right]=2R$$

$(\omega_2-\omega_1)[L+L]=2R$ [By using equation (3)]

$$(\omega_2-\omega_1)2L=2R$$

$$\omega_2-\omega_1=\frac{R}{L}$$

$$\frac{1}{\omega_2-\omega_1}=\frac{L}{R} \text{ ---------------- (4)}$$

$\omega_2-\omega_1=2\Delta\omega$ is the band width

$$\therefore Q=\frac{\omega_r}{2\Delta\omega}=\frac{\omega_r}{\omega_2-\omega_1}=\frac{\omega_r L}{R} \text{ ---------------- (5)} \quad \text{[By using equation (4)]}$$

$$\because \omega_r L=\frac{1}{\omega_r C} \quad \text{and} \quad \omega_r=\frac{1}{\sqrt{LC}}$$

Equation (5) can be written as

$$Q=\frac{1}{\omega_r CR}=\frac{1}{\omega_r}\cdot\frac{1}{CR}=\sqrt{LC}\cdot\frac{1}{CR}=\frac{1}{R}\sqrt{\frac{L}{C}}$$

Again equation (5) can also be written as

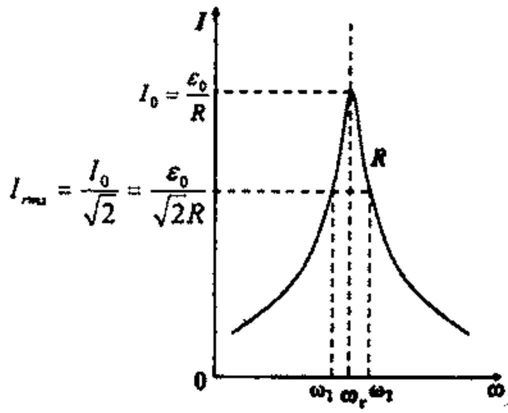
$$Q=\frac{\omega_r L I_{rms}}{R I_{rms}}=\frac{V_L}{V_R}=\frac{\text{Voltage drop across L}}{\text{Applied voltage}}$$

$$= \text{Voltage magnification}$$

Or

Expression for Q-factor: Clearly at ω_r, the impedance (i.e., the resistance of the L-C-R circuit) is equal to R, while at $\omega_1=(\omega_r-\Delta\omega)$ and $\omega_2=(\omega_r+\Delta\omega)$ its value is $\sqrt{2}$ R.

$$\sqrt{R^2+\left(\omega_1 L-\frac{1}{\omega_1 C}\right)^2}=\sqrt{2}R$$



Squaring both sides we have

$$R^2+\left(\omega_1 L-\frac{1}{\omega_1 C}\right)^2=2R^2$$

$$\left(\omega_1 L-\frac{1}{\omega_1 C}\right)^2=R^2$$

$$\omega_1 L-\frac{1}{\omega_1 C}=-R$$

$$(\omega_r-\Delta\omega)L-\frac{1}{(\omega_r-\Delta\omega)C}=-R$$

$$\omega_r L\left(1-\frac{\Delta\omega}{\omega_r}\right)-\frac{1}{\omega_r C\left(1-\frac{\Delta\omega}{\omega_r}\right)}=-R$$

$$\omega_r L\left(1-\frac{\Delta\omega}{\omega_r}\right)-\frac{\omega_r L}{\left(1-\frac{\Delta\omega}{\omega_r}\right)}=-R \qquad \left[\because \omega_r L=\frac{1}{\omega_r C}\right]$$

$$\omega_r L\left(1-\frac{\Delta\omega}{\omega_r}\right)-\omega_r L\left(1-\frac{\Delta\omega}{\omega_r}\right)^{-1}=-R$$

$$\omega_r L\left(1-\frac{\Delta\omega}{\omega_r}\right)-\omega_r L\left(1+\frac{\Delta\omega}{\omega_r}\right)=-R$$

$$\omega_r L-\omega_r L\frac{\Delta\omega}{\omega_r}-\omega_r L-\omega_r L\frac{\Delta\omega}{\omega_r}=-R$$

$$-2\omega_r L\frac{\Delta\omega}{\omega_r}=-R$$

$$\omega_r L\times\frac{2\Delta\omega}{\omega_r}=R$$

$$\Delta\omega=\frac{R}{2L}$$

Therefore, sharpness of resonance or Q-factor is

$$Q=\frac{\omega_r}{2\Delta\omega}=\frac{\omega_r}{2}\cdot\frac{2L}{R}=\frac{\omega_r L}{R} \text{ ------------------ (1)}$$

$$\because \omega_r L=\frac{1}{\omega_r C} \quad \text{and} \quad \omega_r=\frac{1}{\sqrt{LC}}$$

Equation (1) can be written as

$$Q=\frac{1}{\omega_r CR}=\frac{1}{\omega_r}\cdot\frac{1}{CR}=\sqrt{LC}\cdot\frac{1}{CR}=\frac{1}{R}\sqrt{\frac{L}{C}}$$

Power in an A.C circuit: The rate at which electric energy is consumed in an electric circuit is called its power.

In d.c. circuit, Power = Voltage × current

In a.c. circuit, Power = Instantaneous voltage × Instantaneous current

Suppose in an a.c. circuit, the voltage and current at any instant are given by

$$\varepsilon=\varepsilon_o\sin\omega t$$

$$I=I_o\sin(\omega t+\phi)$$

where ϕ is the phase angle by which the current I leads the voltage ε the instantaneous power is given by

$$P=\varepsilon I=\varepsilon_o I_o\sin\omega t\cdot\sin(\omega t+\phi)$$

$$P=\varepsilon_o I_o\sin\omega t[\sin\omega t\cos\phi+\cos\omega t\sin\phi]$$

$$P=\varepsilon_o I_o[\sin^2\omega t\cos\phi+\sin\omega t\cos\omega t\sin\phi]$$

$$P=\frac{\varepsilon_o I_o}{2}[(1-\cos2\omega t)\cos\phi+\sin2\omega t\sin\phi]$$

$$P=\frac{\varepsilon_o I_o}{2}[\cos\phi-(\cos2\omega t\cos\phi-\sin2\omega t\sin\phi)]$$

$$P=\frac{\varepsilon_o I_o}{2}[\cos\phi-\cos(2\omega t+\phi)]$$

If the instantaneous power is assumed to be constant for an infinitesimally small time dt, then the work done is

$$dW=Pdt$$

The total work done over a complete cycle is

$$W = \int_0^T P dt$$

Hence average power dissipated in the circuit over a complete cycle is

$$P_{av} = \frac{W}{T} = \frac{1}{T}\int_0^T P dt$$

$$P_{av} = \frac{1}{T}\int_0^T \frac{\varepsilon_o I_o}{2}[\cos\phi - \cos(2\omega t + \phi)]dt$$

$$P_{av} = \frac{\varepsilon_o I_o}{2T}\int_0^T [\cos\phi - \cos(2\omega t + \phi)]dt$$

$$P_{av} = \frac{\varepsilon_o I_o}{2T}\left[\int_0^T \cos\phi \, dt - \int_0^T \cos(2\omega t + \phi)\, dt\right]$$

$$P_{av} = \frac{\varepsilon_o I_o}{2T}[\cos\phi \left| t \right|_0^T - 0]$$

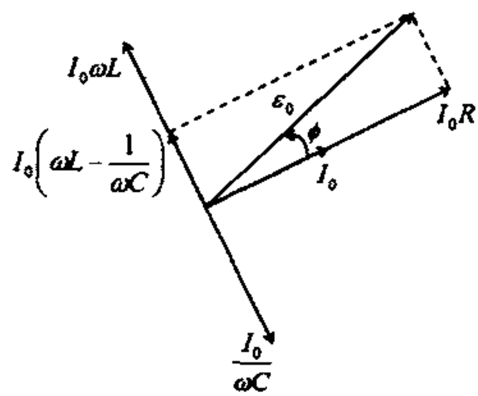
$$P_{av} = \frac{\varepsilon_o I_o}{2T}\cos\phi \, T$$

$$P_{av} = \frac{\varepsilon_o I_o}{2}\cos\phi$$

$$P_{av} = \frac{\varepsilon_o I_o}{\sqrt{2}\sqrt{2}}\cos\phi = \frac{\varepsilon_o}{\sqrt{2}} \cdot \frac{I_o}{\sqrt{2}}\cos\phi$$

$$P_{av} = \varepsilon_{rms} I_{rms} \cos\phi$$

$$P_{av} = \varepsilon_{rms} I_{rms} \frac{R}{Z} = \varepsilon_{rms} I_{rms} \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



$\cos\phi$, $\frac{R}{Z}$ and $\frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$ are called power factors

$\varepsilon_{rms} I_{rms}$ is called the apparent power

Note: $\cos\phi = \frac{I_o R}{\varepsilon_o} = \frac{I_o R}{Z I_o} = \frac{R}{Z}$

Power in a.c. circuit with R:

In R current and EMF are in phase, $\phi = 0^0$

$$P_{av} = \varepsilon_{rms} I_{rms} \cos\phi = \varepsilon_{rms} I_{rms} \cos 0^0 = \varepsilon_{rms} I_{rms}$$

Power in a.c. circuit with *L*:

In *L* the current lags behind EMF by $\frac{\pi}{2}$, $\quad \phi = -\frac{\pi}{2}$

$$P_{av} = \varepsilon_{rms} I_{rms} \cos\phi = \varepsilon_{rms} I_{rms} \cos\left(-\frac{\pi}{2}\right) = 0$$

Power in a.c. circuit with *C*:

In C the current leads EMF by $\frac{\pi}{2}$, $\phi = \frac{\pi}{2}$

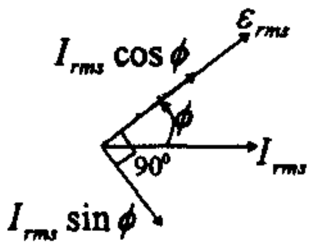
$$P_{av} = \varepsilon_{rms} I_{rms} \cos\phi = \varepsilon_{rms} I_{rms} \cos\left(\frac{\pi}{2}\right) = 0$$

Power (Energy) is not dissipated in Inductor and Capacitor and hence they find a lot of practical applications and in devices using alternating current.

Wattless Current or Idle Current: The current in a.c. circuit is said to be wattless if the average power consumed in the circuit is zero. The average power of an a.c. circuit is given by

$$P_{av} = \varepsilon_{rms} I_{rms} \cos\phi$$

Here P_{av} is associated with $I_{rms}\cos\phi$ not with $I_{rms}\sin\phi$. Hence $I_{rms}\sin\phi$ is called wattless current.



Or

The component $I_{rms}\cos\phi$ generates power with ε_{rms}. However, the component $I_{rms}\sin\phi$ does not contribute to power along ε_{rms} and hence power generated is zero. This component of current is called wattless or idle current.

LC oscillations: When a charged capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called LC-oscillations. Let U_E and U_B are the electric and magnetic energy at the capacitor and inductor respectively, then

(1) At $t = 0$, U_E = Max. & $U_B = 0$

(2) At $t = T/8$, $U_E = U_B$

(3) At $t = 2T/8$, $U_E = 0$ & U_B = Max.

(4) At $t = 3T/8$, $U_E = U_B$

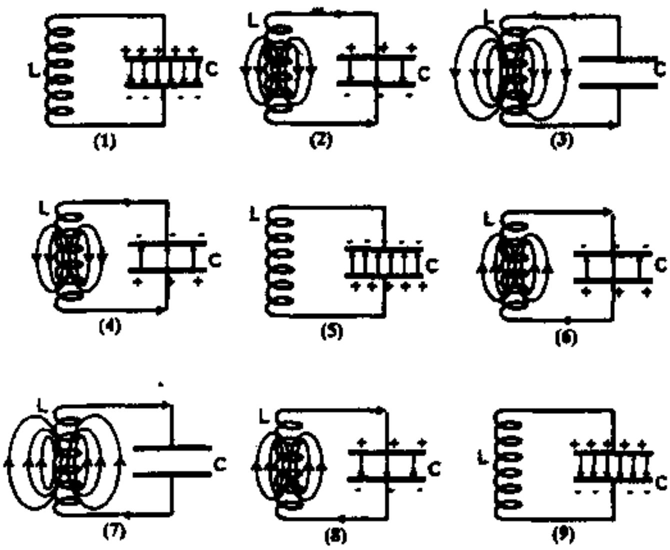
(5) At $t = 4T/8$, U_E = Max. & $U_B = 0$

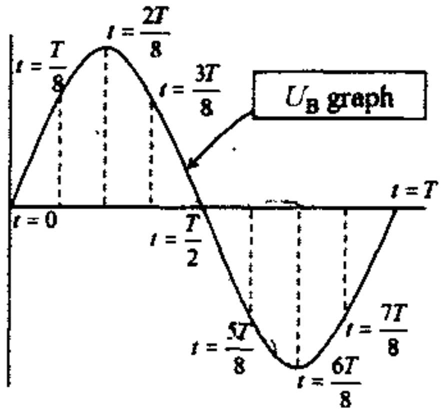
(6) At $t = 5T/8$, $U_E = U_B$

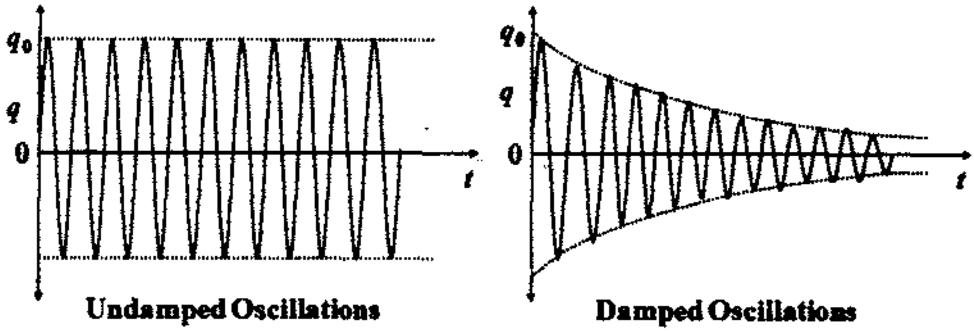
(7) At $t = 6T/8$, $U_E = 0$ & U_B = Max.

(8) At $t = 7T/8$, $U_E = U_B$

(9) At $t = T$, U_E = Max. & $U_B = 0$







If LC circuit does not have any resistance, the amplitude of oscillations will remain constant. Such oscillations are called undamped oscillations. However if the amplitude of oscillating current decreases gradually, and eventually becomes zero, such oscillations are called damped oscillations.

Mathematical treatment of LC-oscillations: If q be the charge on the capacitor at any time t and $\frac{dI}{dt}$ the rate of change of current, then

$$-L\frac{dI}{dt}+\frac{q}{C}=0 \qquad \text{[Since there is no resistor]}$$

$$-L\left[\frac{d}{dt}\left(-\frac{dq}{dt}\right)\right]+\frac{q}{C}=0$$

Here $I=-\frac{dq}{dt}$ in the present case (as q decreases, I increases).

$$L\left(\frac{d^2q}{dt^2}\right)+\frac{q}{C}=0$$

$$\frac{d^2q}{dt^2}+\frac{q}{LC}=0$$

Putting $\frac{1}{LC}=\omega_0^2$

$$\frac{d^2q}{dt^2}+\omega_o^2q=0$$

The above equation represents Simple Harmonic Electrical Oscillation with ω_o as angular frequency i.e., the charge, oscillates with a natural frequency ω_o

The solution of this equation is $q = A\cos\omega_o t + B\sin\omega_o t$ where A and B are constants

(i) At $t = 0$, $q = q_0$

$$q = A\cos\omega_o t + B\sin\omega_o t$$

$$\therefore q_o = A\cos\omega_o(0) + B\sin\omega_o(0) \qquad \Rightarrow q_o = A$$

(ii) $$I = \frac{dq}{dt} = \frac{d}{dt}(A\cos\omega_o t + B\sin\omega_o t)$$

$$I = -A\omega\sin\omega_o t + B\omega\cos\omega_o t$$

At $t = 0$, $I = 0$

$$\therefore 0 = -q_o\omega\sin\omega_o(0) + B\omega\cos\omega_o(0) \qquad \Rightarrow B = 0$$

Putting the values of A and B in the general solution we get

$$q = q_o\cos\omega_o t + (0)\sin\omega_o t$$

Or $q = q_o\cos\omega_o t$

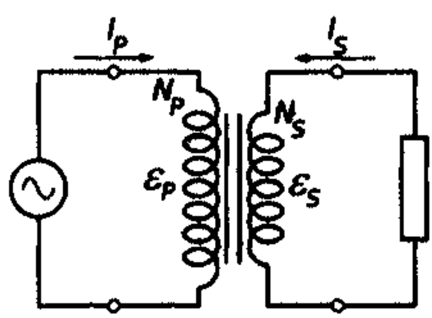
And $$I = -\frac{dq}{dt} = -\frac{d}{dt}(q_o\cos\omega_o t) = \omega_o q_o\sin\omega_o t$$

(a) The maximum value of varying charge on the capacitor is q_0

(b) The maximum value of varying current in the inductor is $I_o = \omega_o q_o = q_o\frac{1}{\sqrt{LC}}$

(c) T frequency of oscillating charge or current is given by $f = \frac{1}{2\pi\sqrt{LC}}$

Transformer: Transformer is a device which converts lower alternating voltage at higher current into higher alternating voltage at lower current



Principle: Transformer is based on the principle of mutual Induction. It is the phenomenon of inducing EMF in the secondary coil due to change in current in the primary coil and hence the change in magnetic flux in the secondary coil.

Theory: Let N_P and N_S be the number of turns in the primary and secondary coil of the transformer respectively and ϕ be the magnetic flux in the iron core linked with each coil.

The induced EMF in the primary is

$$\varepsilon_p = -N_p\frac{d\phi}{dt}$$

The induced EMF in the secondary is

$$\varepsilon_s = -N_s\frac{d\phi}{dt}$$

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p} = K$$

(where K is called Transformation Ratio or Turns Ratio)

For an ideal transformer,
Output Power = Input Power

$\varepsilon_s I_s = \varepsilon_p I_p$

$\frac{\varepsilon_s}{\varepsilon_p} = \frac{I_p}{I_s}$

$\frac{\varepsilon_s}{\varepsilon_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$

Efficiency (η): Efficiency of a transformer is the ratio of output power to input power i.e.,

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{\varepsilon_s I_s}{\varepsilon_p I_p}$$

For an ideal transformer η is 100%

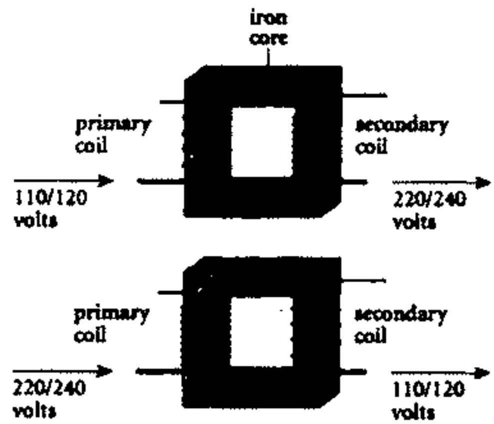
Step up and step down transformer:

$N_s > N_p$ i.e., $K > 1$

$\varepsilon_s > \varepsilon_p$ and $I_s < I_p$

$N_s < N_p$ i.e., $K < 1$

$\varepsilon_s < \varepsilon_p$ and $I_s > I_p$



Energy losses in transformer:

(1) Copper Loss: Heat is produced due to the resistance of the copper windings of Primary and Secondary coils when current flows through them. This can be avoided by using thick wires for winding.

(2) Flux Loss: In actual transformer coupling between Primary and Secondary coil is not perfect. So, a certain amount of magnetic flux is wasted. Linking can be maximised by winding the coils over one another.

(3) Iron Losses:

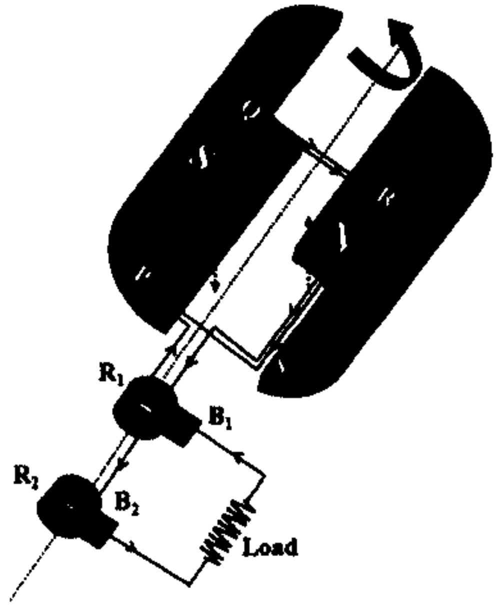
(a) Eddy Currents Losses: When a changing magnetic flux is linked with the iron core, eddy currents are set up which in turn produce heat and energy is wasted. Eddy currents are reduced by using laminated core instead of a solid iron block because in laminated core the eddy currents are confined within the lamination and they do not get added up to produce larger current. In other words their paths are broken instead of continuous ones.

(b) Hysteresis Loss: When alternating current is passed, the iron core is magnetised and demagnetised repeatedly over the cycles and some energy is being lost in the process. This can be minimised by using suitable material with thin hysteresis loop.

(4) Losses due to vibration of core: Some electrical energy is lost in the form of mechanical energy due to vibration of the core and humming noise due to magnetostriction effect.

Note: Magnetostriction is a property of ferromagnetic materials that causes them to change their shape or dimensions during the process of magnetization.

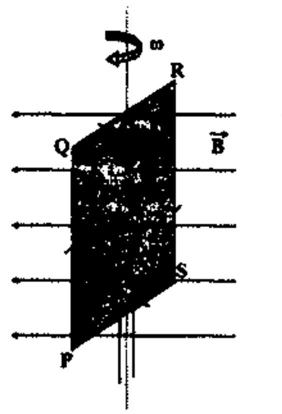
Alternating current Generator or Alternator: Alternating current Generator or a.c. Dynamo or Alternator is a device which converts mechanical energy into alternating current (electrical energy).



Principle: a.c. Generator is based on the principle of Electromagnetic Induction.

Construction:
(1) Field Magnet with poles N and S
(2) Armature (Coil) PQRS
(3) Slip Rings (R_1 and R_2)
(4) Brushes (B_1 and B_2)
(5) Load

Working:
Let the armature be rotated in such a way that the arm PQ goes down and RS comes up from the plane of the diagram. Induced EMF and hence current is set up in the coil. By Fleming's Right Hand Rule, the direction of the current is $PQRSR_2B_2B_1R_1P$.
After half the rotation of the coil, the arm PQ comes up and RS goes down into the plane of the diagram. By Fleming's Right Hand Rule, the direction of the current is $PR_1B_1B_2R_2SRQP$.
If one way of current is taken positive, then the reverse current is taken negative. Therefore the current is said to be alternating and the corresponding wave is sinusoidal.



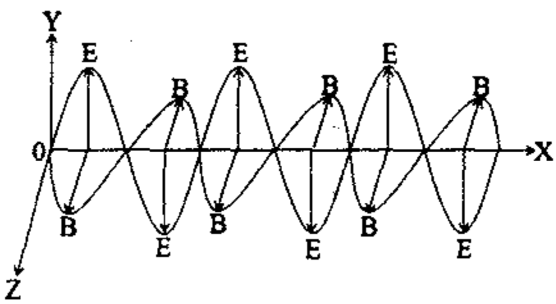
Theory: Let N be the number of turns in the coil of area A, and B is a strong uniform magnetic field. The flux ϕ linked with the coil at any instant is given by

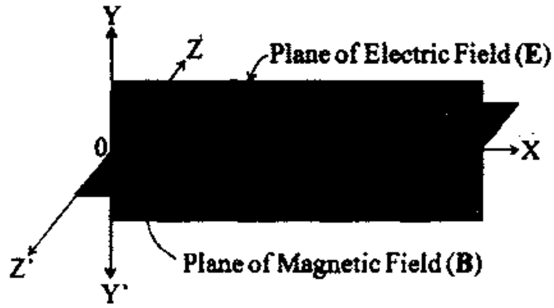
$$\phi = N\,B\,A\cos\theta$$

At time t, with angular velocity ω,
$\theta = \omega t$ (at $t = 0$, loop is assumed to be perpendicular to the magnetic field and $\theta = 0°$)
$\therefore \phi = N\,B\,A\cos\omega t$
Differentiating with respect to t,
$d\phi/dt = -NBA\,\omega\sin\omega t$
$\varepsilon = -d\phi/dt$
$\varepsilon = NBA\,\omega\sin\omega t$
$\varepsilon = \varepsilon_0\sin\omega t$ (where $\varepsilon_0 = NBA\omega$)

ELECTROMAGNETIC WAVES

Electromagnetic waves: Electromagnetic waves are produced when an electric charge vibrates or accelerates. In other words, electromagnetic waves are produced in constantly changing fields.
Electromagnetic waves consist of sinusoidal variation of electric and magnetic fields at right angles to each other and to the direction of propagation of the wave. Both these fields vary with time and space and have the same frequency.





Characteristics of electromagnetic waves:

(1) Electromagnetic waves are produced by accelerated charges.
(2) They do not require any material medium for propagation.
(3) In an electromagnetic wave, the electric ($\vec{E}$) and magnetic ($\vec{B}$) field vectors are at right angles to each other and to the direction of propagation. Hence electromagnetic waves are transverse in nature.
(4) Variation of maxima and minima in both $\vec{E}$ and $\vec{B}$ occur simultaneously.
(5) They travel in vacuum or free space with a velocity 3×10^8 m s^{-1} given by the relation

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$ (μ_0 - permeability of free space and ε_0 - permittivity of free space)

(6) The energy in an electromagnetic wave is equally divided between electric and magnetic field vectors.
(7) The electromagnetic waves being charge less, are not deflected by electric and magnetic fields.
(8) The electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ are related by $c = \frac{E_0}{B_0}$ where E_0 and B_0 are the amplitudes of the respective fields and c is speed of light.
(9) The velocity of electromagnetic waves in a material medium is $\frac{1}{\sqrt{\mu\varepsilon}}$ where μ and ε are absolute permeability and absolute permittivity of the material medium.
(10) Electromagnetic waves obey the principle of superposition.
(11) Electromagnetic waves can transfer energy as well as momentum to objects placed on their paths.
(12) For discussion of optical effects of electromagnetic wave, more significance is given to Electric Field (E). Therefore, electric field is called 'light vector'.

Electromagnetic spectrum: The orderly classification of electromagnetic waves according to their wavelength or frequency is called the electromagnetic spectrum.

Electromagnetic Spectrum:

Sl. no	EM Wave	Range of λ	Range of ν	Source	Use
1	Radio wave	A few km to 0.3 m	A few Hz to 10^9 Hz	Oscillating electronic circuits	Radio and TV broadcasting
2	Microwave	0.3 m to 10^{-3} m	10^9 Hz to 3×10^{11} Hz	Oscillating electronic circuits	Radar, analysis of fine details of atomic and molecular structures & Microwave oven
3	Infra Red wave	10^{-3} m to 7.8×10^{-7} m	3×10^{11} Hz to 4×10^{14} Hz	Molecules and hot bodies	Industry, medicine, astronomy, night vision device, green house, revealing secret writings on ancient walls, etc.
4	Light or Visible Spectrum	7.8×10^{-7} m to 3.8×10^{-7} m	4×10^{14} Hz to 8×10^{14} Hz	Atoms and molecules when electrons are excited	Optics and Optical Instruments, Vision, photography, etc.
5	Ultra Violet Rays	3.8×10^{-7} m to 6×10^{-10} m	8×10^{14} Hz to 3×10^{17} Hz	Carbon-arc lamp, electric spark, discharge tube, hot bodies and sun.	Medical application, sterilization, killing bacteria and germs in food stuff, detection of invisible writing, forged documents, finger print, etc.
6	X - Rays	10^{-9} m to 6×10^{-12} m	3×10^{17} Hz to 5×10^{19} Hz	Inner or more tightly bound electrons in atoms.	X-ray photography, treatment of cancer, skin disease & tumor, locating cracks and flaws in finished metallic objects, detection of smuggled goods in bags of a person, study of crystal structure, etc.
7	γ-Rays	They overlap the upper limit of the X-Ray. 10^{-10} m to 10^{-14} m	3×10^{18} Hz to 3×10^{22} Hz	Radioactive substances	Information about structure of nuclei, astronomical research, etc.

Energy distribution between electric and magnetic field of electromagnetic waves: An electromagnetic wave can transfer energy from one point to another. The energy stored in a unit volume is called energy density. Let the electric and magnetic field vectors are

$$E = E_0 \sin(kx - \omega t) \quad \text{and}$$
$$B = B_0 \sin(kx - \omega t)$$

In vacuum, the electric energy density and the magnetic energy density are given by

$$u_E = \frac{1}{2}\varepsilon_0 E_{rms}^2 \qquad \text{and} \qquad u_B = \frac{1}{2}\cdot\frac{B_{rms}^2}{\mu_0}$$

$$\because u_E = \frac{1}{2}\varepsilon_0 E_{rms}^2 = \frac{1}{2}\varepsilon_0\left(\frac{E_0}{\sqrt{2}}\right)^2$$

Or $u_E = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}\varepsilon_0 (cB_0)^2 \qquad \left[\because \frac{E_0}{B_0} = c\right]$

Or $u_E = \frac{1}{4}\varepsilon_0 B_0^2 c^2 = \frac{1}{4}\varepsilon_0 B_0^2\left(\frac{1}{\mu_0\varepsilon_0}\right) \qquad \left[\because c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$

Or $u_E = \frac{1}{4}\frac{\varepsilon_0 B_0^2}{\mu_0\varepsilon_0} = \frac{1}{4}\frac{B_0^2}{\mu_0}$

Or $u_E = \frac{1}{2\mu_0}\frac{B_0^2}{2} = \frac{1}{2\mu_0}\left(\frac{B_0}{\sqrt{2}}\right)^2$

Or $u_E = \frac{1}{2}\cdot\frac{B_{rms}^2}{\mu_0} = u_B$

Therefore the energy is equally distributed in electric and magnetic field of an electromagnetic wave.

The total energy density is

$u = u_E + u_B = u_E + u_E = 2u_E \qquad [\because u_B = u_E]$

Or $u = u_E + u_B = u_B + u_B = 2u_B \qquad [\because u_E = u_B]$

Hence $u = 2u_E = 2\cdot\frac{1}{2}\varepsilon_0 E_{rms}^2 = \varepsilon_0 E_{rms}^2$

Or $u = 2u_E = 2\cdot\frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2$

Also $u = 2u_B = 2\cdot\frac{1}{2}\cdot\frac{B_{rms}^2}{\mu_0} = \frac{B_{rms}^2}{\mu_0}$

Or $u = 2u_B = 2\cdot\frac{1}{2\mu_0}\frac{B_0^2}{2} = \frac{1}{2}\frac{B_0^2}{\mu_0}$

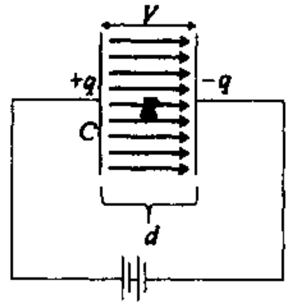
Momentum and Radiation pressure of electromagnetic wave: The waves that can transport energy can also transport linear momentum. Thus the electromagnetic waves exert pressure on the objects on its path and this pressure is called radiation pressure.

Displacement current: Displacement current is that current which comes into existence, in addition to the conduction current, whenever the electric field and hence the electric flux changes with time.

Mathematical analysis of Displacement current:

Consider a capacitor C being charged from a battery as shown in the figure. The charge q on the capacitor at any time is $q = CV$ where V is the potential difference between the plates at that instant.

Now $V = Ed$ where d is the plate separation and E is the electric field between the plates.



Also $C = \frac{\varepsilon_0 A}{d}$

where A is the area of the plate

$$\therefore q = CV = \left(\frac{\varepsilon_0 A}{d}\right)(Ed)$$

Or $q = \varepsilon_0 AE$

Or $\frac{dq}{dt} = \varepsilon_0 A \frac{dE}{dt}$

Or $I_D = \varepsilon_0 A \frac{dE}{dt}$ where I_D is the displacement current.

Again $\phi_E = EA$ Or $E = \frac{\phi_E}{A}$ where ϕ_E is the electric flux.

$$\therefore I_D = \varepsilon_0 A \frac{d\phi_E}{A\,dt}$$

Or $I_D = \varepsilon_0 \frac{d\phi_E}{dt}$

Equality of The conduction current (I) and the displacement current (I_D):

$\because I = \frac{dq}{dt}$,

$\phi_E = \frac{q}{\varepsilon_0} \quad \Rightarrow q = \varepsilon_0 \phi_E$

$\frac{dq}{dt} = \varepsilon_0 \frac{d\phi_E}{dt}$

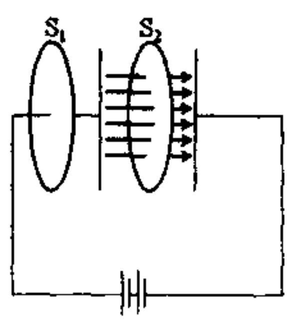
$\therefore I = I_D$

Generalised form of Ampere's Law:

$\because$ ampere's Law is $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ where I is the conduction current.

The modified form of Ampere's Law is $\oint \vec{B}\cdot d\vec{l} = \mu_0 (I + I_D)$

where I_D is the displacement current.



Or $\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$

For surface S_1, $I_D = 0$ $\quad \therefore \oint \vec{B}\cdot d\vec{l} = \mu_0 I$

For surface S_2, $I = 0$ $\quad \therefore \oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$

GEOMETRICAL OPTICS

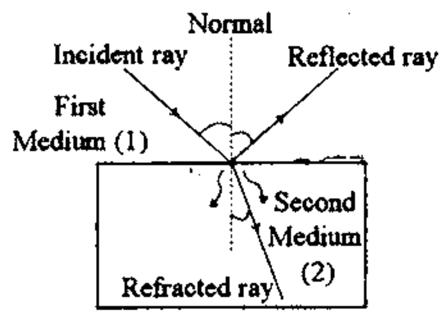
Optics: Optics is the branch of Physics which deals with the study of nature, production and propagation of light. Optics can be divided into two main branches.

(1) Ray optics: It concerns itself with the particle nature of light and is based on the
(i) Rectilinear propagation of light
(ii) Law of reflection and refraction of light.

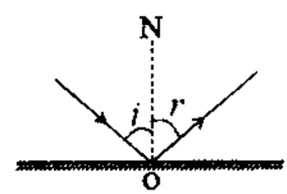
(2) Wave optics: It concerns itself with the wave nature of light and is based on the phenomena like
(i) Interference
(ii) Diffraction and
(iii) Polarisation of light

Behaviour of light at the interface of two media: When light travelling in one medium falls on the surface of the second medium, the following effects may occur:

(1) A part of the incident light is turned back into the first medium. This is called Reflection of light.
(2) A part of the incident light is transmitted into the second medium along a changed direction. This is called Refraction of light.
(3) A part of the incident light is absorbed by the second medium. This is called Absorption of light



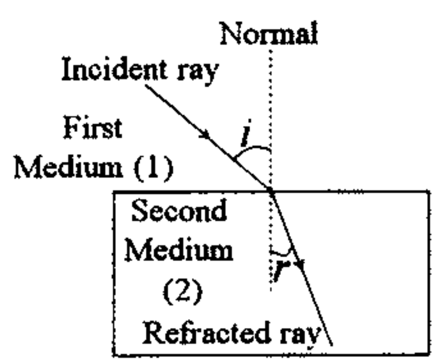
(1) Laws of Reflection of light: Reflection of light takes place according to the following two laws
First Law: The angle of incidence is equal to the angle of reflection. i.e., $\angle i = \angle r$
Second Law: The incident ray the reflected ray and the normal at the point of incidence all lie in the same plane.



(2) Laws of Refraction of light: The phenomenon of refraction of light obeys the following two laws
First Law: The incident ray, the refracted ray and the normal to the interface at the point of incidence all lie in the same plane.
Second Law: The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for a given pair of media.

Mathematically, ${}^1\mu_2 = \frac{\sin i}{\sin r}$

This law is known as Snell's law of refraction. ${}^1\mu_2$ is called the refractive index of the second medium with respect to the first medium.

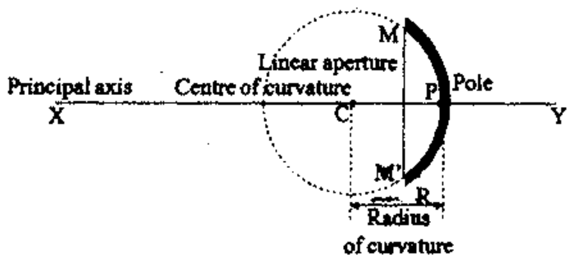


Spherical mirrors: A spherical mirror is a reflecting surface which forms part of a hollow sphere. Spherical mirrors are of two types:

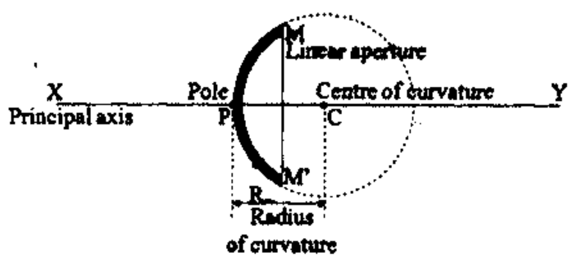
Concave mirror **Convex mirror**

Concave mirror: A spherical mirror in which the outer bulged surface is silvered polished and the reflection of light takes place from the inner hollow surface is called a concave mirror.
Convex mirror: A spherical mirror in which the inner hollow surface is silvered polished and the reflection of light takes place from the outer bulged surface is called a convex mirror.

Definitions in connection with spherical mirrors:

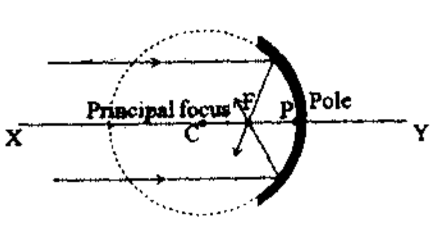


Concave mirror

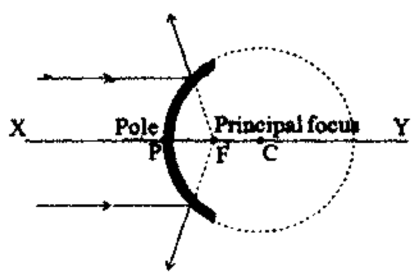


Convex mirror

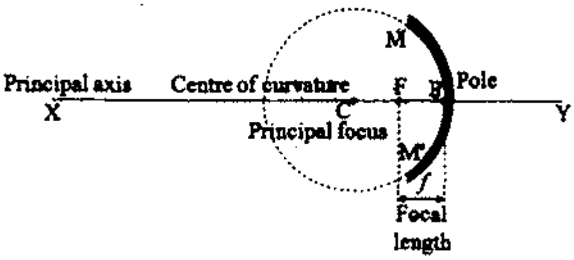
Pole: It is the middle point 'P' of the spherical mirror.
Centre of curvature: It is the centre 'C 'of the sphere of which the mirror forms a part.
Radius of curvature: It is the radius '$R = CP$ 'of the sphere of which the mirror forms a part.
Principal axis: The line 'XY 'passing through the pole and the centre of curvature of the mirror is called its principal axis
Linear aperture: It is the distance between the extreme points M and M' (diameter of the circular boundary) on the periphery of the spherical mirror.
Angular aperture: It is the angle MCM' (solid angle) subtended by the boundary of the spherical mirror at its centre of curvature 'C '
Principal focus: A narrow beam of light parallel to the principal axis either actually converges to or appears to diverge from a point 'F 'on the principal axis after reflection from the spherical mirror. This point is called the principal focus of the mirror. A concave mirror has a real focus while a convex mirror has a virtual focus.



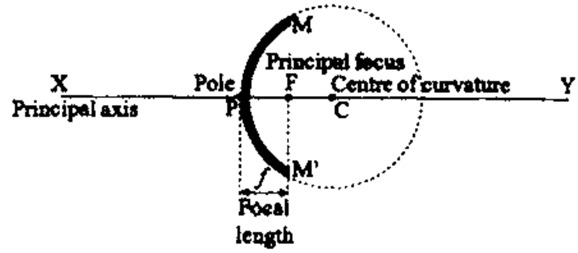
Concave mirror



Convex mirror



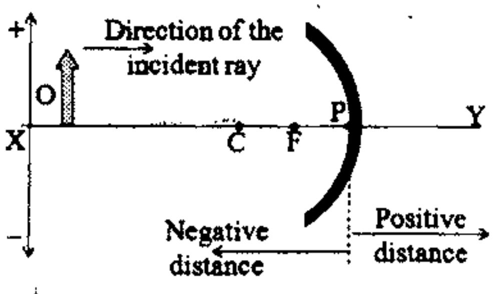
Concave mirror



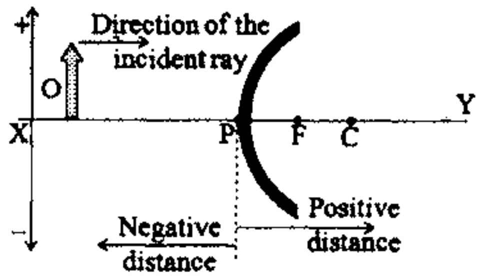
Convex mirror

Focal length: It is the distance $f = PF$ between the focus and the pole of the mirror.
Focal plane: The vertical plane passing through the principal focus and perpendicular to the principal axis is called focal plane.

New Cartesian Sign convention for Spherical mirrors:
(1) All ray diagrams are drawn with the incident light travelling from left to right.
(2) All distances are measured from the pole of the mirror.
(3) All distances measured in the direction of incident light are taken to be positive.
(4) All distances measured in the opposite direction of incident light are taken to be negative.
(5) Heights measured upward and perpendicular to the principal axis are taken positive.
(6) Heights measured downward and perpendicular to the principal axis are taken negative



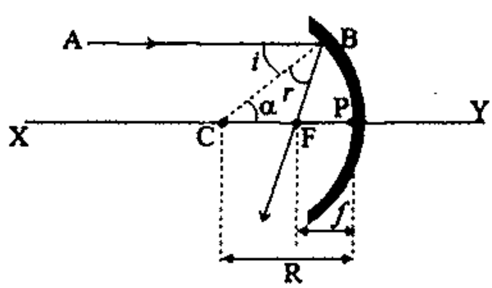
Concave mirror



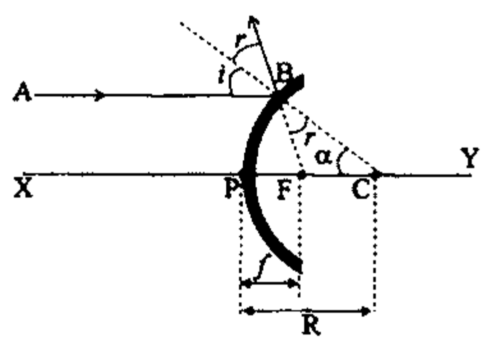
Convex mirror

Note: According to the sign convention, the focal length (f) and radius of curvature (R) are negative for concave mirror and positive for convex mirror.

Relation between f and R:



Concave mirror



Convex mirror

According to the law of reflection $\angle i = \angle r$.
As AB is parallel to PC, $\angle i = \angle\alpha$

$$\therefore \angle r = \angle\alpha$$

Hence CF = FB
For a mirror of small aperture, FP ≈ FB

$$\text{Or } CF = FP$$

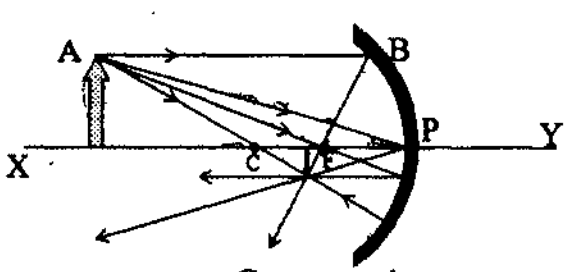
$$\text{Hence } CP = CF + FP = FP + FP$$

$$\text{Or } R = f + f = 2f$$

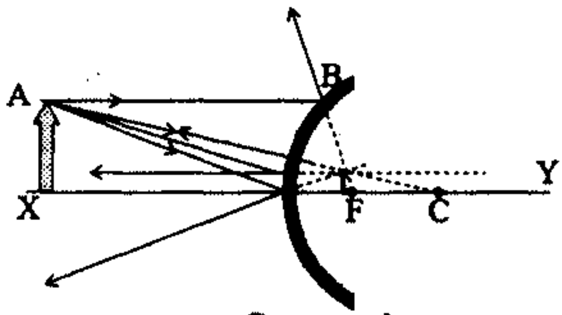
$$\therefore R = 2f \qquad \Rightarrow f = \frac{R}{2}$$

$$\text{Or Focal length} = \frac{1}{2} \times \text{Radius of curvature.}$$

Rules for drawing images formed by spherical mirrors:

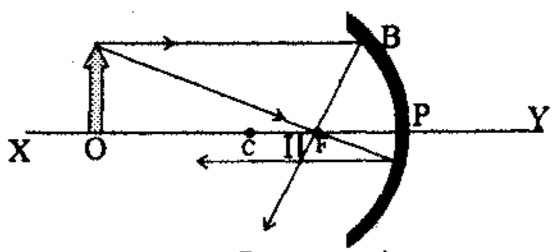


Concave mirror

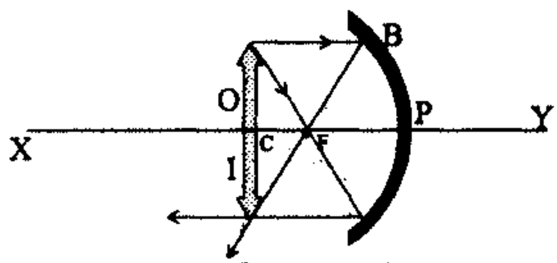


Convex mirror

Formation of images by concave mirror:

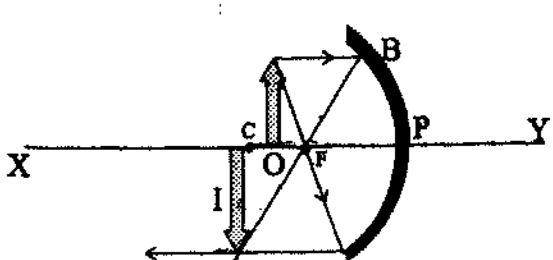


Concave mirror

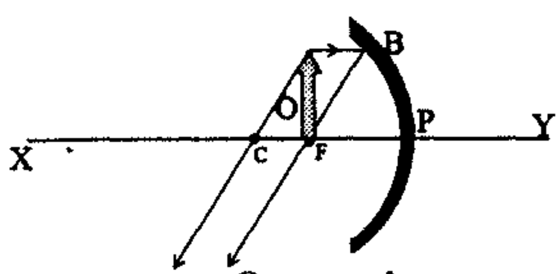


Concave mirror

Object beyond C. The Image is	Object at C. The Image is
Between C and F Real Inverted Smaller than object	At C Real Inverted Same size as object

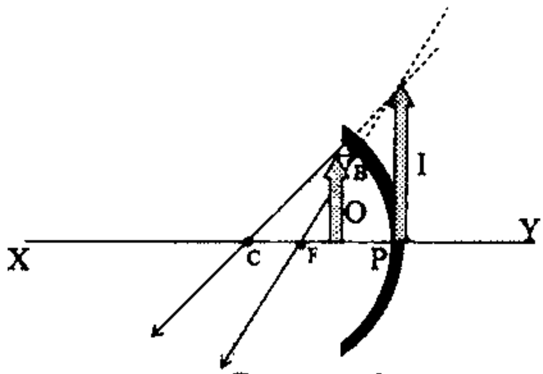


Concave mirror



Concave mirror

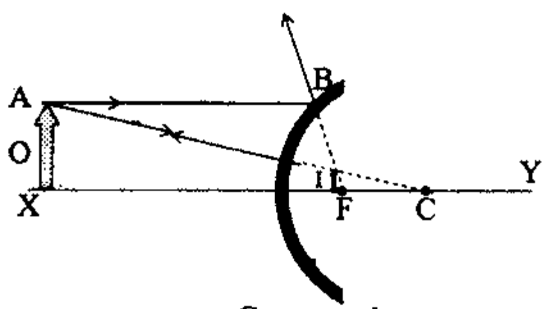
Object between F and C. The Image is	Object at F. The Image is
Beyond C Real Inverted Larger than object	At infinity Real Inverted Extremely large



Concave mirror

Object between F and P. The Image is
Behind the mirror
Virtual
Erect
Larger than object

Formation of images by convex mirror:



Convex mirror

For any position of the object , the Image is
Behind the mirror
Virtual
Erect
Smaller than object

Derivation of a mirror formula for a concave mirror when its forms a real image:

Using new Cartesian sign convention, we find

Focal length FP = $-f$

Image distance IP = $-v$

Radius of curvature CP = $-R = -2f$

Object distance OP = $-u$

$\Delta OAP \approx \Delta IBP$

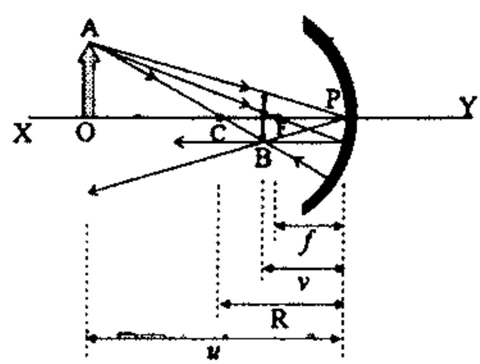
$\because \angle AOP = \angle BIP = 90°$, and $\angle APO = \angle BPI$

$$\therefore \frac{OA}{IB} = \frac{OP}{IP} = \frac{-u}{-v} = \frac{u}{v} \text{ ------- (1)}$$

Again, $\Delta OAC \approx \Delta IBC$

$\because \angle AOC = \angle BIC = 90°$, and $\angle ACO = \angle BCI$

$$\therefore \frac{OA}{IB} = \frac{OC}{IC} = \frac{OP - CP}{CP - IP} = \frac{-u - (-R)}{-R - (-v)} = \frac{-u + R}{-R + v} \text{ ------- (2)}$$



Concave mirror

Equating equation (1) and equation (2) we have

$$\frac{u}{v}=\frac{-u+R}{-R+v}$$

Or $-uR + uv = -uv + vR$

Or $uv + uv = vR + uR$

Or $2uv = R(u + v)$

$$uv=\frac{R}{2}(u+v)=f(u+v)$$

Divide both sides by uv we have

$$1=f\left(\frac{1}{v}+\frac{1}{u}\right) \qquad \Rightarrow \frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

This is the mirror formula for a concave mirror

Derivation of a mirror formula for a concave mirror when its forms a virtual image:

Using new Cartesian sign convention, we find

Focal length FP = $-f$

Image distance IP = v

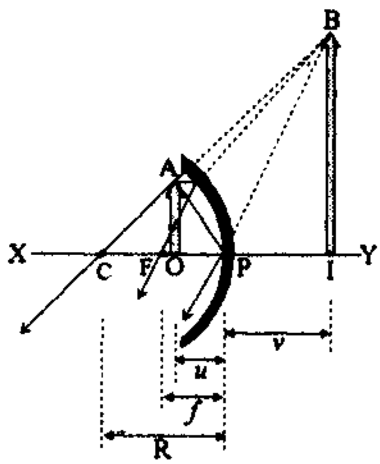
Radius of curvature CP = $-R = -2f$

Object distance OP = $-u$

$\Delta OAP \approx \Delta IBP$

$\because \angle AOP = \angle BIP = 90°$, and $\angle APO = \angle BPI$

$$\therefore \frac{OA}{IB}=\frac{OP}{IP}=\frac{-u}{v} \text{ -------- (1)}$$



Concave mirror

Again, $\Delta OAC \approx \Delta IBC$

$\because \angle AOC = \angle BIC = 90°$, and $\angle ACO = \angle BCI$

$$\therefore \frac{OA}{IB}=\frac{OC}{IC}=\frac{PC-PO}{PC+PI}=\frac{-R-(-u)}{-R+v}=\frac{-R+u}{-R+v} \text{ ------- (2)}$$

Equating equation (1) and equation (2) we have

$$\frac{-u}{v}=\frac{-R+u}{-R+v}$$

Or $uR - uv = -vR + uv$

Or $uv + uv = vR + uR$

Or $2uv = R(u + v)$

$$uv=\frac{R}{2}(u+v)=f(u+v)$$

Divide both sides by uv we have

$$1=f\left(\frac{1}{v}+\frac{1}{u}\right) \qquad \Rightarrow \frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

This is the mirror formula for a concave mirror

Derivation of a mirror formula for a convex mirror when its forms a virtual image:

Using new Cartesian sign convention, we find

Focal length PF = f

Image distance PI = v

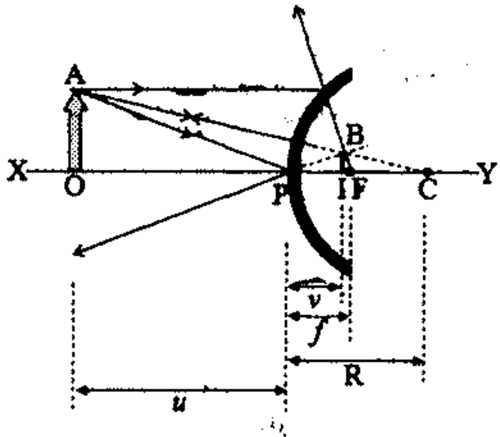
Radius of curvature PC = R = $2f$

Object distance PO = $-u$

$\Delta OAP \approx \Delta IBP$

$\because \angle AOP = \angle BIP = 90°$, and $\angle APO = \angle BPI$

$\therefore \frac{OA}{IB} = \frac{PO}{PI} = \frac{-u}{v}$ -------- (1)



Convex mirror

Again, $\Delta OAC \approx \Delta IBC$

$\because \angle AOC = \angle BIC = 90°$, and $\angle ACO = \angle BCI$

$\therefore \frac{OA}{IB} = \frac{OC}{IC} = \frac{PO + PC}{PC - PI} = \frac{-u + R}{R - v}$ -------- (2)

Equating equation (1) and equation (2) we have

$$\frac{-u}{v} = \frac{-u + R}{R - v}$$

Or $\quad -uR + uv = -uv + vR$

Or $\quad uv + uv = vR + uR$

Or $\quad 2uv = R(u + v)$

$$uv = \frac{R}{2}(u + v) = f(u + v)$$

Divide both sides by uv we have

$$1 = f\left(\frac{1}{v} + \frac{1}{u}\right) \qquad \Rightarrow \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

This is the mirror formula for a convex mirror

Linear magnification:

The ratio of the height of the image to that of the object is called linear or transverse magnification or just magnification and is denoted by m

$$m = \frac{\text{Height of the Image}}{\text{Height of the object}} = \frac{h_2}{h_1}$$

Linear magnification m for Concave mirror when the image is real:

$\because \Delta OAP \approx \Delta IBP$

$\therefore \frac{IB}{OA} = \frac{PI}{PO}$

Using new Cartesian sign convention, we find

Image distance PI = $-v$

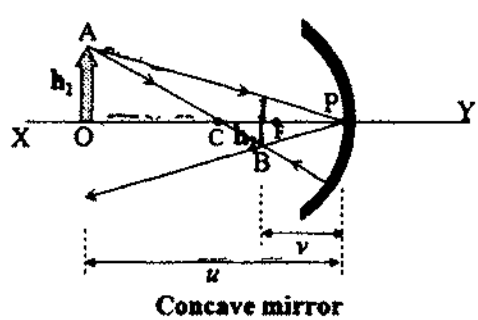
Object distance PO = $-u$

Height of the Image IB = $-h_2$

Height of the Object OA = h_1

Or $\frac{-h_2}{h_1} = \frac{-v}{-u}$

Or $\frac{-h_2}{h_1} = \frac{v}{u} \qquad \Rightarrow \frac{h_2}{h_1} = -\frac{v}{u}$



Concave mirror

$\therefore$ Magnification $m = \frac{h_2}{h_1} = -\frac{v}{u}$

Linear magnification *m* for Concave mirror when the image is virtual:

$\because \Delta OAP \approx \Delta IBP$

$\therefore \frac{IB}{OA} = \frac{PI}{PO}$

Using new Cartesian sign convention, we find

Image distance PI = v

Object distance PO = $-u$

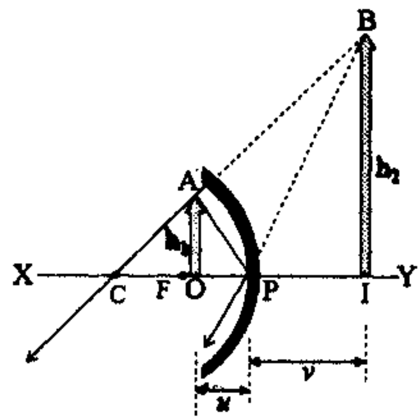
Height of the Image IB = h_2

Height of the Object OA = h_1

Or $\frac{h_2}{h_1} = \frac{v}{-u}$

Or $\frac{h_2}{h_1} = -\frac{v}{u}$

$\therefore$ Magnification $m = \frac{h_2}{h_1} = -\frac{v}{u}$



Concave mirror

Linear magnification *m* for Convex mirror:

$\because \Delta OAP \approx \Delta IBP$

$\therefore \frac{IB}{OA} = \frac{PI}{PO}$

Using new Cartesian sign convention, we find

Image distance PI = v

Object distance PO = $-u$

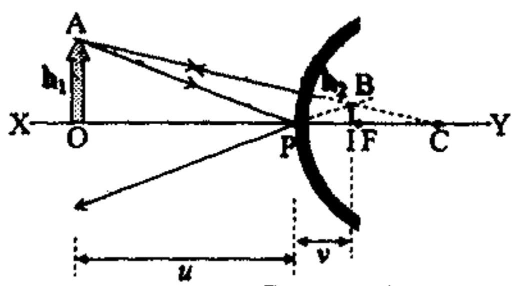
Height of the Image IB = h_2

Height of the Object OA = h_1

Or $\frac{h_2}{h_1} = \frac{v}{-u}$

Or $\frac{h_2}{h_1} = -\frac{v}{u}$

$\therefore$ Magnification $m = \frac{h_2}{h_1} = -\frac{v}{u}$



Convex mirror

Linear magnification in terms of *v* and *f*:

The mirror formula is $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

Multiplying both sides by v, we get

$$\frac{1}{f} \times v = \frac{1}{v} \times v + \frac{1}{u} \times v$$

$$\frac{v}{f} = 1 + \frac{v}{u}$$

Or $-\frac{v}{u} = 1 - \frac{v}{f} = \frac{f - v}{f}$

$\therefore$ Magnification $m = -\frac{v}{u} = \frac{f-v}{f}$

Linear magnification in terms of *u* and *f*:

The mirror formula is $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

Multiplying both sides by *u*, we get

$$\frac{1}{f} \times u = \frac{1}{v} \times u + \frac{1}{u} \times u$$

$$\frac{u}{f} = \frac{u}{v} + 1$$

Or $-\frac{u}{v} = 1 - \frac{u}{f} = \frac{f-u}{f}$

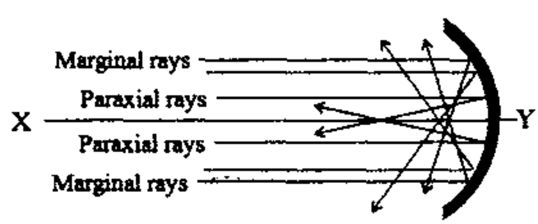
Or $-\frac{v}{u} = \frac{f}{f-u}$

$\therefore$ Magnification $m = -\frac{v}{u} = \frac{f}{f-u}$

Spherical aberration: The inability of a spherical mirror of large aperture to bring all the rays of wide beam of light falling on it to focus at a single point is called spherical aberration.

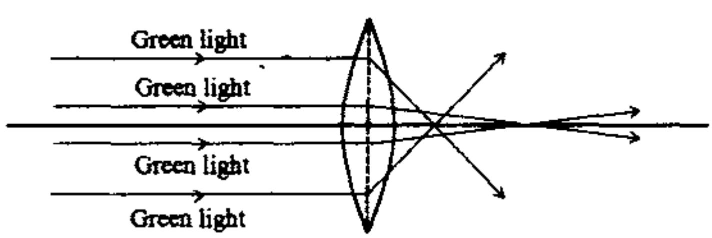
Note:

(1) When parallel rays of light, and parallel to the principal axis fall on the spherical mirror of large aperture, they will converge to different points on the principal axis. This defect can be reduced by decreasing the aperture of the mirror.

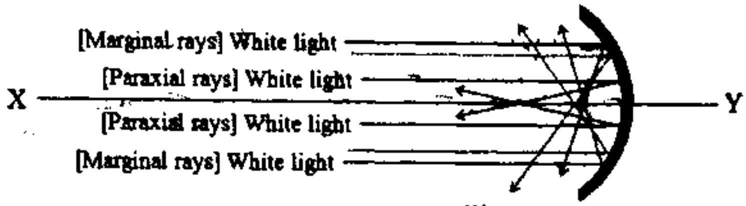


Concave mirror

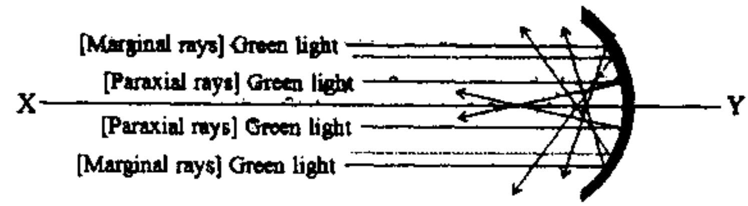
(2) Spherical aberrations are defects due to large aperture of lenses and mirrors.



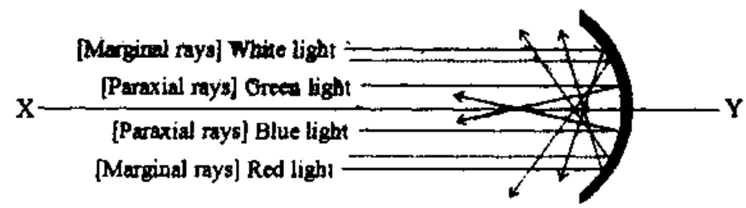
CONVEX LENS



(1)



(2)

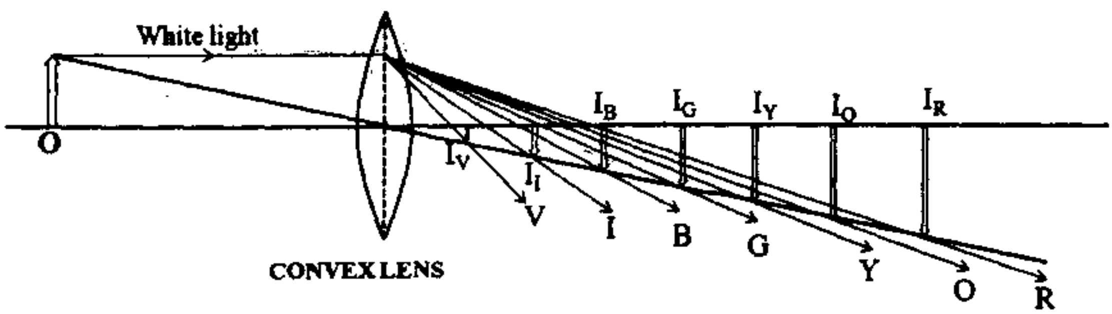


(3)

CONCAVE MIRROR

(2) No spherical aberration in a parabolic mirror.

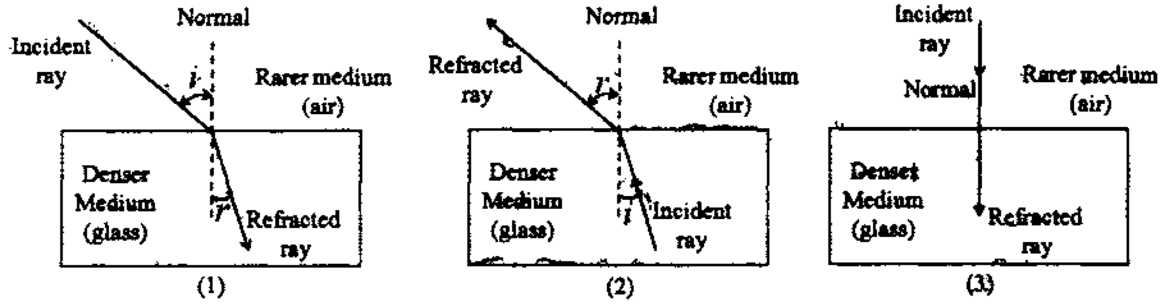
Chromatic aberrations: Chromatic aberrations are defects due to dispersion of light by lens. All types of mirrors are free from Chromatic aberrations.



Refraction of light: The bending of light as it passes obliquely from one transparent medium to another is called refraction of light.

It is observed that:

(1) When a ray of light passes from an optically rarer medium to an optically denser medium, it bends towards the normal i.e., the angle of incidence $\angle i$ is greater than the angle of refraction $\angle r$ as shown in figure (1).



(2) When a ray of light passes from an optically denser medium to an optically rarer medium, it bends away from the normal i.e., the angle of incidence $\angle i$ is less than the angle of refraction $\angle r$ as shown in figure (2).

(3) A ray of light travelling along the normal passes undeflected i.e., $\angle i = \angle r = 0^\circ$ as shown in figure (3).

Laws of refraction of light:

First law: The incident ray, the refracted ray and the normal to the interface at the point of incidence all lie in the same plane.

Second law: The ratio of the sine of angle of incidence to the sine of angle of refraction is constant for a given pair of media.

Mathematically, $\frac{\sin i}{\sin r} = {}^1\mu_2$

${}^1\mu_2$ is called the refractive index of the second medium with respect to the first medium. This law is also known as Snell's law of refraction.

Third law: When light goes from one medium to another, the frequency of light does not change. However, the velocity and wavelength of light change.

Refractive index: A property of a material that changes the speed of light is called refractive index

Refractive index in terms of speed of light: The refractive index of a medium may be defined as the ratio of the speed of light in vacuum to its speed in that medium.

$$\text{Refractive index} = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}}$$

$$\text{Or } \mu = \frac{c}{v}$$

Refractive index of a medium with respect to vacuum is called absolute refractive index.

Refractive index in terms of wavelength: The refractive index of a medium may be defined as the ratio of the wavelength of light in vacuum to its wavelength in that medium.

$$\mu = \frac{c}{v} = \frac{\lambda_{vacuum} \times \nu}{\lambda_{medium} \times \nu} = \frac{\lambda_{vacuum}}{\lambda_{medium}} \qquad \Rightarrow \mu \propto \frac{1}{\lambda}$$

Where ν is the frequency and λ is the wavelength of light.

Factors on which the refractive index of a medium depends:

(1) Nature of the medium.
(2) Wavelength of light used.
(3) Temperature.
(4) Nature of the surrounding medium.

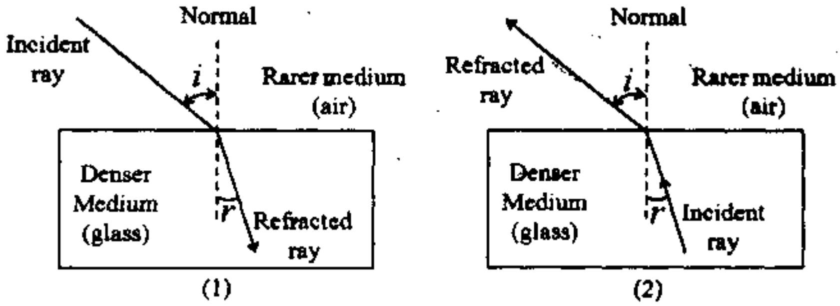
Physical significance of refractive index:

(1) The value of refractive index gives information about the direction of bending of refracted ray. It tells whether the ray will bend towards or away from the normal.

(2) The refractive index of the medium is related to the speed of light.

For example, refractive index of glass is 3/2. This indicates that the ratio of the speed of light in glass to that in vacuum is 2 : 3 or the speed of light in glass is two-third of its speed in vacuum

Principle of reversibility of light: This principle states that if the final path of the ray of light after it has suffered several reflections and refractions is reversed, it retraces its path exactly.



In figure (1) $^{a}\mu_{g} = \dfrac{\sin i}{\sin r}$ ------- (1)

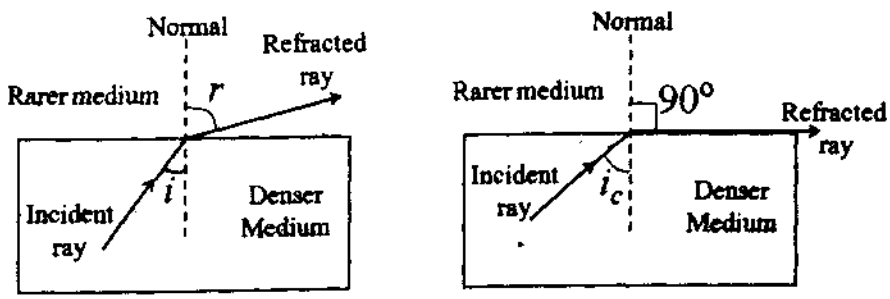
In figure (2) $^{g}\mu_{a} = \dfrac{\sin r}{\sin i}$ ------- (2)

Multiplying (1) and (2) we have

$$^{a}\mu_{g} \cdot {}^{g}\mu_{a} = \frac{\sin i}{\sin r} \cdot \frac{\sin r}{\sin i} = 1$$

$$\therefore {}^{a}\mu_{g} = \frac{1}{^{g}\mu_{a}}$$

Critical angle: The angle of incidence in the denser medium for which the angle of reflection in the rarer medium is 90° is called critical angle of the denser medium, and is denoted by i_c.



Relation between critical angle and refractive index:

$$\mu_{glass} \sin i_c = \mu_{air} \sin 90^o$$

$$\frac{\mu_{glass}}{\mu_{air}} = \frac{\sin 90^o}{\sin i_c} = \frac{1}{\sin i_c}$$

$$\frac{\mu_{glass}}{1} = \frac{1}{\sin i_c} \qquad \Rightarrow \mu_{glass} = \frac{1}{\sin i_c}$$

$$\sin i_c = \frac{1}{\mu_{glass}} \qquad \Rightarrow i_c = \sin^{-1}\left(\frac{1}{\mu_{glass}}\right)$$

Or

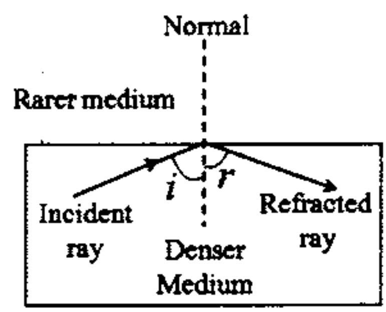
$$^{denser}\mu_{rarer} = \frac{\sin i_c}{\sin 90^o}$$

$$^{denser}\mu_{rarer} = \frac{\sin i_c}{1} = \sin i_c$$

$$\sin i_c = \frac{1}{^{rarer}\mu_{denser}} \qquad \Rightarrow i_c = \sin^{-1}\left(\frac{1}{^{rarer}\mu_{denser}}\right)$$

$$^{rarer}\mu_{denser} = \frac{1}{\sin i_c}$$

Total internal reflection: The phenomenon in which a ray of light travelling at an angle of incidence greater than the critical angle from denser to a rarer medium is totally reflected back into the denser medium is called total internal reflection.



Necessary conditions for total internal reflection:
(1) Light must travel from an optically denser medium to an optically rarer medium.
(2) The angle of incidence in the denser medium must be greater than the critical angle for the two media.

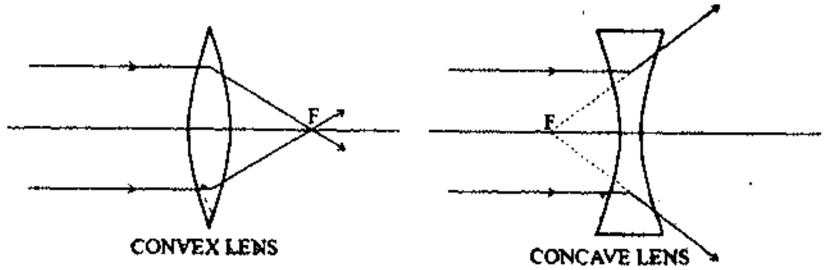
Applications of total internal reflection:
(1) Sparkling of diamond
(2) Mirage
(3) Optical fibres
(4) Totally reflecting prism

Applications of optical fibres: Fibre optic cables find many uses in a wide variety of industries and applications. Some uses of fibre optic cables include:

Medical: Used as light guides, imaging tools and also as lasers for surgeries
Defense/Government: Used as hydrophones for seismic and SONAR uses, as wiring in aircraft, submarines and other vehicles and also for field networking .
Data Storage: Used for data transmission
Telecommunications: Fiber is laid and used for transmitting and receiving purposes
Networking: Used to connect users and servers in a variety of network settings and help increase the speed and accuracy of data transmission

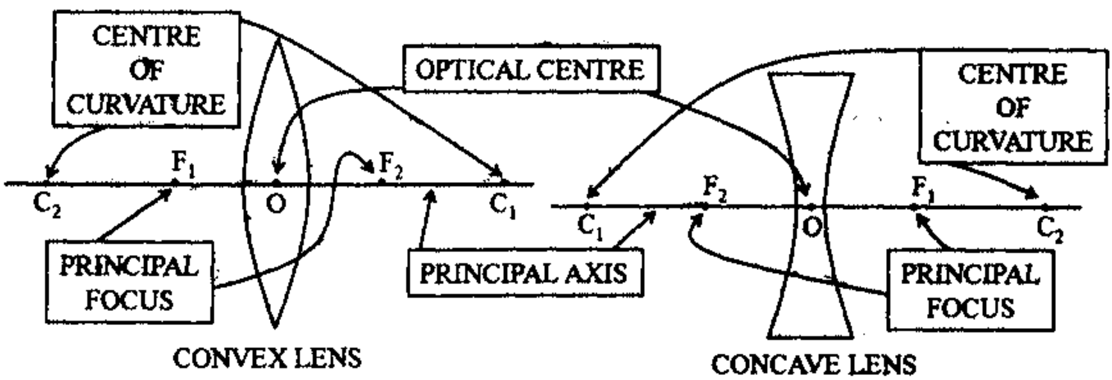
Spherical lenses or lenses: A lens is a piece of refracting medium bounded by two surfaces, at least one of which is a curved surface. Lens can be divided into two categories.
(1) Convex or converging lenses
(2) Concave or diverging lenses

Convex or converging lens: It is thicker at the centre than at the edges. It is converges a parallel beam of light on refraction through it. It has real focus.

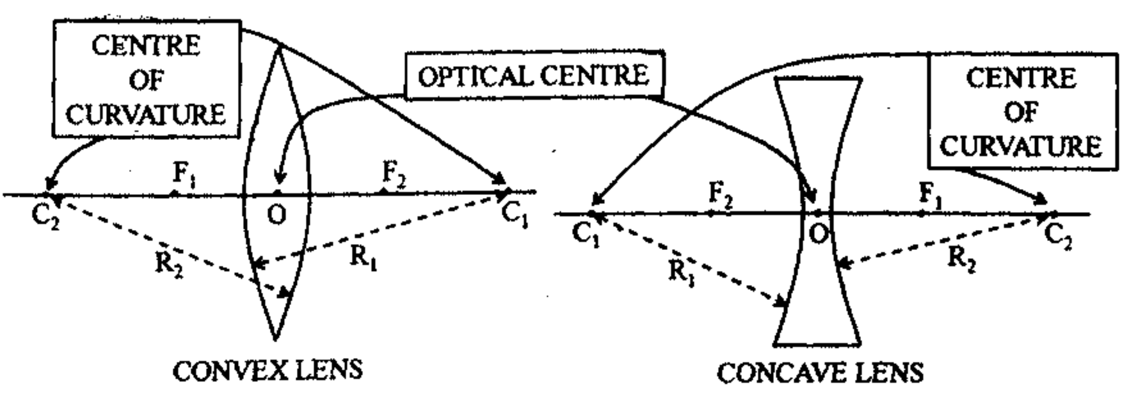


Concave or diverging lens: It is thinner at the centre than at the edges. It is diverges a parallel beam of light on refraction through it. It has virtual focus.

Definitions in connection with Spherical lenses:



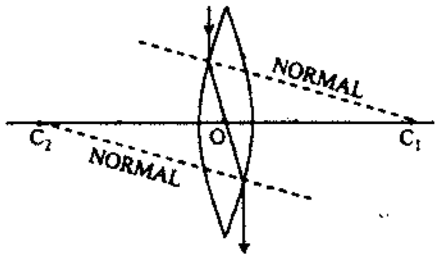
Centre of curvature (C): The centre of curvature of the surface of a lens is the centre of the sphere of which it forms a part. Because a lens has two surfaces, so it has two centres of curvature C_1 and C_2.



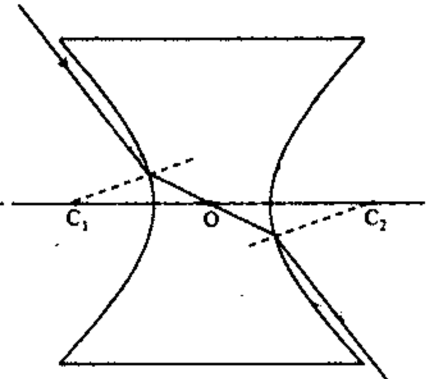
Radius of curvature (R): The radius of curvature of the surface of a lens is the radii of the sphere of which the surface forms a part.

Principal axis (C_1C_2): It is the line passing through the two centres of curvature of the lens

Optical centre (O): If a ray of light is incident on a lens such that after refraction through the lens the emergent ray is parallel to the incident ray, then the point at which the refracted ray intersects the principal axis is called the optical centre of the lens



CONVEX LENS

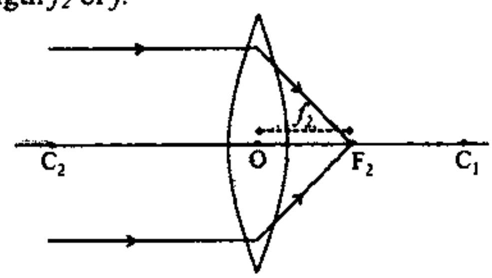


CONCAVE LENS

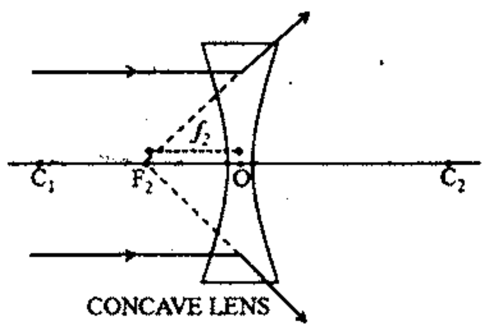
Or

The centre of the lens where the principal axis passes through, is called the optical centre

Second principal focus (F_2 or F): It is a fixed point on the principal axis such that the light rays incident parallel to the principal axis, after refraction through the lens, either converge to this point (in convex lens) or appear to diverge from this point (in concave lens). The distance between the second principal focus (F_2 or F) and the optical centre (O) is called the second focal length f_2 or f.

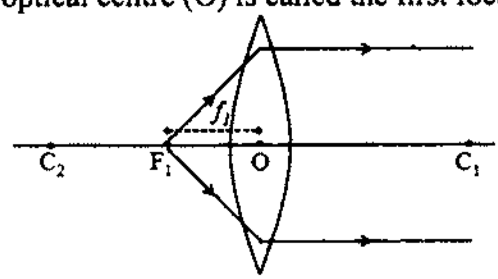


CONVEX LENS

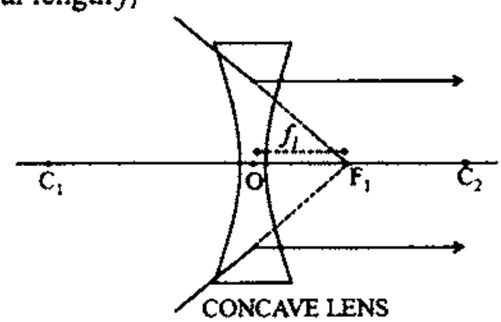


CONCAVE LENS

First principal focus (F_1): It is a fixed point on the principal axis such that rays starting from this point (in convex lens) or appearing to go towards this point, after refraction from the lens, become parallel to the principal axis. The distance between the first principal focus (F_1) and the optical centre (O) is called the first focal length f_1

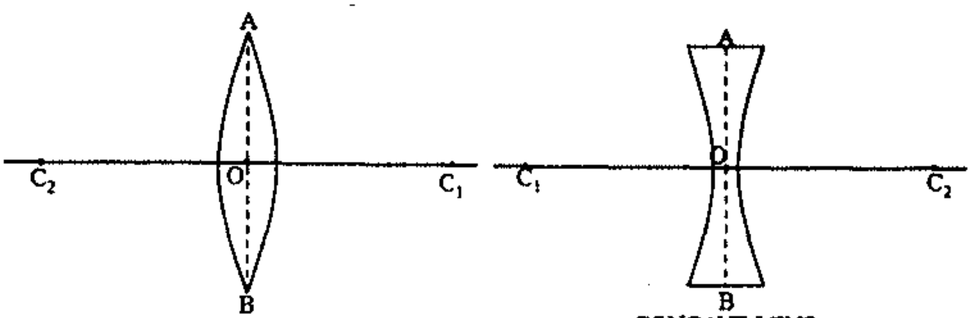


CONVEX LENS



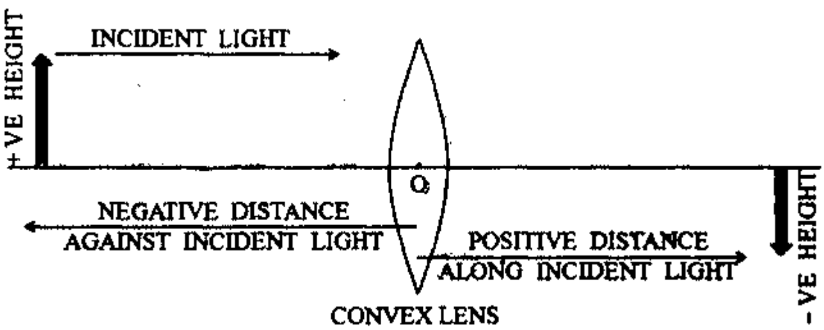
CONCAVE LENS

Aperture: It is the diameter AB of the circular boundary of the lens

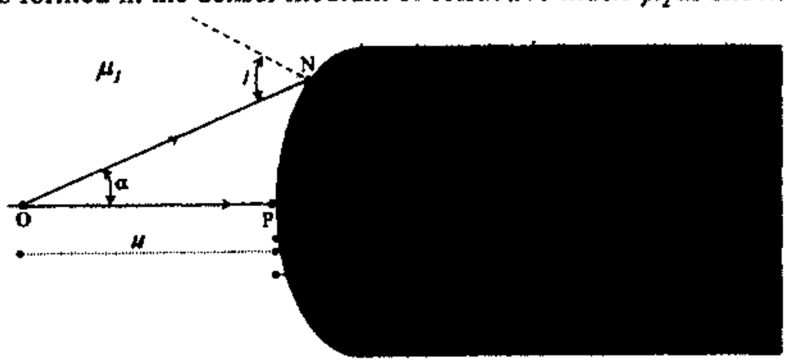


CONVEX LENS CONCAVE LENS

New Cartesian sign convention for Spherical lenses: All distances are measured from the optical centre (O) of the lens.



Refraction at the convex Spherical surface (Object lies in rarer medium, Image in the denser medium): Let O be a point object situated in the rarer medium of refractive index μ_1 whose image I is formed in the denser medium of refractive index μ_2 as shown in the figure.



$$\because \mu_1 \sin i = \mu_2 \sin r \text{ ------- } (1)$$

In ΔNOC

$$i = \alpha + \gamma$$

$$\sin i = \sin\alpha + \sin\gamma$$

$$\sin i = \frac{NM}{ON} + \frac{NM}{CN}$$

If α and γ are very small, then we have

$$\sin i = \frac{NM}{OP} + \frac{NM}{CP} = \frac{NM}{-u} + \frac{NM}{R}$$

$$\sin i = NM\left(\frac{1}{-u} + \frac{1}{R}\right) \text{ ------- } (2)$$

In ΔNCI

$$\gamma = r + \beta$$

$$r = \gamma - \beta$$

$$\sin r = \sin \gamma - \sin \beta$$

$$\sin r = \frac{NM}{CN} - \frac{NM}{NI}$$

If γ and β are very small, then we have

$$\sin r = \frac{NM}{CP} - \frac{NM}{IP} = \frac{NM}{R} - \frac{NM}{v}$$

$$\sin r = NM\left(\frac{1}{R} - \frac{1}{v}\right) \text{------- (3)}$$

Putting (2) and (3) in (1) we have

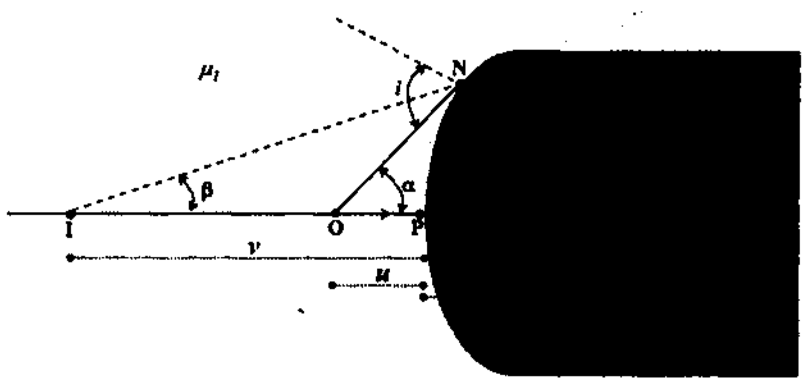
$$\mu_1 NM\left(\frac{1}{-u} + \frac{1}{R}\right) = \mu_2 NM\left(\frac{1}{R} - \frac{1}{v}\right)$$

$$-\frac{\mu_1}{u} + \frac{\mu_1}{R} = \frac{\mu_2}{R} - \frac{\mu_2}{v}$$

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2}{R} - \frac{\mu_1}{R}$$

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

Refraction at the convex Spherical surface (Object lies in rarer medium, Image in the rarer medium): Let O be a point object situated in the rarer medium of refractive index μ_1 whose image I is also formed in the same rarer medium. The refractive index of the denser medium is μ_2 as shown in the figure.



$\because \mu_1 \sin i = \mu_2 \sin r$ ------- (1)

In ΔNOC

$$i = \alpha + \gamma$$

$$\sin i = \sin \alpha + \sin \gamma$$

$$\sin i = \frac{NM}{ON} + \frac{NM}{CN}$$

If α and γ are very small, then we have

$$\sin i = \frac{NM}{OP} + \frac{NM}{CP} = \frac{NM}{-u} + \frac{NM}{R}$$

$$\sin i = NM\left(\frac{1}{-u}+\frac{1}{R}\right) \text{-------- (2)}$$

In ΔNCI

$$r = \gamma + \beta$$

$$\sin r = \sin\gamma + \sin\beta$$

$$\sin r = \frac{NM}{CN} + \frac{NM}{NI}$$

If γ and β are very small, then we have

$$\sin r = \frac{NM}{CP} + \frac{NM}{IP} = \frac{NM}{R} + \frac{NM}{-v} = \frac{NM}{R} - \frac{NM}{v}$$

$$\sin r = NM\left(\frac{1}{R}-\frac{1}{v}\right) \text{-------- (3)}$$

Putting (2) and (3) in (1) we have

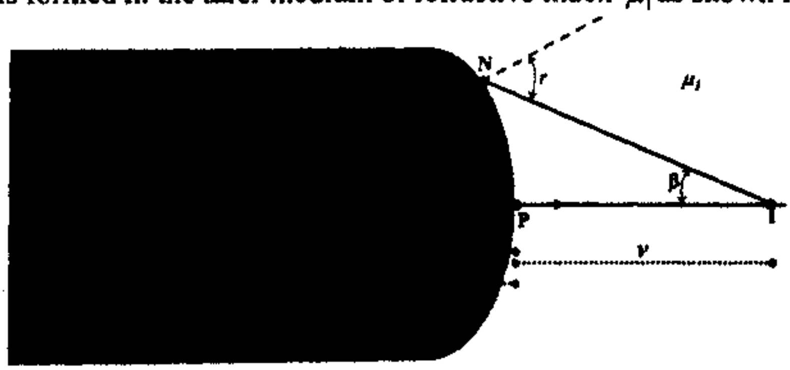
$$\mu_1 NM\left(\frac{1}{-u}+\frac{1}{R}\right) = \mu_2 NM\left(\frac{1}{R}-\frac{1}{v}\right)$$

$$-\frac{\mu_1}{u}+\frac{\mu_1}{R} = \frac{\mu_2}{R}-\frac{\mu_2}{v}$$

$$-\frac{\mu_1}{u}+\frac{\mu_2}{v} = \frac{\mu_2}{R}-\frac{\mu_1}{R}$$

$$-\frac{\mu_1}{u}+\frac{\mu_2}{v} = \frac{\mu_2-\mu_1}{R}$$

Refraction at the convex Spherical surface (Object lies in denser medium, Image in the rarer medium): Let O be a point object situated in the denser medium of refractive index μ_2 whose image I is formed in the rarer medium of refractive index μ_1 as shown in the figure.



$$\because \mu_2 \sin i = \mu_1 \sin r \text{ -------- (1)}$$

In ΔNOC

$$\gamma = \alpha + i$$

$$i = \gamma - \alpha$$

$$\sin i = \sin\gamma - \sin\alpha$$

$$\sin i = \frac{NM}{CN} - \frac{NM}{NO}$$

If γ and α are very small, then we have

$$\sin i = \frac{NM}{CP} - \frac{NM}{OP} = \frac{NM}{-R} - \frac{NM}{-u} = \frac{NM}{u} - \frac{NM}{R}$$

$$\sin i = NM\left(\frac{1}{u}-\frac{1}{R}\right) \text{--------} (2)$$

In ΔNCI

$$r = \gamma + \beta$$

$$\sin r = \sin\gamma + \sin\beta$$

$$\sin r = \frac{NM}{CN} + \frac{NM}{NI}$$

If γ and β are very small, then we have

$$\sin r = \frac{NM}{CP} + \frac{NM}{IP} = \frac{NM}{-R} + \frac{NM}{v}$$

$$\sin r = NM\left(\frac{1}{v}-\frac{1}{R}\right) \text{-------} (3)$$

Putting (2) and (3) in (1) we have

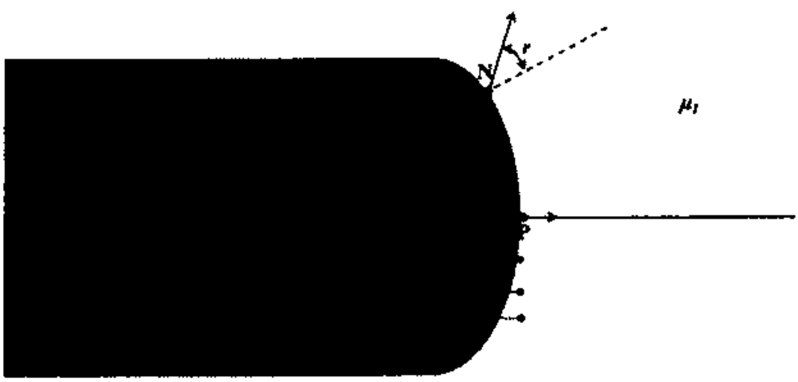
$$\mu_2 NM\left(\frac{1}{u}-\frac{1}{R}\right) = \mu_1 NM\left(\frac{1}{v}-\frac{1}{R}\right)$$

$$\frac{\mu_2}{u} - \frac{\mu_2}{R} = \frac{\mu_1}{v} - \frac{\mu_1}{R}$$

$$-\frac{\mu_1}{v} + \frac{\mu_2}{u} = \frac{\mu_2}{R} - \frac{\mu_1}{R}$$

$$-\frac{\mu_1}{v} + \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

Refraction at the convex Spherical surface (Object lies in denser medium, Image in the denser medium): Let O be a point object situated in the denser medium of refractive index μ_2 whose image I is also formed in the same denser medium. The refractive index of the rarer medium is μ_1 as shown in the figure.



$$\because \mu_2 \sin i = \mu_1 \sin r \text{ -------} (1)$$

In ΔNOC

$$\alpha = i + \gamma$$

$$i = \alpha - \gamma$$

$$\sin i = \sin\alpha - \sin\gamma$$

$$\sin i = \frac{NM}{NO} - \frac{NM}{CN}$$

If α and γ are very small, then we have

$$\sin i = \frac{NM}{OP} - \frac{NM}{CP} = \frac{NM}{-u} - \frac{NM}{-R} = \frac{NM}{R} - \frac{NM}{u}$$

$$\sin i = NM\left(\frac{1}{R} - \frac{1}{u}\right) \text{ ------- } (2)$$

In ΔNCI

$$\beta = r + \gamma$$

$$r = \beta - \gamma$$

$$\sin r = \sin\beta - \sin\gamma$$

$$\sin r = \frac{NM}{NI} - \frac{NM}{CN}$$

If β and γ are very small, then we have

$$\sin r = \frac{NM}{IP} - \frac{NM}{CP} = \frac{NM}{-v} - \frac{NM}{-R} = \frac{NM}{R} - \frac{NM}{v}$$

$$\sin r = NM\left(\frac{1}{R} - \frac{1}{v}\right) \text{ ------- } (3)$$

Putting (2) and (3) in (1) we have

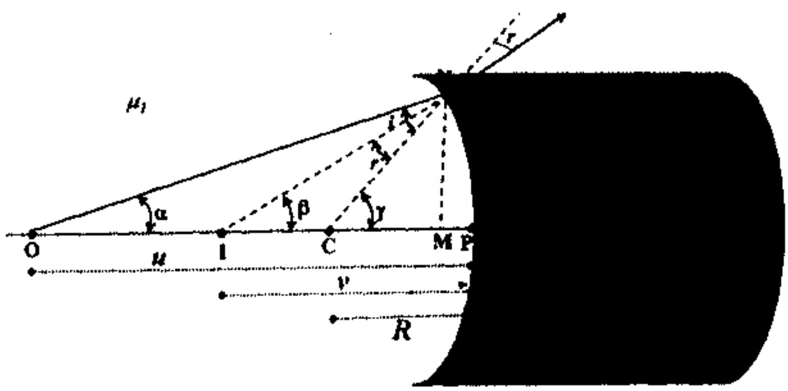
$$\mu_2 NM\left(\frac{1}{R} - \frac{1}{u}\right) = \mu_1 NM\left(\frac{1}{R} - \frac{1}{v}\right)$$

$$\frac{\mu_2}{R} - \frac{\mu_2}{u} = \frac{\mu_1}{R} - \frac{\mu_1}{v}$$

$$-\frac{\mu_1}{v} + \frac{\mu_2}{u} = \frac{\mu_2}{R} - \frac{\mu_1}{R}$$

$$-\frac{\mu_1}{v} + \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

Refraction at the concave Spherical surface (Object lies in rarer medium): Let O be a point object situated in the rarer medium of refractive index μ_1 whose image I is also formed in the same rarer medium. The refractive index of the denser medium is μ_2 as shown in the figure.



$\because \mu_1 \sin i = \mu_2 \sin r$ ------- (1)

In ΔNOC

$$\gamma = i + \alpha$$

$$i = \gamma - \alpha$$

$$\sin i = \sin\gamma - \sin\alpha$$

$$\sin i = \frac{NM}{CN} - \frac{NM}{ON}$$

If γ and α are very small, then we have

$$\sin i = \frac{NM}{CP} - \frac{NM}{OP} = \frac{NM}{-R} - \frac{NM}{-u} = \frac{NM}{u} - \frac{NM}{R}$$

$$\sin i = NM\left(\frac{1}{u} - \frac{1}{R}\right) \text{------- (2)}$$

In ΔNCI

$$\gamma = r + \beta$$

$$r = \gamma - \beta$$

$$\sin r = \sin\gamma - \sin\beta$$

$$\sin r = \frac{NM}{CN} - \frac{NM}{NI}$$

If γ and β are very small, then we have

$$\sin r = \frac{NM}{CP} - \frac{NM}{IP} = \frac{NM}{-R} - \frac{NM}{-v} = \frac{NM}{v} - \frac{NM}{R}$$

$$\sin r = NM\left(\frac{1}{v} - \frac{1}{R}\right) \text{------- (3)}$$

Putting (2) and (3) in (1) we have

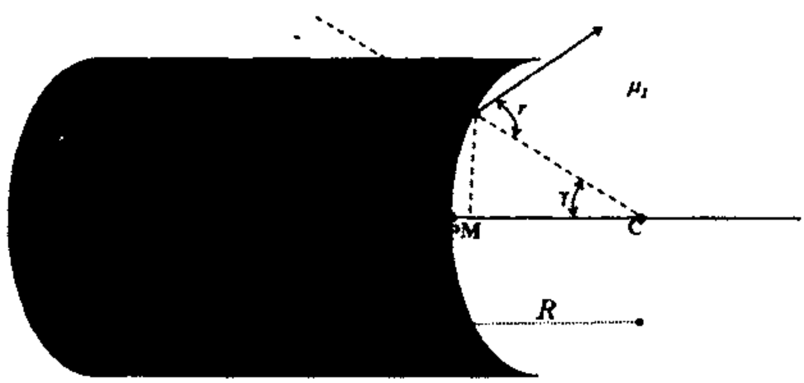
$$\mu_1 NM\left(\frac{1}{u} - \frac{1}{R}\right) = \mu_2 NM\left(\frac{1}{v} - \frac{1}{R}\right)$$

$$\frac{\mu_1}{u} - \frac{\mu_1}{R} = \frac{\mu_2}{v} - \frac{\mu_2}{R}$$

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2}{R} - \frac{\mu_1}{R}$$

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

Refraction at the concave Spherical surface (Object lies in rarer medium): Let O be a point object situated in the denser medium of refractive index μ_2 whose image I is also formed in the same denser medium. The refractive index of the rarer medium is μ_1 as shown in the figure.



$$\because \mu_2 \sin i = \mu_1 \sin r \text{ ------- (1)}$$

In ΔNOC

$$i = \alpha + \gamma$$

$$\sin i = \sin\alpha + \sin\gamma$$

$$\sin i = \frac{NM}{NO} + \frac{NM}{CN}$$

If α and γ are very small, then we have

$$\sin i = \frac{NM}{OP} + \frac{NM}{CP} = \frac{NM}{-u} + \frac{NM}{R} = \frac{NM}{R} - \frac{NM}{u}$$

$$\sin i = NM\left(\frac{1}{R} - \frac{1}{u}\right) \text{-------- (2)}$$

In ΔNCI

$$r = \beta + \gamma$$

$$\sin r = \sin\beta + \sin\gamma$$

$$\sin r = \frac{NM}{NI} + \frac{NM}{CN}$$

If β and γ are very small, then we have

$$\sin r = \frac{NM}{IP} + \frac{NM}{CP} = \frac{NM}{-v} + \frac{NM}{R} = \frac{NM}{R} - \frac{NM}{v}$$

$$\sin r = NM\left(\frac{1}{R} - \frac{1}{v}\right) \text{-------- (3)}$$

Putting (2) and (3) in (1) we have

$$\mu_2 NM\left(\frac{1}{R} - \frac{1}{u}\right) = \mu_1 NM\left(\frac{1}{R} - \frac{1}{v}\right)$$

$$\frac{\mu_2}{R} - \frac{\mu_2}{u} = \frac{\mu_1}{R} - \frac{\mu_1}{v}$$

$$-\frac{\mu_1}{v} + \frac{\mu_2}{u} = \frac{\mu_2}{R} - \frac{\mu_1}{R}$$

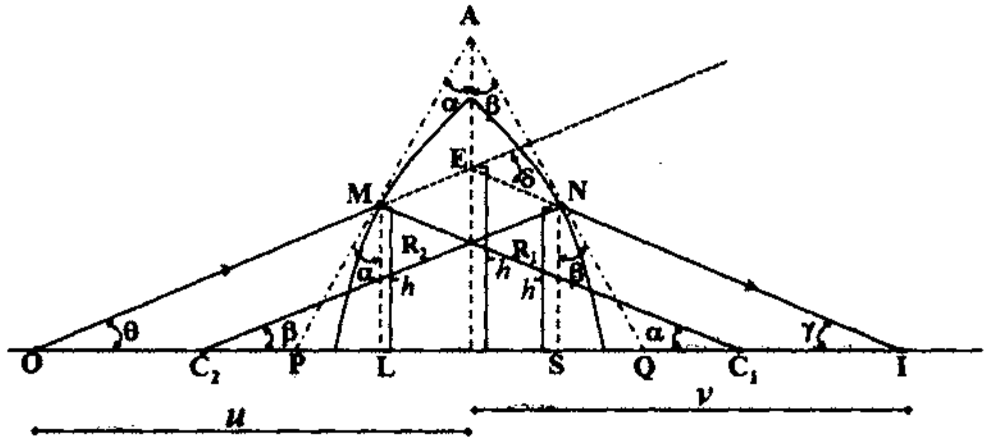
$$-\frac{\mu_1}{v} + \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

Note:

(1) When object lies in rarer medium, we have $-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$

(2) When object lies in denser medium, we have $-\frac{\mu_1}{v} + \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$

Lens maker's formula:



Let us consider the upper half of the lens which is equivalent to a prism. Its refracting angle is at the top. Let OM be the incident ray and NI be the emergent ray. AP and AQ are tangents at M and N respectively.

In ΔEOI,

The angle of deviation $\delta = \theta + \gamma$

For small angles θ and γ, $\theta = \tan\theta$ and $\gamma = \tan\gamma$

$\therefore \delta = \tan\theta + \tan\gamma$

Or $\delta = \frac{h}{-u} + \frac{h}{v} = h\left(\frac{1}{v} - \frac{1}{u}\right) = \frac{h}{f}$ ------- (1)

But we know that

$\delta = (\mu - 1)A = (\mu - 1)(\alpha + \beta)$ ------- (2)

Let ML = NS = h, and also for small angles α and β, $\alpha = \sin\alpha$ and $\beta = \sin\beta$

$\alpha + \beta = \sin\alpha + \sin\beta = \frac{h}{R_1} + \frac{h}{-R_2}$ ------- (3)

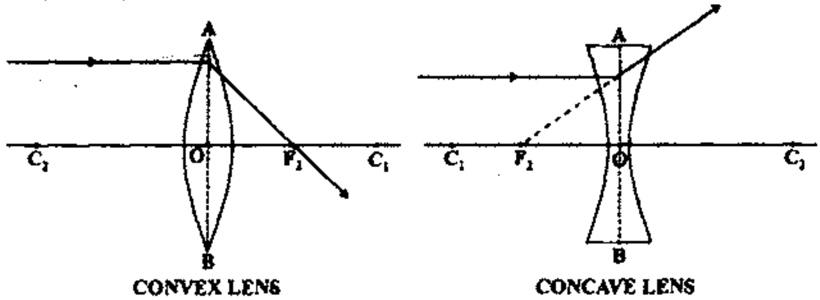
Where R_1 and R_2 are the radii of curvatures.

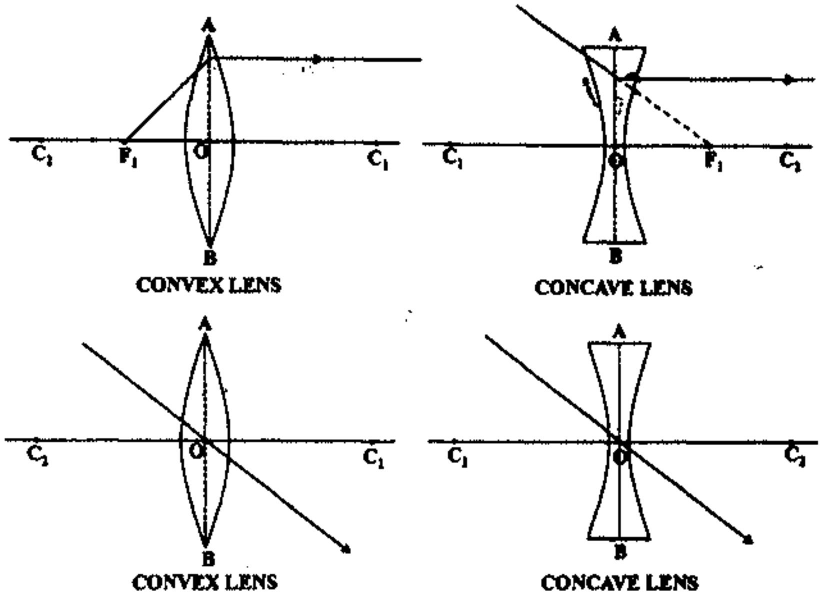
Putting equations (1) and (3) in equation (2) we have

$$\frac{h}{f} = (\mu - 1)\left(\frac{h}{R_1} - \frac{h}{R_2}\right)$$

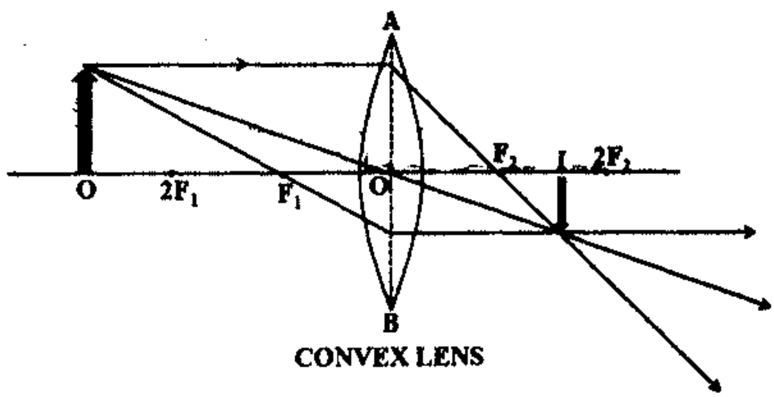
$$\therefore \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Rules for drawing images formed by Spherical lenses:



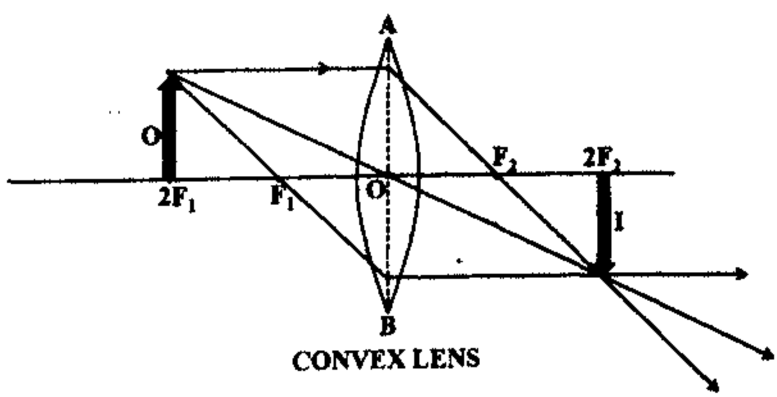


Formation of images by Spherical lenses:



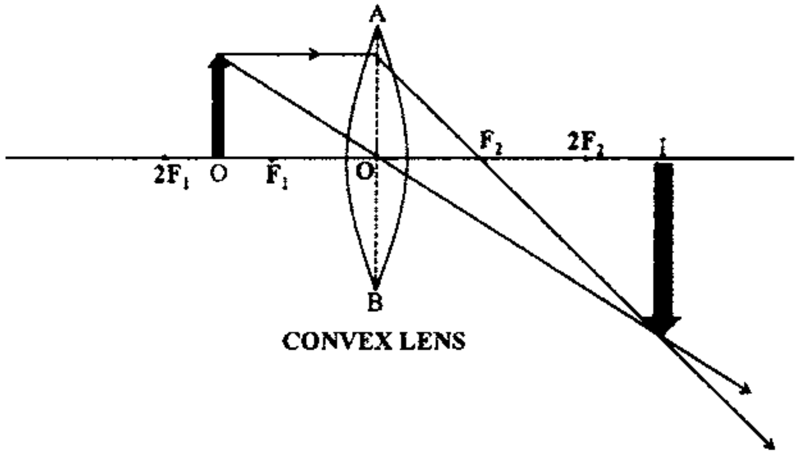
Object beyond 2F. The image is

(i) Between F and 2F
(ii) real
(iii) Inverted
(iv) smaller



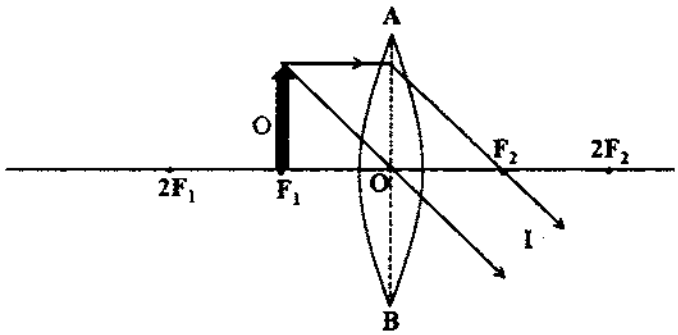
Object at 2F. The image is

(i) at 2F
(ii) real
(iii) Inverted
(iv) same size



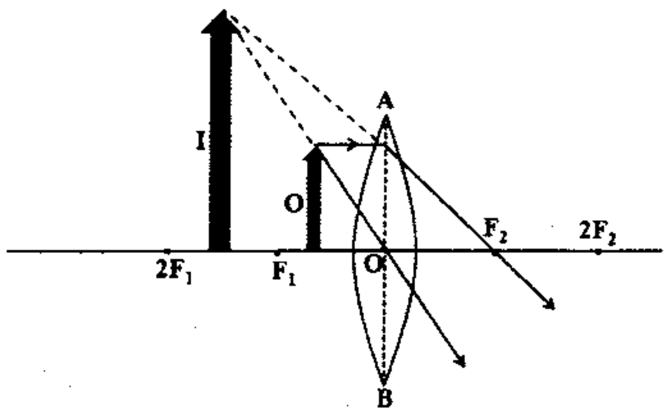
Object between 2F and F. The image is

(i) beyond 2F
(ii) real
(iii) Inverted
(iv) larger



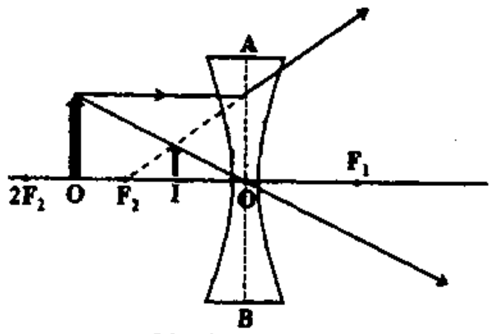
Object at F. The image is

(i) at infinity
(ii) real
(iii) inverted
(iv) larger



Object between F and O. The image is

(i) behind object
(ii) virtual
(iii) erect
(iv) larger



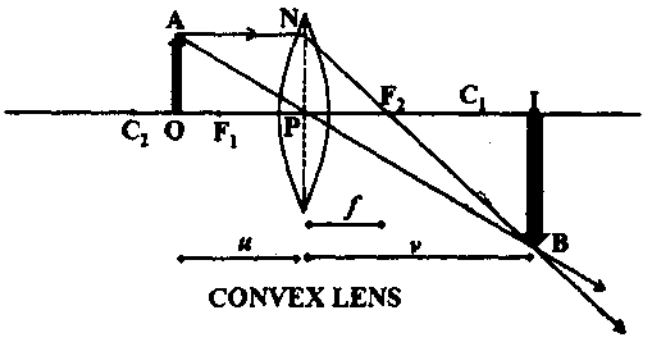
CONCAVE LENS

Object in any position. The image is

(i) In front of object (ii) virtual

(iii) erect (iv) smaller

Thin lens formula for a convex lens when it forms a real image:



CONVEX LENS

Consider an object OA placed perpendicular to the principal axis of a thin convex lens between F_1 and C_2. A real, inverted and magnified image IB is formed beyond C_1 on the other side of the lens.

ΔAOP and ΔBIP are similar triangles.

$$\therefore \frac{AO}{BI} = \frac{OP}{IP} = \frac{-u}{v} \text{ ---------------- } (1)$$

Also ΔNPF_2 and ΔBIF_2 are similar triangles.

$$\therefore \frac{NP}{BI} = \frac{F_2P}{F_2I} = \frac{F_2P}{IP - F_2P} = \frac{f}{v-f} \text{ ---------------- } (2)$$

$\because$ NP = AO

Equating equation (1) and equation (2) gives

$$\frac{-u}{v} = \frac{f}{v-f}$$

$$-uv + uf = vf$$

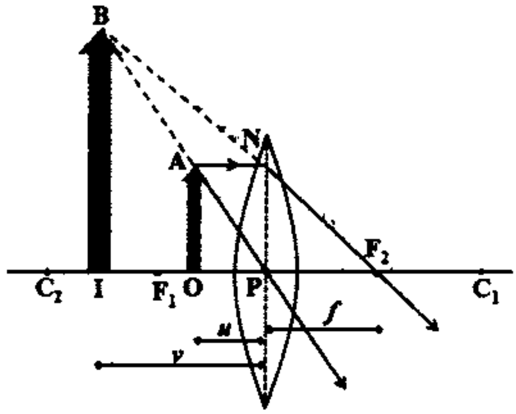
Divide both sides by uvf we have

$$-\frac{uv}{uvf} + \frac{uf}{uvf} = \frac{vf}{uvf} \qquad \Rightarrow \quad -\frac{1}{f} + \frac{1}{v} = \frac{1}{u}$$

$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This proves the lens formula for a convex lens when it forms a real image

Thin lens formula for a convex lens when it forms a virtual image:



CONVEX LENS

Consider an object OA placed perpendicular to the principal axis of a thin convex lens between F_1 and P. A virtual, erect and magnified image IB is formed on the side of the object.

ΔAOP and ΔBIP are similar triangles.

$$\therefore \frac{AO}{BI} = \frac{OP}{IP} = \frac{-u}{-v} = \frac{u}{v} \text{ ------------------ (1)}$$

Also ΔNPF_2 and ΔBIF_2 are similar triangles.

$$\therefore \frac{NP}{BI} = \frac{F_2P}{F_2I} = \frac{F_2P}{IP + F_2P} = \frac{f}{-v+f} \text{ ------------------ (2)}$$

$\because$ NP = AO

Equating equation (1) and equation (2) gives

$$\frac{u}{v} = \frac{f}{-v+f}$$

$$-uv + uf = vf$$

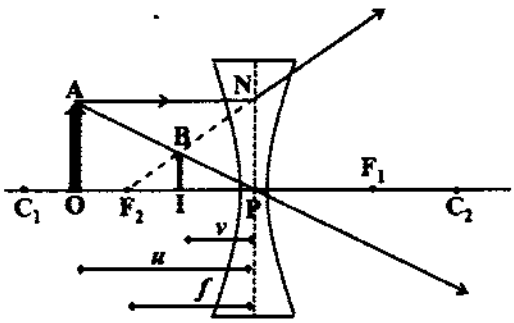
Divide both sides by uvf we have

$$-\frac{uv}{uvf} + \frac{uf}{uvf} = \frac{vf}{uvf} \qquad \Rightarrow -\frac{1}{f} + \frac{1}{v} = \frac{1}{u}$$

$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This proves the lens formula for a convex lens when it forms a virtual image

Thin lens formula for a concave lens when it forms a virtual image:



CONCAVE LENS

Consider an object OA placed perpendicular to the principal axis of a thin concave lens. A virtual, erect and diminished image IB is formed on the side of the object.

ΔAOP and ΔBIP are similar triangles.

$$\therefore \frac{AO}{BI} = \frac{OP}{IP} = \frac{-u}{-v} = \frac{u}{v} \text{ ------------------ (1)}$$

Also ΔNPF_2 and ΔBIF_2 are similar triangles.

$$\therefore \frac{NP}{BI} = \frac{F_2P}{F_2I} = \frac{F_2P}{F_2P - IP} = \frac{-f}{-f-(-v)} = \frac{-f}{-f+v} \text{ ------------------ (2)}$$

$\because NP = AO$

Equating equation (1) and equation (2) gives

$$\frac{u}{v} = \frac{-f}{-f+v}$$

$$-uf + uv = -vf$$

Divide both sides by uvf we have

$$-\frac{uf}{uvf} + \frac{uv}{uvf} = -\frac{vf}{uvf} \qquad \Rightarrow -\frac{1}{v} + \frac{1}{f} = -\frac{1}{u}$$

$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

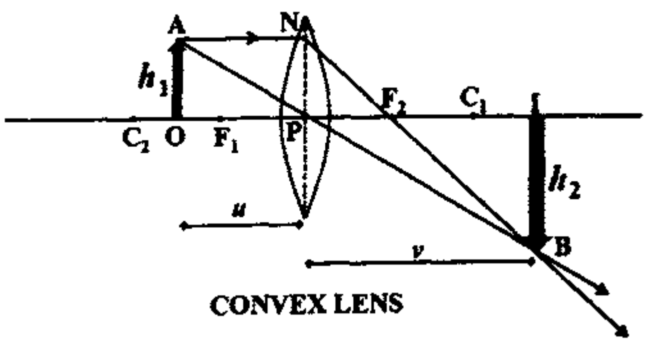
This proves the lens formula for a concave lens when it forms a virtual image

Linear magnification:

The linear magnification produced by a lens is defined as the ratio of the size of the image formed by the lens to the size of the object. It is denoted by m. Thus

$$m = \frac{\text{size of the image}}{\text{size of the object}} = \frac{h_2}{h_1}$$

Linear magnification for a convex lens:



CONVEX LENS

The linear magnification of a lens can be expressed in terms of object distance u and image distance v.

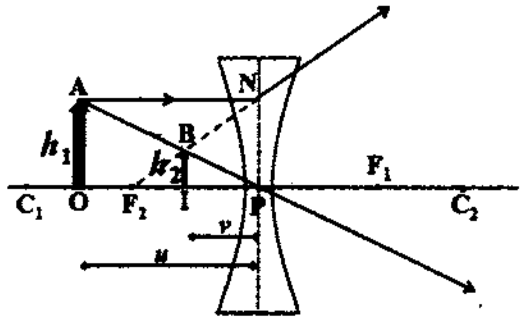
ΔAOP and ΔBIP are similar triangles.

$$\therefore \frac{BI}{AO} = \frac{IP}{OP}$$

$$\frac{-h_2}{+h_1} = \frac{+v}{-u} \qquad \Rightarrow -\frac{h_2}{h_1} = -\frac{v}{u}$$

$$\therefore m = \frac{h_2}{h_1} = \frac{v}{u}$$

Linear magnification for a concave lens:



CONCAVE LENS

ΔAOP and ΔBIP are similar triangles.

$$\therefore \frac{BI}{AO} = \frac{IP}{OP}$$

$$\frac{+h_2}{+h_1} = \frac{-v}{-u} \qquad \Rightarrow \frac{h_2}{h_1} = \frac{v}{u}$$

$$\therefore m = \frac{h_2}{h_1} = \frac{v}{u}$$

Linear magnification in terms of *u* and *f*:

$$\because \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Multiplying both sides by *u*, we get

$$\frac{1}{v}\cdot u - \frac{1}{u}\cdot u = \frac{1}{f}\cdot u \qquad \Rightarrow \frac{u}{v} - 1 = \frac{u}{f}$$

$$\frac{u}{v} = 1 + \frac{u}{f} = \frac{f+u}{f}$$

$$\therefore m = \frac{v}{u} = \frac{f}{f+u}$$

Linear magnification in terms of *v* and *f*:

$$\because \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

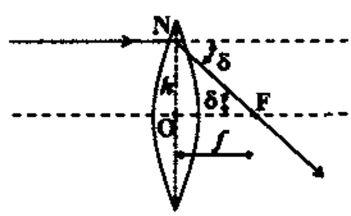
Multiplying both sides by *v*, we get

$$\frac{1}{v}\cdot v - \frac{1}{u}\cdot v = \frac{1}{f}\cdot v \qquad \Rightarrow 1 - \frac{v}{u} = \frac{v}{f}$$

$$\frac{v}{u} = 1 - \frac{v}{f} = \frac{f-v}{f}$$

$$\therefore m = \frac{v}{u} = \frac{f-v}{f}$$

Power of lens: The power of a lens is defined as the tangent of the angle at which it converges or diverges a beam of light falling at unit distance from the optical centre.



CONVEX LENS

Smaller the focal length of the lens, more is the ability to bend light rays and greater is its power.

Or

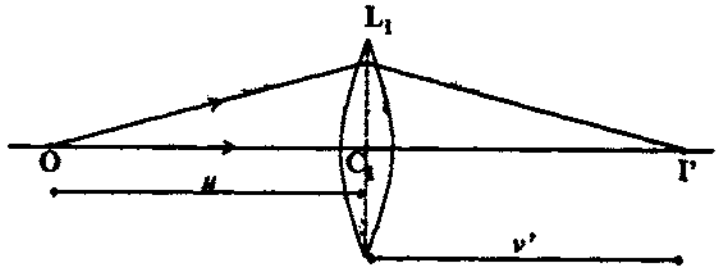
The power of a lens may also be defined as the reciprocal of its focal length

$$\text{Clearly } \tan\delta = \frac{h}{f}$$

$$\text{If } h = 1, \quad \text{then } \tan\delta = \frac{1}{f} \qquad \Rightarrow P = \frac{1}{f}$$

$$\text{If } f = 1m, \quad \text{then } P = \frac{1}{1m} = 1\text{dioptre (D)}$$

Combination of thin lenses in contact:

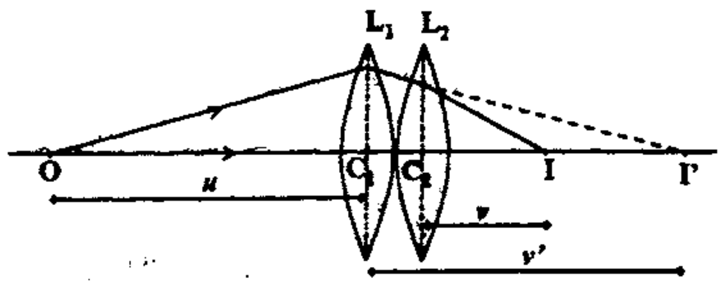


CONVEX LENS

Let L_1 and L_2 be two thin lenses of focal length f_1 and f_2 respectively. Let O be a point object on the principal axis of the lens system.

The first lens L_1 will form real image I' of O. Using thin lens formula we have

$$\frac{1}{f_1} = \frac{1}{v'} - \frac{1}{u} \text{ ----------------- (1)}$$



CONVEX LENSES

The image I' acts as the virtual object ($u = v'$) for the second lens L_2 which finally forms its real image I at distance v. Thus

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v'} \text{ ----------------- (2)}$$

Adding equation (1) and equation (2) we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v'} - \frac{1}{u} + \frac{1}{v} - \frac{1}{v'}$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} \text{ ----------------- (3)}$$

If f is the equivalent focal length, then

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \text{ ----------------- (4)}$$

From equation (3) and equation (4) we find that

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

$\therefore$ Equivalent Power, $P = P_1 + P_2$

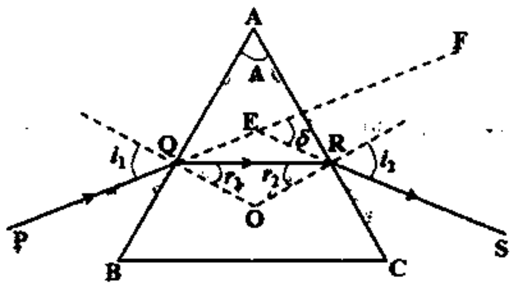
For n thin lenses in contact, we have

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots\frac{1}{f_n}$$

$\therefore$ Equivalent Power, $P = P_1 + P_2 + P_3 + \ldots P_n$

Prism: A prism is a transparent medium bounded by the three plane faces. Out of the three faces, one is grounded and the other two are polished. The polished faces are called refracting faces. The angle between the refracting faces is called angle of prism, or the refracting angle. The third face is called base of the prism.

Refraction of light through a Prism:



The figure shows the cross section of a triangular prism ABC, placed in air. Let A be the refracting angle of the prism. A ray of light PQ incident on the refracting face AB, gets refracted along QR and emerges along RS. The angle of incidence and refraction at the two faces are i_1, r_1, r_2 and i_2 respectively. The angle between the incident ray PQ and the emergent ray RS is called angle of deviation, δ.

In the quadrilateral AQOR, we have

$$A + \angle AQO + O + \angle ARO = 360^\circ$$
$$\text{Or } A + 90^\circ + O + 90^\circ = 360^\circ$$
$$\text{Or } A + O + 180^\circ = 360^\circ$$
$$\text{Or } A + O = 360^\circ - 180^\circ$$
$$\text{Or } A + O = 180^\circ \text{ ---------------- (1)}$$

In the triangle QOR, we have

$$r_1 + O + r_2 = 180^\circ \text{ ---------------- (2)}$$

Equating equation (1) and equation (2) we have

$$r_1 + O + r_2 = A + O$$
$$\text{Or } r_1 + r_2 = A \text{ ---------------- (3)}$$

In the triangle EOR, we have

$$\angle REF = \angle EQR + \angle ERQ$$
$$\text{Or } \delta = (i_1 - r_1) + (i_2 - r_2)$$
$$\text{Or } \delta = (i_1 + i_2) - (r_1 + r_2)$$

From equation (3) we have

$$\delta = (i_1 + i_2) - A$$
$$\text{Or } i_1 + i_2 = \delta + A \text{ ---------------- (4)}$$

At minimum deviation,

$$r_1 = r_2 = r \quad \text{and} \quad i_1 + i_2 = i$$

$\therefore$ From equation (3) we have $2r = A$ or $r = \frac{A}{2}$

And from equation (4) we have $2i = \delta + A$ or $i = \frac{\delta + A}{2}$

The refractive index is $\mu = \frac{\sin i}{\sin r}$

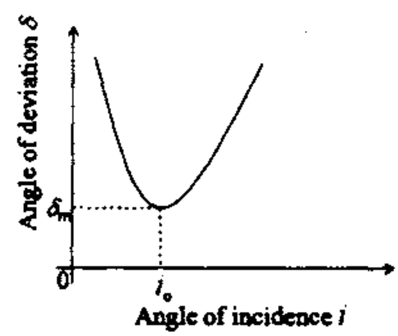
$$\therefore \mu = \frac{\sin \frac{\delta + A}{2}}{\sin \frac{A}{2}}$$

Angle of deviation in a prism (δ): The angle between the emergent ray RS and the direction of the incident ray PQ is called the angle of deviation, δ.

Angle of minimum deviation in a prism (δ_m): The angle between the emergent ray RS and the direction of the incident ray PQ is called the angle of deviation δ. At minimum deviation δ_m, the refracted ray inside the prism becomes parallel to its base.

Plot of angle of deviation (δ) versus angle of incidence (i) for a triangular prism:

As the angle of incidence i gradually increases, the angle of deviation δ decreases, reaches a minimum value δ_m and then increases. δ_m is called the angle of minimum deviation. It will be seen from the graph that there is only one angle of incidence i_o for which the deviation is a minimum.



Deviation produced by thin Prism:

If μ is the refractive index of the prism material, then according to snell's law,

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{i_1}{r_1} \qquad \Rightarrow i_1 = \mu r_1$$

Similarly $\mu = \frac{\sin i_2}{\sin r_2} = \frac{i_2}{r_2} \qquad \Rightarrow i_2 = \mu r_2$

Now $\delta + A = i_1 + i_2$

$\delta + A = \mu r_1 + \mu r_2 = \mu(r_1 + r_2)$

$\delta + A = \mu A$

Or $\delta = \mu A - A$

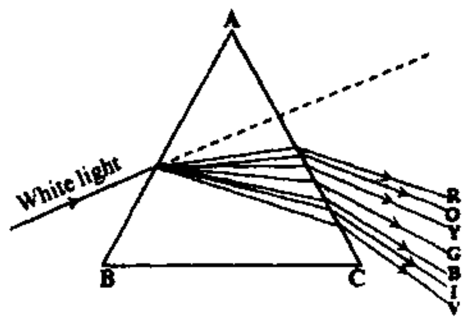
Or $\delta = (\mu - 1) A$

Thus the deviation δ produced by a thin prism depends upon the refractive index μ of the prism material and the angle of the prism A.

Note:

$\delta = (\mu - 1) A$ for small A up to 10°

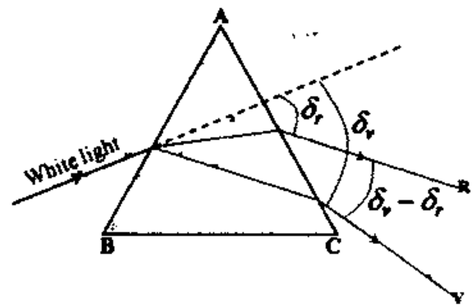
$\delta = (i_1 + i_2) - A$ for A greater than 10°



Dispersion of white light: Dispersion is the splitting of white light into its constituent colours. This band of colours of light is called its spectrum.
In the visible region of spectrum, the spectral lines are seen in the order from violet to red. The colours are given by the word VIBGYOR (Violet, Indigo, Blue, Green, Yellow, Orange and Red)

Dispersion takes place because the refractive index of the material of the prism is different for different colours (wavelengths). The deviation and hence the refractive index is more for violet rays of light than the corresponding values for red rays of light.

Angular dispersion: If δ_v and δ_r are the deviations produced for the violet and red rays and μ_v and μ_r are the corresponding refractive indices of the material of the small angled prism then,



For violet light, $\delta_v = (\mu_v - 1)A$
For red light, $\delta_r = (\mu_r - 1)A$

$\therefore \delta_v - \delta_r = (\mu_v - 1)A - (\mu_r - 1)A$

$\delta_v - \delta_r = (\mu_v - \mu_r)A$

The angular separation between the two extreme colours (violet and red) in the spectrum is called the angular dispersion.
$\therefore$ Angular dispersion = $\delta_v - \delta_r$

Angular dispersion depends upon
(i) angle of the prism and
(ii) nature of the material of the prism

Dispersive power: Dispersive power is the ability of the prism material to cause dispersion. It is defined as the ratio of the angular dispersion to the mean deviation.

$$\therefore \text{Dispersive power} = \frac{\text{angular dispersion}}{\text{mean deviation}}$$

$$\omega = \frac{\delta_v - \delta_r}{\delta} \qquad \Rightarrow \omega = \frac{\mu_v - \mu_r}{\mu - 1}$$

Note: The dispersive power ω of a prism depends only on the nature of the material of the prism. However, angular dispersion and mean deviation both depend on the nature of prism material and the angle of prism.

Scattering of light: When a beam of light falls on an atom, it causes the electrons in the atom to vibrate. The vibrating electrons, in turn, re-emit light in all directions. This process is called scattering.
According to Rayleigh law, the intensity of scattered light (I) varies inversely as the fourth power of the wavelength of light i.e.,

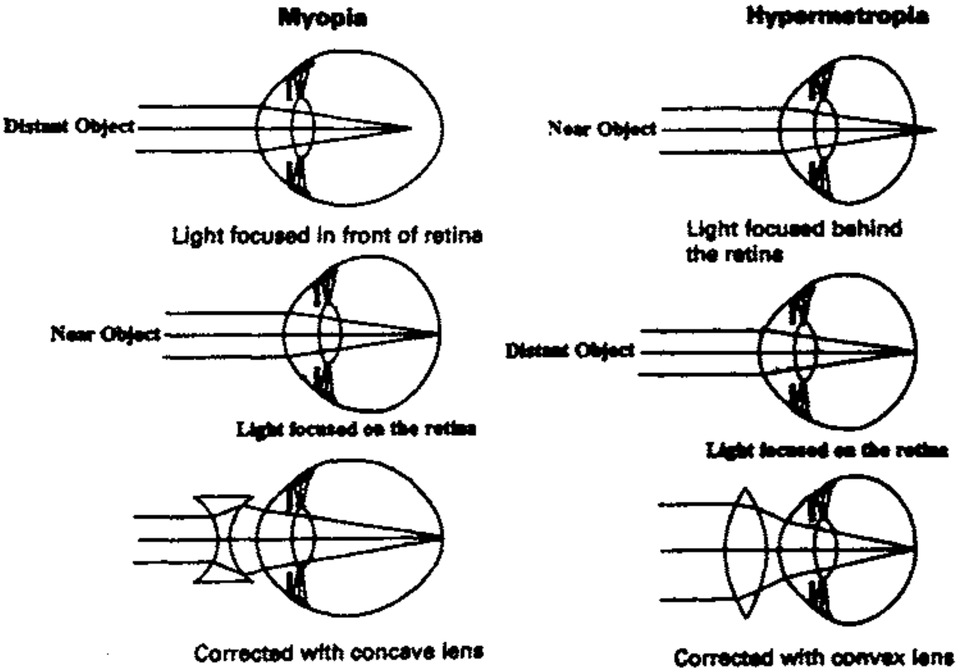
$$I \propto \frac{1}{\lambda^4}$$

Blue colour of the sky: As sunlight travels through the earth's atmosphere, it gets scattered by the atmospheric particles. Light of shorter wavelengths is scattered much more than light of longer wavelengths. Hence, the bluish colour predominates in a clear sky, since blue has a shorter wavelength than red and is scattered much more strongly.

Reddish appearance of the sun at sunrise or sunset: At sunrise or sunset, the sun's rays have to pass through a larger distance in the atmosphere. Most of the blue and other shorter wavelengths are removed by scattering. The least scattered light reaching our eyes, therefore, the sun looks reddish. This explains the reddish appearance of the sun and full moon near the horizon.

The Human eye: The closest point at which the object is seen most clearly without strain is called the near point of the eye. This limiting distance is known as, least distance of distinct vision (D). For an adult with normal eye, this distance is taken to be 25 cm by convention.

Defects of vision and their correction:

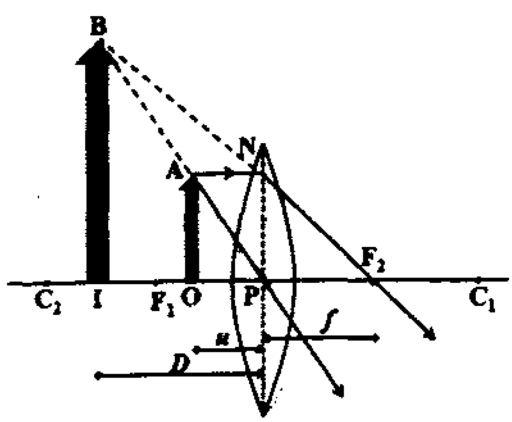


In addition to myopia and hypermetropia, there are also other types of defects in human eye. These are presbyopia and astigmatism.

Simple Microscope or Magnifying Glass: A simple microscope or a magnifying glass is just a convex lens of short focal length, held closed to the eye.

Principle: A simple microscope is based on the principle that a converging lens can form magnified images when the object is inside the focal length of the lens. The image formed is virtual, erect and magnified.

When image is formed at the near point: When an object OA is placed between the focus F_1 and the optical centre P of the convex lens, a virtual, erect and magnified image IB is formed behind the object. The image is seen most clearly when it is at the near point.



$$\text{Magnification } m = \frac{v}{u} = \frac{f-v}{f} = 1 - \frac{v}{f}$$

Here $v = -D$

$$\therefore m = 1 - \frac{-D}{f} = 1 + \frac{D}{f}$$

When image is formed at infinity: When an object OA is placed at the focus F_1 of the convex lens, a virtual, erect and magnified image IB is formed behind the object.

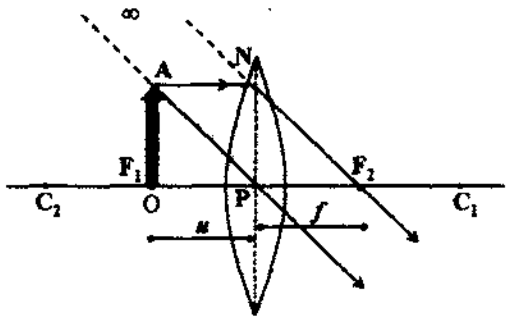
Magnification $m = \frac{v}{u} = \frac{f - v}{f} = 1 - \frac{v}{f}$

Here $v = \infty = -D$

$$m = 1 - \frac{-D}{f} = 1 + \frac{D}{f}$$

$\because D >> f$ thus $\frac{D}{f} >> 1$

$$\therefore m = \frac{D}{f}$$



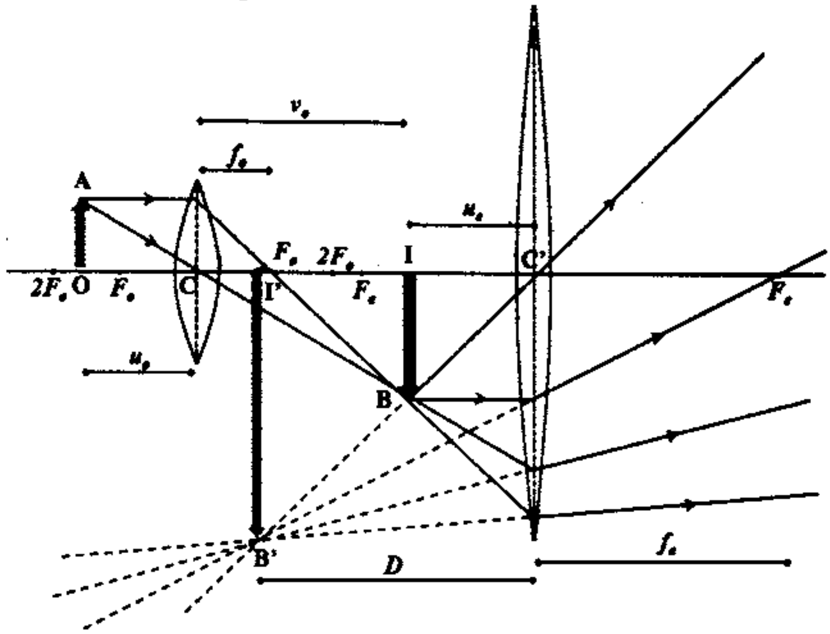
Note: The maximum angular magnification is produced when the image is at the near point and minimum angular magnification is produced when the image is at infinity.

Uses of magnifying glass:
(i) Jewellers and watch makers use the magnifying glass to obtain a magnified view of tiny parts of jewellery and watch parts.
(ii) In science laboratories, a magnifying glass is used for reading vernier scales etc.

Compound Microscope: A compound microscope makes use of two converging lenses. Therefore, its magnifying power is much greater than that of the simple microscope. The lens nearer to the object is called the objective lens and forms a real image of the object. The lens through which the final image is viewed is called the eyepiece. The image formed by the objective lens becomes the object for the eyepiece.

Principle: A compound microscope is based on the principle that a converging lens can form magnified images in the following two ways
(i) When the object is inside the focal length of the lens, the image formed is virtual, erect and magnified as in a simple microscope.
(ii) When the object is between the focal length f_o and $2f_o$ from the lens, the image formed is real, inverted and magnified.

When image is formed at near point:



Magnification $m = \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$

Since the focal length of the objective lens is very small, $u_o \approx f_o$. Again the focal length of the eyepiece is also very short so that $v_o \approx L$ where L is equal to the length of the microscope tube.

$$\therefore m = \frac{L}{f_o}\left(1+\frac{D}{f_e}\right)$$

When image is formed at infinity:

Magnification $m = \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$

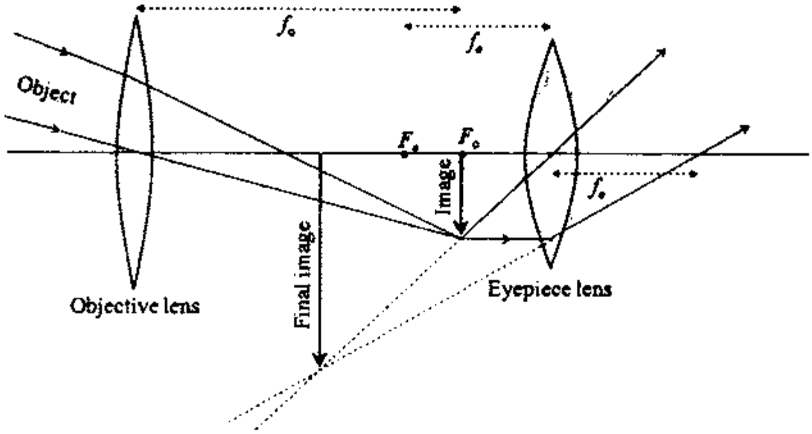
At infinity, D = ∞, so $\frac{D}{f_e} >> 1$

$$\therefore m = \frac{v_o}{u_o} \times \frac{D}{f_e}$$

Also $\quad m = \frac{L}{f_o} \times \frac{D}{f_e}$

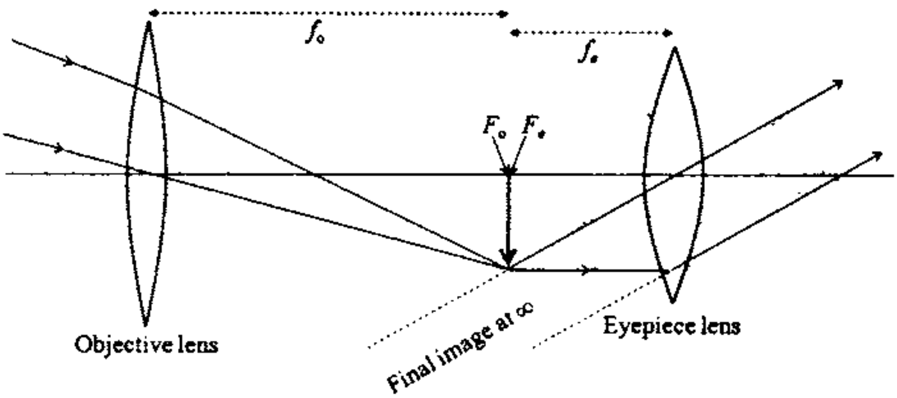
Astronomical Telescope: An astronomical telescope is used for seeing heavenly bodies such as sun and stars. The astronomical refracting telescope consists of an objective lens of long focal length f_o and an eyepiece lens of short focal length f_e. Both lenses are converging.

Final image at the near point:



Magnification $m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

Final image at infinity (i.e., normal adjustment):



Magnification $m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

At infinity, D = ∞, so $\frac{f_e}{D} << 1$

$$\therefore\ m = \frac{f_o}{f_e}$$

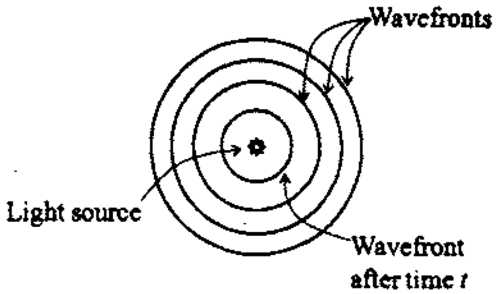
Length of tube of the telescope $L = f_o + f_e$

PHYSICAL OPTICS

Wavefront: Particles of a light wave which are equidistant from the light source and vibrate in the same phase constitute a wavefront.

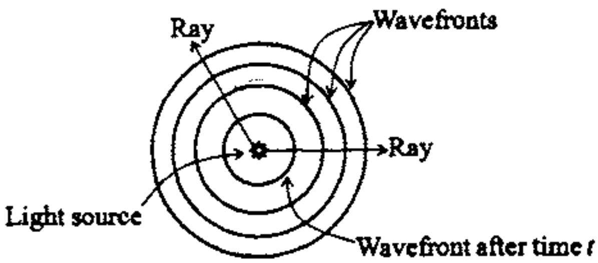
Or

A wavefront is the locus of points (wavelets) having the same phase of oscillations.

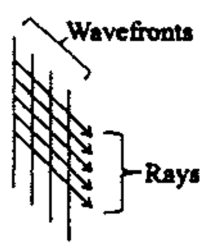


Wavelet: A wavelet is the point of disturbance due to propagation of light.

Ray: An arrow drawn perpendicular to a wavefront in the direction of propagation is called a 'ray'.

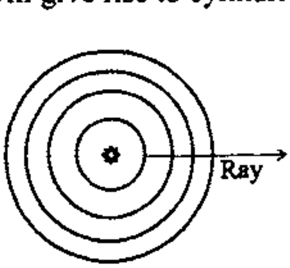


Spherical wavefronts

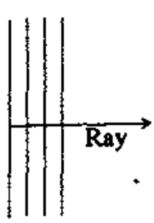


Plane wavefronts

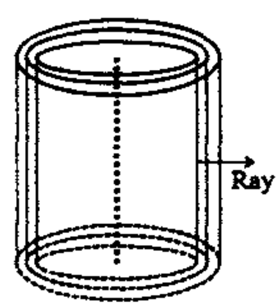
Types of wavefront: A point source of light at a finite distance in an isotropic medium (a medium in which the waves travel with the same speed in all directions) emits a spherical wave front (figure a). A point source of light in an isotropic medium at infinite distance will give rise to plane wavefront (figure b). A linear source of light such as a slit illuminated by a lamp, will give rise to cylindrical wavefront (figure c).



(a)

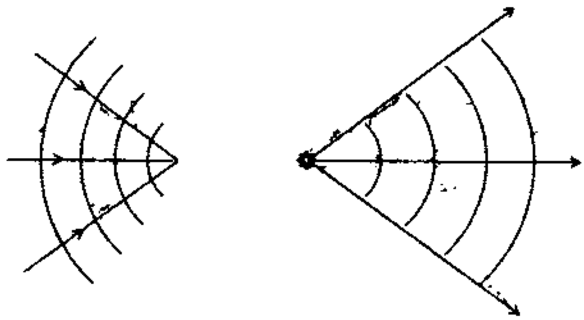


(b)



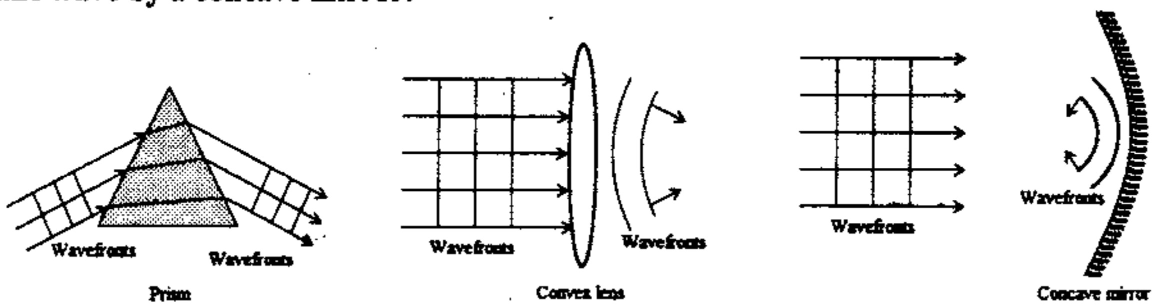
(c)

A spherical wavefront can be converging or diverging.



Converging wavefronts Diverging wavefronts

Refraction of a plane wave by a thin prism, a convex lens and Reflection of a plane wave by a concave mirror:

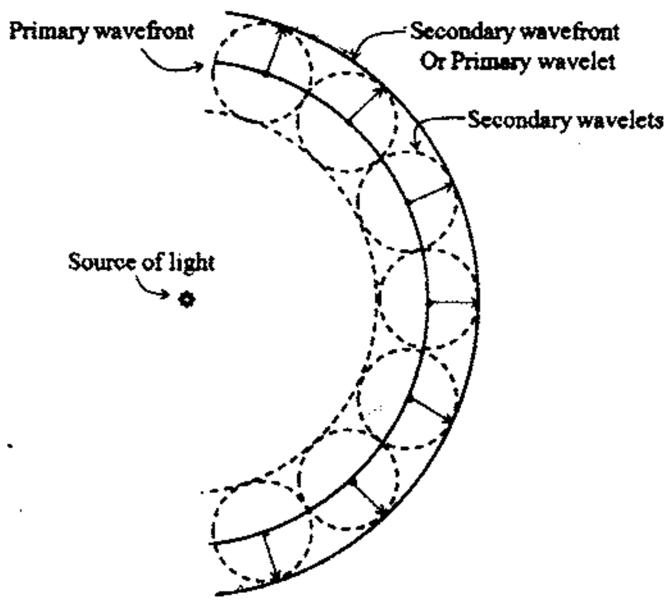


Huygen's principle:

According to Huygens' principle,

1) Each and every point on the given wavefront, called "primary wavefront," acts as a source of new disturbances called "secondary wavelets" that travel in all directions with the velocity of light in the medium.

2) A surface touching these secondary wavelets tangentially in the forward direction at any instant gives a new wavefront at that instant, which is known as the secondary wavefront.



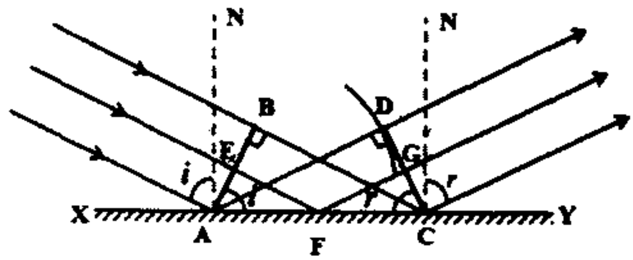
Note:

i) Every point in the primary wavefront serves as the source of spherical secondary wavelets, such that the primary wavelet at the later time is the envelope of these secondary wavelets.

ii) Backward wavefront is rejected. Why?

Amplitude of secondary wavelet is proportional to $\frac{1}{2}(1+\cos\theta)$. Obviously, for the backward wavelet $\theta = 180°$ and $(1+\cos\theta)$ is 0. So the amplitude of the backward wavefront is zero.

Laws of Reflection at a Plane Surface base on Huygens' Principle:
If c be the speed of light in air, t be the time taken by light to go from B to C or A to D or E to G through F, then



$$t = \frac{EF}{c} + \frac{FG}{c}$$

$$t = \frac{AF\sin i}{c} + \frac{FC\sin r}{c}$$

$$t = \frac{AF\sin i + FC\sin r}{c}$$

$$t = \frac{AF\sin i + (AC - AF)\sin r}{c}$$

$$t = \frac{AF\sin i + AC\sin r - AF\sin r}{c}$$

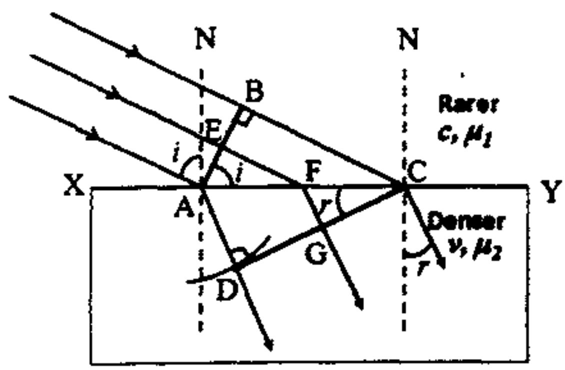
$$t = \frac{AC\sin r + AF(\sin i - \sin r)}{c}$$

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the reflected wavefront.
So, t should not depend upon AF. This is possible only if

$$\sin i - \sin r = 0.$$

i.e., $\sin i = \sin r$ or $i = r$

Laws of Refraction at a Plane Surface base on Huygens' Principle:
If c be the speed of light in air, v be the speed of light in the medium, t be the time taken by light to go from B to C or A to D or E to G through F, then



$$t = \frac{EF}{c} + \frac{FG}{v}$$

$$t = \frac{AF\sin i}{c} + \frac{FC\sin r}{v}$$

$$t = \frac{AF\sin i}{c} + \frac{(AC - AF)\sin r}{v}$$

$$t = \frac{AF\sin i}{c} + \frac{AC\sin r}{v} - \frac{AF\sin r}{v}$$

$$t = \frac{AC\sin r}{v} + AF\left(\frac{\sin i}{c} - \frac{\sin r}{v}\right)$$

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the refracted wavefront.
So, t should not depend upon AF. This is possible only if

$$\frac{\sin i}{c} - \frac{\sin r}{v} = 0$$

Or $$\frac{\sin i}{c} = \frac{\sin r}{v}$$

$$\text{Or } \frac{\sin i}{\sin r} = \frac{c}{v} = \mu$$

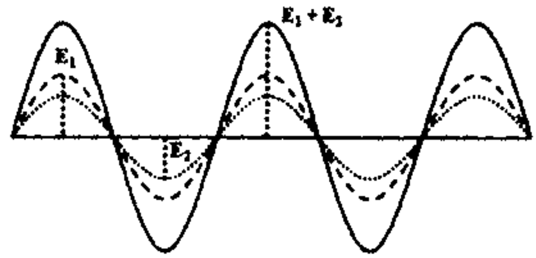
Coherent sources of light: Sources of light that emit continuous light waves having same wavelength, same frequency, and in same phase or having a constant phase difference are known as coherent sources of light.
Two independent sources of light cannot be coherent. Two coherent sources of light can be obtained from a single source of light, by reflection, refraction, etc.

Coherent sources can be produced by two methods:
1) By division of wavefront (Young's Double Slit Experiment, Fresnel's Biprism and Lloyd's Mirror)
2) By division of amplitude (Partial reflection or refraction)

Incoherent sources of light: When the phase difference between the two vibrating sources changes rapidly with time, the two sources are known as incoherent sources of light.

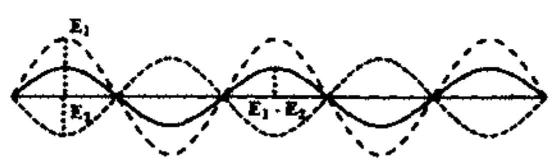
Interference of light waves: The redistribution of light energy on account of superposition of light waves from two coherent sources of light is known as interference of light.

Constructive interference: For constructive interference at a point, the phase difference between the two waves reaching that point should be zero or an even integral multiple of π. In other words, the path difference between the two waves reaching the point should be zero or an integral multiple of wavelength λ.



Constructive Interference $E = E_1 + E_2$

Destructive interference: For destructive interference at a point, the phase difference between the two waves reaching that particular point should be an odd integral multiple of π. In other words, the path difference between the two waves reaching the point should be an odd integral multiple of half-wavelength $\lambda/2$.



Destructive Interference $E = E_1 - E_2$

Theory of Interference of Waves Or Analytical treatment of interference:
The waves are with same speed, wavelength, frequency, time period, nearly equal amplitudes, travelling in the same direction with constant phase difference of ϕ. ω is the angular frequency of the waves, a, b are the amplitudes and E_1, E_2 are the instantaneous values of Electric displacement.

$$E_1 = a \sin \omega t$$
$$E_2 = b \sin (\omega t + \phi)$$

Applying superposition principle, the magnitude of the resultant displacement of the waves is

$$E = E_1 + E_2$$
$$E = a \sin \omega t + b \sin (\omega t + \phi)$$

$E = a \sin \omega t + b \sin \omega t \cos \phi + b \cos \omega t \sin \phi$
$E = (a + b \cos \phi) \sin \omega t + b \sin \phi \cos \omega t$

Putting $a + b \cos \phi = A \cos \theta$ ----------(1)
$b \sin \phi = A \sin \theta$ ----------(2)
We get $E = A \sin (\omega t + \theta)$
where E is the resultant displacement, A is the resultant amplitude and θ is the resultant phase difference.
Squaring and adding equation (1) and equation (2) we have

$$(A \cos \theta)^2 + (A \sin \theta)^2 = (a + b \cos \phi)^2 + (b \sin \phi)^2$$
$$A^2 (\cos^2 \theta + \sin^2 \theta) = a^2 + b^2 \cos^2 \phi + 2\,a\,b \cos \phi + b^2 \sin^2 \phi$$
$$A^2 = a^2 + 2\,a\,b \cos \phi + b^2 (\sin^2 \phi + \cos^2 \phi)$$
$$A^2 = a^2 + 2\,a\,b \cos \phi + b^2$$
$$\therefore A = \sqrt{a^2 + b^2 + 2\,a\,b \cos \phi}$$

Dividing equation (2) by equation (1) we have

$$\frac{A \sin\theta}{A \cos\theta} = \frac{b \sin\phi}{a + b\cos\phi}$$
$$\tan\theta = \frac{b \sin\phi}{a + b\cos\phi}$$
$$\therefore \theta = \tan^{-1}\left(\frac{b \sin\phi}{a + b\cos\phi}\right)$$

Intensity I is proportional to the square of the amplitude of the wave:

$$\because I \propto A^2$$
$$\therefore\ I \propto a^2 + b^2 + 2\,a\,b \cos\phi$$

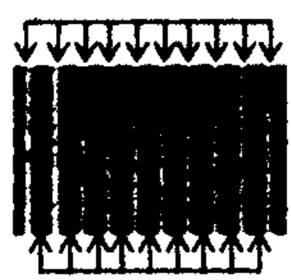
Condition for Constructive Interference of Waves:
For constructive interference, I should be maximum which is possible only if $\cos \phi = +1$

$$\because I \propto a^2 + b^2 + 2\,a\,b \cos\phi$$
$$I_{max} \propto a^2 + b^2 + 2\,a\,b = (a+b)^2$$
$$\therefore\ \phi = 2n\pi \quad \text{where } n = 0, 1, 2, 3, \ldots$$

And the corresponding path difference is

$$\frac{\Delta}{\lambda} = \frac{\phi}{2\pi} \qquad \Rightarrow \Delta = \frac{\lambda}{2\pi}\phi$$

Or $\Delta = \frac{\lambda}{2\pi} 2n\pi = n\lambda$



Condition for Destructive Interference of Waves:
For destructive interference, I should be minimum which is possible only if $\cos \phi = -1$

$$\because I \propto a^2 + b^2 + 2\,a\,b \cos\phi$$
$$I_{min} \propto a^2 + b^2 - 2\,a\,b = (a-b)^2$$
$$\therefore\ \phi = (2n-1)\pi \quad \text{where } n = 1, 2, 3, \ldots$$

And the corresponding path difference is

$$\frac{\Delta}{\lambda}=\frac{\phi}{2\pi} \qquad \Rightarrow \Delta=\frac{\lambda}{2\pi}\phi$$

Or $\quad \Delta=\frac{\lambda}{2\pi}(2n-1)\pi=(2n-1)\frac{\lambda}{2}$

Comparison of intensities of maxima and minima:

$$\frac{I_{max}}{I_{min}}=\frac{(a+b)^2}{(a-b)^2}=\frac{\left[b\left(\frac{a}{b}+1\right)\right]^2}{\left[b\left(\frac{a}{b}-1\right)\right]^2}$$

$$\frac{I_{max}}{I_{min}}=\frac{b^2\left(\frac{a}{b}+1\right)^2}{b^2\left(\frac{a}{b}-1\right)^2}=\frac{\left(\frac{a}{b}+1\right)^2}{\left(\frac{a}{b}-1\right)^2}=\frac{(r+1)^2}{(r-1)^2}$$

Where $r=\frac{a}{b}$ is the ratio of the amplitudes.

Resultant Intensity I_R:

$$A^2=a^2+b^2+2ab\cos\phi$$

$$\because I_1\propto a^2 \quad \text{and} \quad I_2\propto b^2$$

$$I_R=A^2=a^2+b^2+2ab\cos\phi$$

$$\therefore I_R=I_1+I_2+2\sqrt{I_1 I_2}\cos\phi$$

If $I_1=I_2=I_0$ and $I_R=I$ then

$I=I_0+I_0+2\sqrt{I_0 I_0}\cos\phi=2I_0+2I_0\cos\phi=2I_0(1+\cos\phi)$

$I=2I_0\left(2\cos^2\frac{\phi}{2}\right)=4I_0\cos^2\frac{\phi}{2}$

The intensity of bright points is $4I_0$ and at dark points is zero. Therefore the average intensity is $I_{av}=4I_0\left(\frac{1}{2}\right)=2I_0$

Note: $\because \sin^2\theta+\cos^2\theta=1$, Therefore the average value of $\sin^2\theta$ and $\cos^2\theta$ is $\frac{1}{2}$

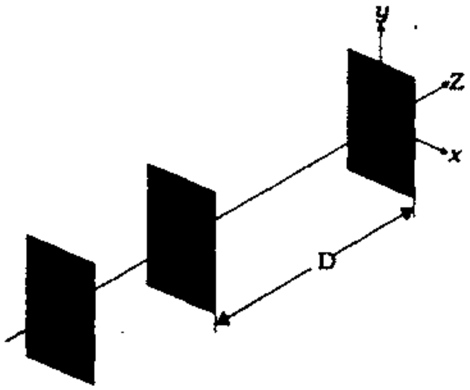
Relation between Intensity (*I*), Amplitude (*a*) of the wave and Width (*w*) of the slit:

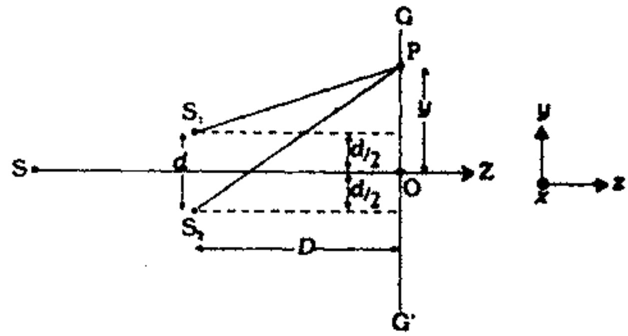
$$\because I\propto a^2$$

And $a\propto\sqrt{w} \qquad \Rightarrow a^2\propto w$

$$\therefore \frac{I_1}{I_2}=\frac{a_1^2}{a_2^2}=\frac{w_1}{w_2}$$

Young's Double Slit Experiment: The waves from S_1 and S_2 reach the point P with some phase difference, and hence path difference is





$\Delta = S_2P - S_1P$

$$\because (S_2P)^2 = [D^2 + \{y + (d/2)\}^2]$$

$$\text{And} \quad (S_1P)^2 = [D^2 + \{y - (d/2)\}^2]$$

$$(S_2P)^2 - (S_1P)^2 = [D^2 + \{y + (d/2)\}^2] - [D^2 + \{y - (d/2)\}^2]$$

$$(S_2P - S_1P)(S_2P + S_1P) = [D^2 + \{y^2 + (d/2)^2 + 2y(d/2)\}] - [D^2 + \{y^2 + (d/2)^2 - 2y(d/2)\}]$$

$$\Delta (S_2P + S_1P) = D^2 + y^2 + (d/2)^2 + 2y(d/2) - D^2 - y^2 - (d/2)^2 + 2y(d/2)$$

$$\Delta (S_2P + S_1P) = 2y(d/2) + 2y(d/2)$$

$$\Delta (S_2P + S_1P) = 2yd$$

If $d << D$ then $S_2P = S_1P \approx D$

$$\therefore \Delta (2D) = 2yd$$

$$\text{Or} \ . \Delta = \frac{yd}{D}$$

Positions of Bright Fringes:

For a bright fringe at P,

$$\Delta = \frac{yd}{D} = n\lambda \qquad \text{where } n = 0, 1, 2, 3, \ldots$$

$$y = \frac{nD\lambda}{d}$$

For $n = 0, \quad y_0 = 0$ central bright fringe (zero order bright fringe)

$n = 1, \quad y_1 = \frac{D\lambda}{d}$ first bright fringe (first order bright fringe)

$n = 2, \quad y_2 = \frac{2D\lambda}{d}$ second bright fringe (second order bright fringe)

$n = 3, \quad y_3 = \frac{3D\lambda}{d}$ third bright fringe (third order bright fringe)

$n = n, \quad y_n = \frac{nD\lambda}{d}$ n^{th} bright fringe (n^{th} order bright fringe)

Positions of Dark Fringes:

For a dark fringe at P,

$$\Delta = \frac{yd}{D} = (2n-1)\frac{\lambda}{2} \qquad \text{where } n = 1, 2, 3, \ldots$$

$$y = \frac{(2n-1)D\lambda}{2d}$$

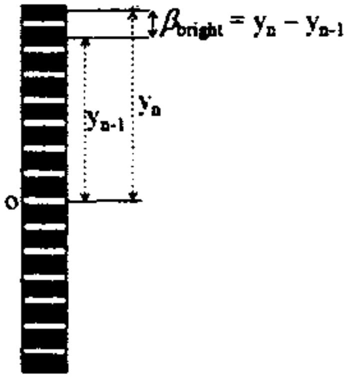
For $n = 1, \quad y_1 = \frac{D\lambda}{2d}$ first dark fringe (first order dark fringe)

$n = 2, \quad y_2 = \frac{3D\lambda}{2d}$ second dark fringe (second order dark fringe)

$n = 3, \quad y_3 = \frac{5D\lambda}{2d}$ third dark fringe (third order dark fringe)

$n = n, \quad y_n = \frac{(2n-1)D\lambda}{2d}$ n^{th} dark fringe (n^{th} order dark fringe)

Expression for Bright Fringe Width (β_{bright}):



$$\beta_{bright} = y_n - y_{n-1} = \frac{(2n-1)D\lambda}{2d} - \frac{\{2(n-1)-1\}D\lambda}{2d}$$

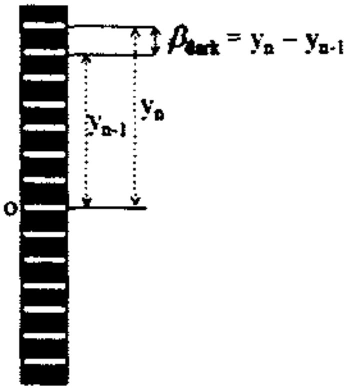
$$\beta_{bright} = \frac{(2n-1)D\lambda}{2d} - \frac{(2n-3)D\lambda}{2d}$$

$$\beta_{bright} = \frac{2nD\lambda}{2d} - \frac{D\lambda}{2d} - \left(\frac{2nD\lambda}{2d} - \frac{3D\lambda}{2d}\right)$$

$$\beta_{bright} = -\frac{D\lambda}{2d} + \frac{3D\lambda}{2d}$$

$$\therefore \beta_{bright} = \frac{D\lambda}{d}$$

Expression for Dark Fringe Width (β_{dark}):



$$\beta_{dark} = y_n - y_{n-1} = \frac{nD\lambda}{d} - \frac{(n-1)D\lambda}{d}$$

$$\beta_{dark} = \frac{nD\lambda}{d} - \left(\frac{nD\lambda}{d} - \frac{D\lambda}{d}\right)$$

$$\therefore \beta_{dark} = \frac{D\lambda}{d}$$

In Young's interference pattern, dark fringes are situated in-between bright fringes and vice-versa. All the bright and dark fringes are of equal width.

Note: When we use white light instead of monochromatic source of light then the interference fringes will be coloured and the central maximum will be white in colour.

Distribution of Intensity:

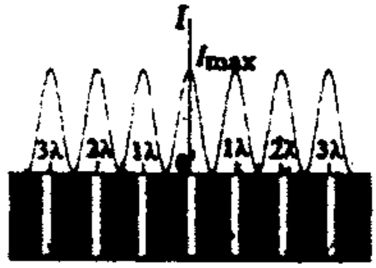
Suppose the two interfering waves have same amplitude say 'a', then

$I_{max} \alpha (a + a)^2$ i.e., $I_{max} \alpha 4a^2$

All the bright fringes have this same intensity.

$I_{min} = 0$

All the dark fringes have zero intensity.



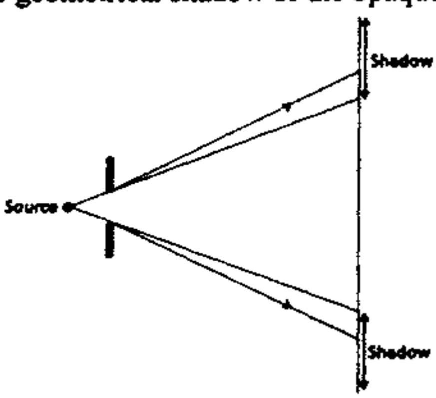
Interference pattern: The pattern of bright and dark fringes on the screen is called an interference pattern.

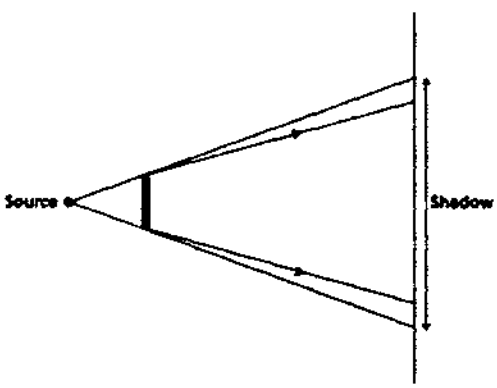
Conditions for sustained interference:

1) The two sources producing interference must be coherent.
2) The two interfering wave trains must have the same plane of polarisation.
3) The two sources must be very close to each other and the pattern must be observed at a larger distance to have sufficient width of the fringe.
4) The sources must be monochromatic. Otherwise, the fringes of different colours will overlap.
5) The two waves must be having same amplitude for better contrast between bright and dark fringes.

Diffraction of light:

The phenomenon of bending of light around the corners and the encroachment of light within the geometrical shadow of the opaque obstacles is called diffraction.





Types of diffraction:

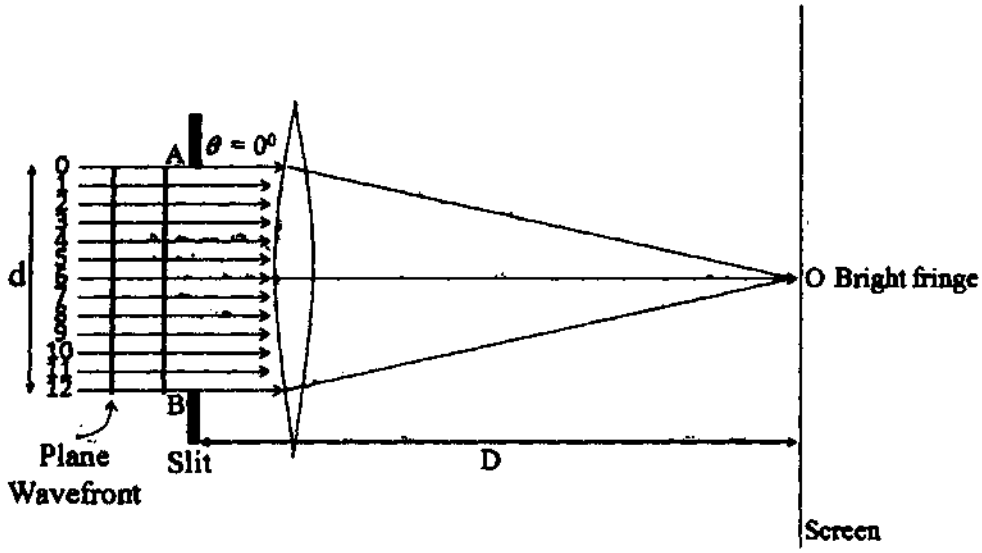
Diffraction of light is of two types viz

(1) Fraunhofer diffraction: In the Fraunhofer diffraction, the source and the screen are at infinite distances from the obstacle producing diffraction. Hence in this case the wavefront undergoing diffraction is plane. The diffracted rays which are parallel to one another are brought to focus with the help of a convex lens.

(2) Fresnel diffraction: In the Fresnel diffraction, the source and the screen are at finite distances from the obstacle producing diffraction. In such a case the wave front undergoing diffraction is either spherical or cylindrical.

Diffraction of light at a single slit (Fraunhofer diffraction):

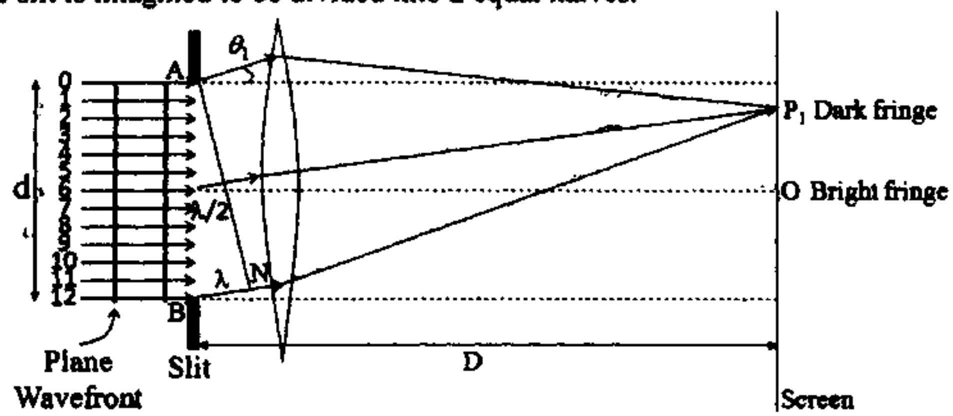
(1) At an angle of diffraction $\theta = 0°$:



The wavelets from the single wavefront reach the centre O on the screen in same phase and hence interfere constructively to give Central or Primary Maximum (Bright fringe).

(2) At an angle of diffraction $\theta = \theta_1$:

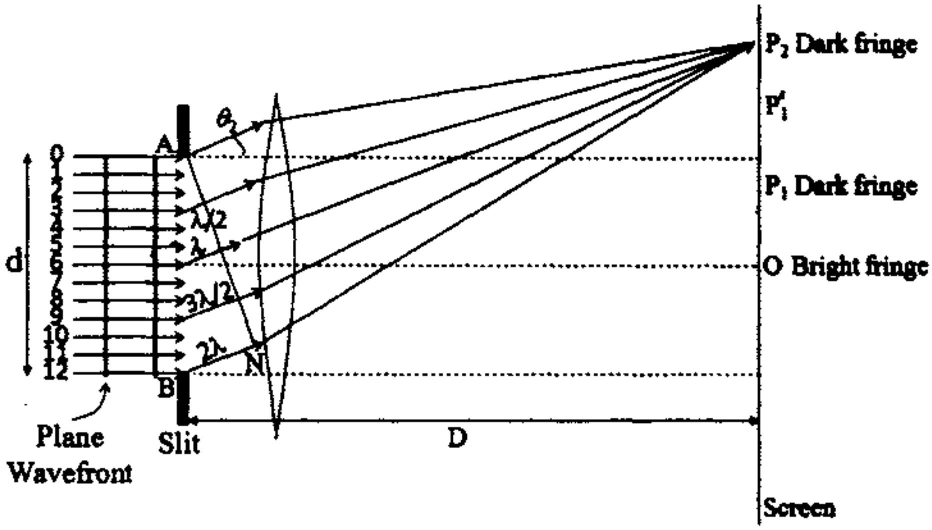
The slit is imagined to be divided into 2 equal halves.



The wavelets from the single wavefront diffract at an angle θ_1 such that BN is λ and reach the point P_1. The pairs (0,6), (1,7), (2,8), (3,9), (4,10), (5,11) and (6,12) interfere destructively with path difference λ/2 and give First Secondary Minimum (Dark fringe).

(3) At an angle of diffraction $\theta = \theta_2$:

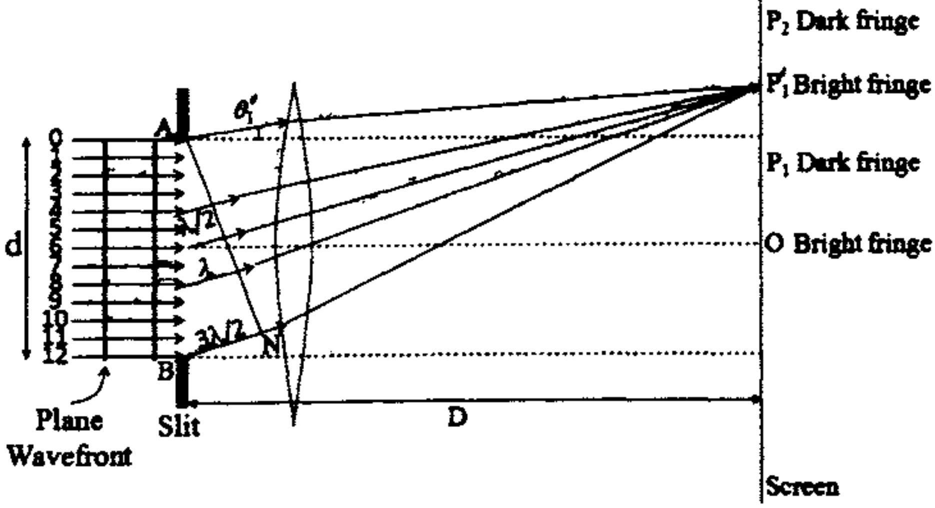
The slit is imagined to be divided into 4 equal parts.



The wavelets from the single wavefront diffract at an angle θ_2 such that BN is 2λ and reach the point P_2. The pairs (0,3), (1,4), (2,5), (3,6), (4,7), (5,8), (6,9), (7,10), (8,11) and (9,12) interfere destructively with path difference λ/2 and give Second Secondary Minimum (Dark fringe).

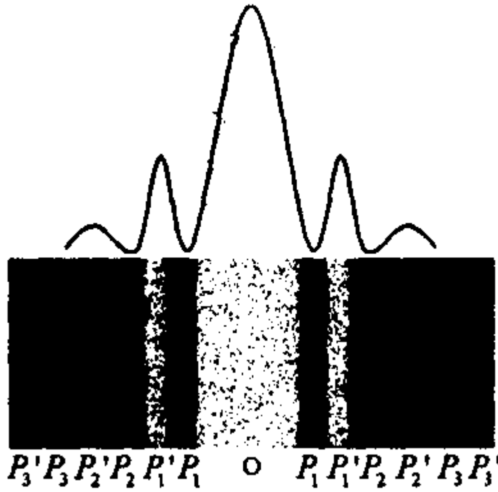
(4) At an angle of diffraction $\theta = \theta_1$':

The slit is imagined to be divided into 3 equal parts.



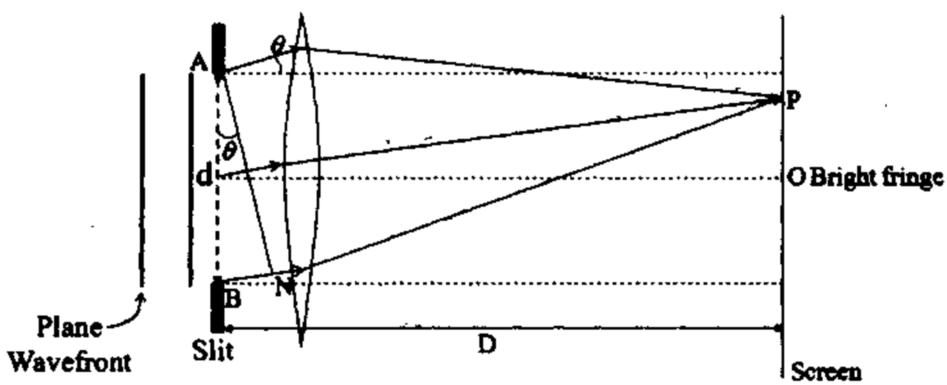
The wavelets from the single wavefront diffract at an angle θ_1' such that BN is 3λ/2 and reach the point P_1'. The pairs (0,8), (1,9), (2,10), (3,11) and (4,12) interfere constructively with path difference λ and (0,4), (1,5), (2,6), …… and (8,12) interfere destructively with path difference λ/2. However due to a few wavelets interfering constructively First Secondary Maximum (Bright fringe) is formed.

Diffraction at various angles: Central Maximum is the brightest fringe. Diffraction is not visible after a few order of diffraction.



Fraunhofer diffraction:

Theory: The path difference between the top wavelet and the bottom wavelet is BN.
If θ is the angle of diffraction and 'd' is the slit width, then BN = d sin θ



To establish the condition for secondary minima, the slit is divided into 2, 4, 6, … equal parts such that corresponding wavelets from successive regions interfere with path difference of λ/2. Or for n^{th} secondary minimum, the slit can be divided into 2n equal parts.

For θ_1, $d \sin \theta_1 = \lambda$

For θ_2, $d \sin \theta_2 = 2\lambda$

For θ_n, $d \sin \theta_n = n\lambda$

Since θ_n is very small,

$$d\,\theta_n = n\lambda$$

$$\theta_n = n\lambda / d \quad (n = 1, 2, 3, \ldots\ldots)$$

To establish the condition for secondary maxima, the slit is divided into 3, 5, 7, … equal parts such that corresponding wavelets from alternate regions interfere with path difference of λ. Or for n^{th} secondary maximum, the slit can be divided into (2n + 1) equal parts.

For θ_1', $d \sin \theta_1' = 3\lambda/2$

For θ_2', $d \sin \theta_2' = 5\lambda/2$

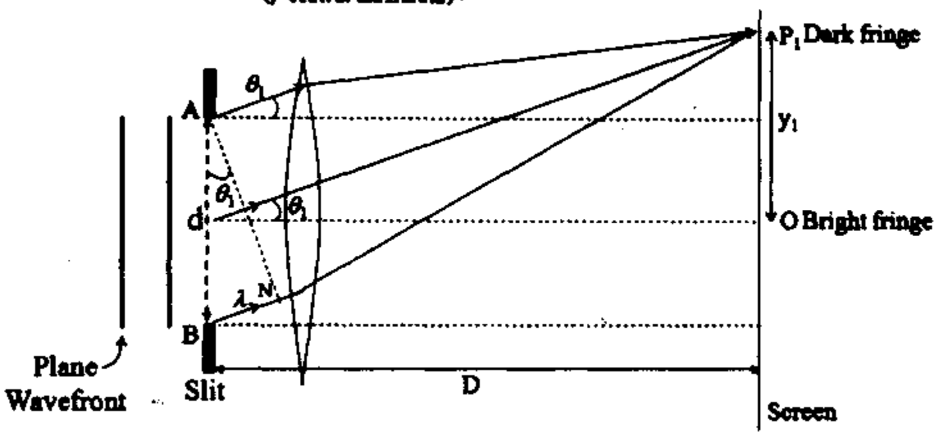
For θ_n', $d \sin \theta_n' = (2n + 1)\lambda/2$

Since θ_n' is very small,

$d\,\theta_n' = (2n + 1)\lambda / 2$

$\theta_n' = (2n + 1)\lambda / 2d \qquad (n = 1, 2, 3, \ldots\ldots)$

Width of Central Maximum ($\beta_{central\ maximum}$):



$\tan\theta_1 = y_1 / D$

or $\theta_1 = y_1 / D$ (since θ_1 is very small)

$d \sin\theta_1 = \lambda$

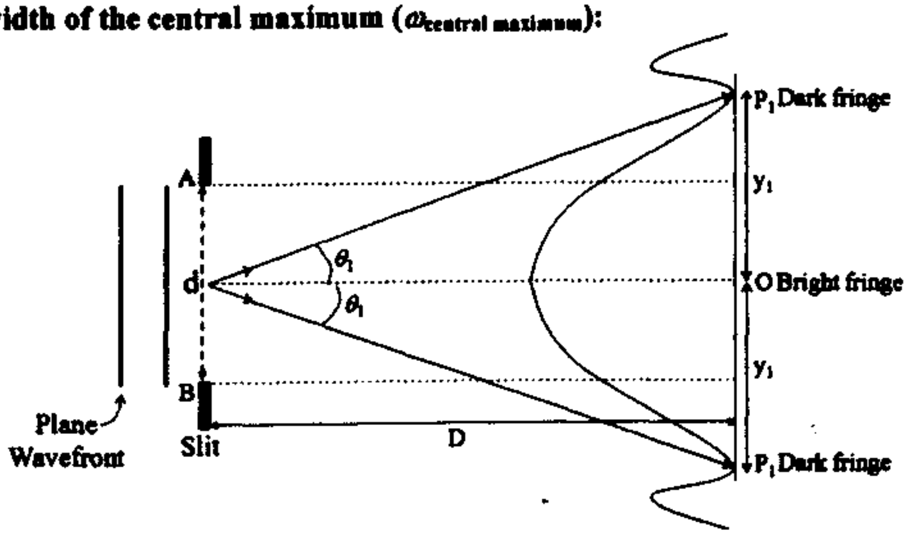
or $\theta_1 = \lambda / d$ (since θ_1 is very small)

$$\frac{y_1}{D} = \frac{\lambda}{d} \qquad \Rightarrow y_1 = \frac{D\lambda}{d}$$

Since the Central Maximum is spread on either side of O, the width is

$$\beta_{Central\ Maximum} = \frac{2D\lambda}{d}$$

Angular width of the central maximum ($\omega_{central\ maximum}$):



$\because d \sin\theta_1 = \lambda$

$$\sin\theta_1 = \frac{\lambda}{d} \qquad \Rightarrow \theta_1 = \sin^{-1}\left(\frac{\lambda}{d}\right)$$

$$\therefore \omega_{Central\ Maximum} = 2\theta_1$$

Or

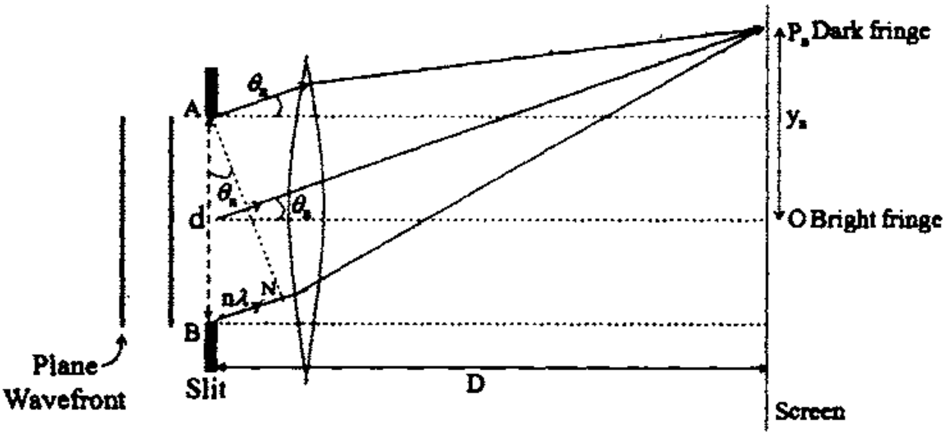
$$\because y_1 = \frac{\lambda D}{d} \quad \text{and} \quad y_1 = D\tan\theta_1$$

$$\therefore D\tan\theta_1 = \frac{\lambda D}{d} \qquad \Rightarrow \tan\theta_1 = \frac{\lambda}{d} \qquad \text{Or } \theta_1 = \tan^{-1}\left(\frac{\lambda}{d}\right)$$

$$\therefore \omega_{\text{Central Maximum}} = 2\theta_1$$

Note: Wavelength of light is $\lambda = d\tan\theta_1$ or $\lambda = d\sin\theta_1$

Position of the secondary minima from the central point (O): As shown in the figure



$\tan\theta_n = y_n / D$

or $\theta_n = y_n / D$ (since θ_n is very small)

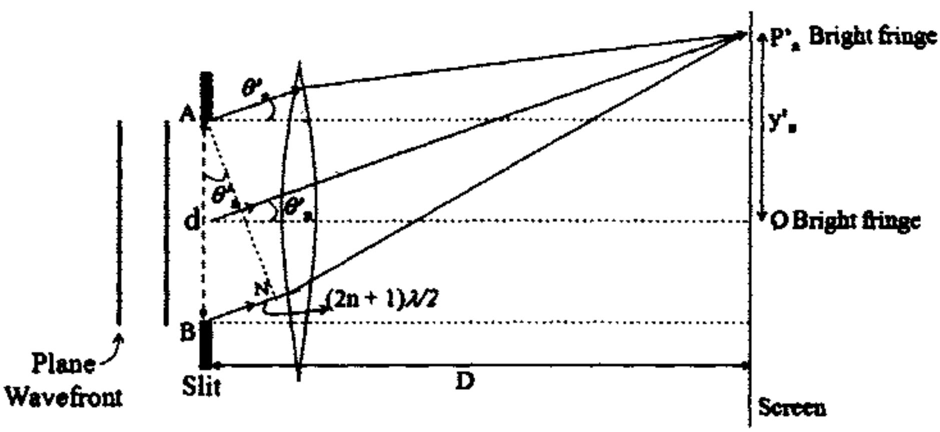
$d\sin\theta_n = n\lambda$

or $\theta_n = n\lambda / d$ (since θ_n is very small)

$$\frac{y_n}{D} = \frac{n\lambda}{d} \qquad \Rightarrow y_n = \frac{nD\lambda}{d}$$

$$\therefore y_2 = \frac{2D\lambda}{d} \qquad y_3 = \frac{3D\lambda}{d} \qquad y_4 = \frac{4D\lambda}{d}\ldots$$

Position of the secondary maxima from the central point (O): As shown in the figure



$\tan\theta_n' = y_n' / D$

or $\theta_n' = y_n' / D$ (since θ_n' is very small)

$d\sin\theta_n' = (2n + 1)\lambda/2$

or $\theta_n' = (2n + 1)\lambda / 2d$ (since θ_n' is very small)

$$\frac{y_n'}{D} = \frac{(2n+1)\lambda}{2d} \qquad \Rightarrow y_n' = \frac{(2n+1)\lambda D}{2d}$$

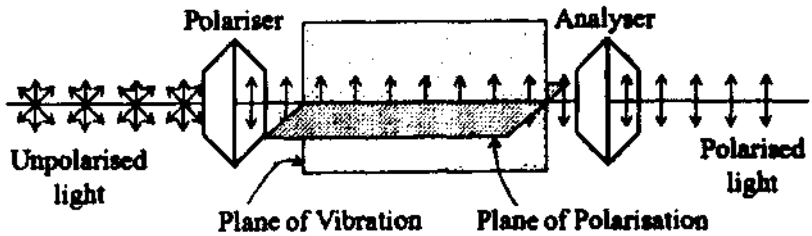
$$\therefore y_1' = \frac{3D\lambda}{2d} \qquad y_2' = \frac{5D\lambda}{2d} \qquad y_3' = \frac{7D\lambda}{2d} \ldots$$

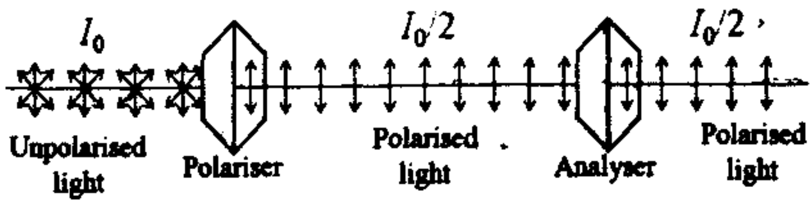
Difference between Interference and Diffraction:

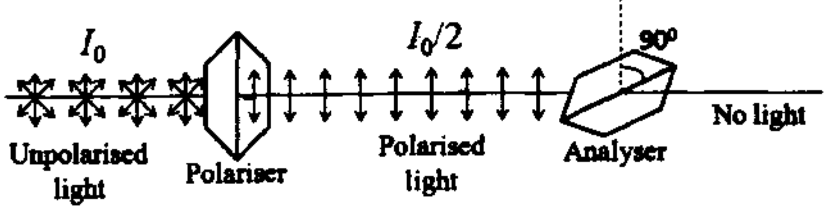
	Interference	Diffraction
1	Interference is due to the superposition of two different wave trains coming from coherent sources.	Diffraction is due to the superposition of secondary wavelets from the different parts of the same wavefront.
2	Fringe width is generally constant.	Fringes are of varying width.
3	All the maxima have the same intensity.	The maxima are of varying intensities.
4	There is a good contrast between the maxima and minima.	There is a poor contrast between the maxima and minima.

Polarization: The phenomenon of restricting the vibrations of light to a single plane is known as polarization of light.

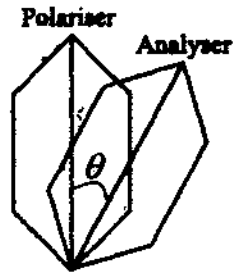
Polarization of Light Waves: When unpolarised light is incident on the polariser, the vibrations parallel to the crystallographic axis are transmitted and those perpendicular to the axis are absorbed. Therefore the transmitted light is plane (linearly) polarised.
The plane which contains the crystallographic axis and vibrations transmitted from the polariser is called plane of vibration.
The plane which is perpendicular to the plane of vibration is called plane of polarisation.







Malus' Law: According to law of Malus, when a beam of plane polarized light is incident on the analyser, the intensity of light I transmitted from the analyser is directly proportional to the square of the cosine of the angle θ between the planes of transmission of the polarizer and analyser. If the transmission axis of an analyser is oriented at an angle θ relative to the transmission axis of the polarizer, Malus' Law is given by



$$I = I_0 \cos^2 \theta$$

where I_0 is the average intensity of the light entering the analyser.

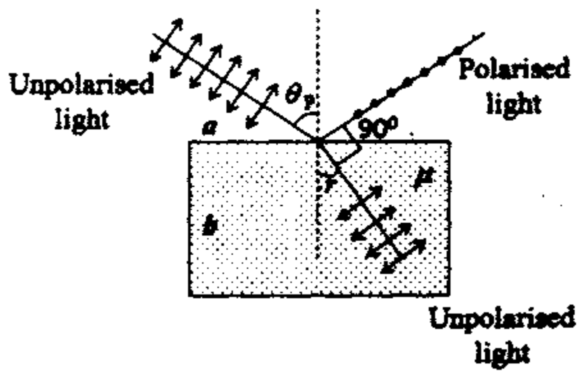
Case I : When $\theta = 0°$ or $180°$, $I = I_0$

Case II : When $\theta = 90°$, $I = 0$

Case III : When unpolarised light is incident on the analyser the intensity of the transmitted light is one-half of the intensity of incident light.
(Since average value of $\cos^2\theta$ is ½)

Polarisation by Reflection:

The incident light wave is made of parallel vibrations (π – components) on the plane of incidence and perpendicular vibrations (σ – components) perpendicular to plane of incidence. At a particular angle θ_P, the parallel components completely refracted, whereas the perpendicular components partially get refracted and partially get reflected, i.e., the reflected components are all in perpendicular plane of vibration and hence plane polarised. The intensity of transmitted light through the medium is greater than that of plane polarised (reflected) light.



Note:

The reflected light is completely plane-polarized in a direction perpendicular to the plane of incidence.

Polarizing angle: The angle of incidence at which the reflected light gets completely plane-polarized is called polarizing angle.

Brewster's Law:

According to Brewster's law, when unpolarized light is incident at polarizing angle on the interface separating air from a medium of refractive index μ, the reflected light is fully polarized provided the refractive index of the medium is equal to the tangent of the polarizing angle i.e.,

$${}_a\mu_b = \tan\theta_P$$

$$\theta_P + r = 90^0$$

$$r = 90^0 - \theta_P$$

$${}_a\mu_b = \frac{\sin\theta_P}{\sin r}$$

$$ {}_a\mu_b = \frac{\sin\theta_P}{\sin(90^0 - \theta_P)} $$

$$ {}_a\mu_b = \frac{\sin\theta_P}{\cos\theta_P} $$

$$ {}_a\mu_b = \tan\theta_P $$

Polaroids:

H – Polaroid is prepared by taking a sheet of polyvinyl alcohol (long chain polymer molecules) and subjecting to a large strain. The molecules are oriented parallel to the strain and the material becomes doubly refracting. When strained with iodine, the material behaves like a dichroic crystal.

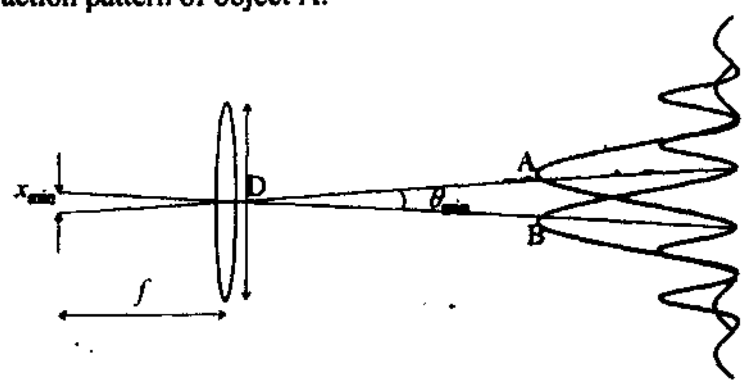
K – Polaroid is prepared by heating a stretched polyvinyl alcohol film in the presence of HCl (an active dehydrating catalyst). When the film becomes slightly darkened, it behaves like a strong dichroic crystal.

Uses of Polaroids:

1) Polaroid Sun Glasses
2) Polaroid Filters
3) For Laboratory Purpose
4) In Head-light of Automobiles
5) In Three – Dimensional Motion Pictures
6) In Window Panes
7) In Wind Shield in Automobiles

Resolving power: The power or ability of an optical instrument to produce distinctly separate images of two closely spaced objects is known as resolving power of the optical instrument.

Rayleigh Criterion: According to Rayleigh, two point objects A and B will be just resolved when the central maximum of diffraction pattern of object B lies on the first secondary minimum of diffraction pattern of object A.

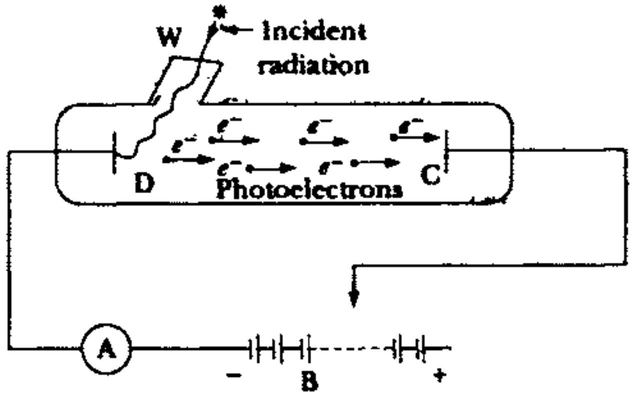


$$ x_{min} = f\theta_{min} = f\left(\frac{1.22\lambda}{D}\right) $$

Photoelectric Effect: When a clean metallic surface is irradiated by monochromatic light of suitable frequency, electrons are emitted. This phenomenon of ejection of electrons from metal surface is called photoelectric effect.
The ejected electrons are called photoelectrons and the current constituted by photoelectrons is known as photoelectric current.

Hertz and Lenard's Observations of Photoelectric Effect:

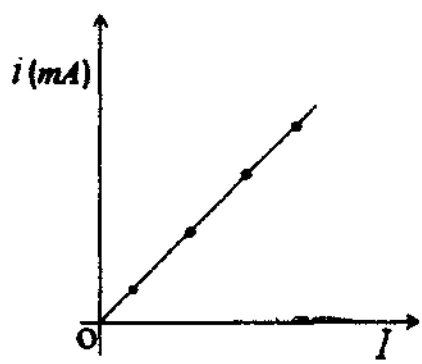
The experimental setup shown in the figure was used to study the photoelectric effect experimentally. In an evacuated glass tube, two zinc plates C and D are enclosed. Plate C acts as the collecting anode and plate D acts as the photosensitive plate. The two plates are connected to a battery B and an ammeter A. If the radiation is incident on the plate D through a quartz window W, electrons are ejected out of the plate and current flows in the circuit. The plate C can be maintained at desired potential (positive or negative) with respect to plate D. With the help of this apparatus, one can study the dependence of the photoelectric effect on the following factors:



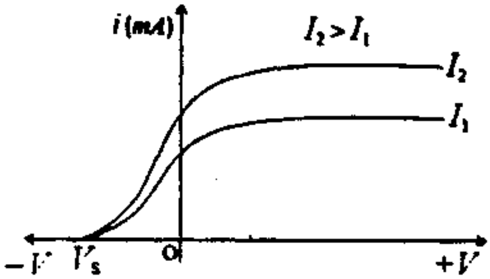
(1) Intensity of incident radiation,
(2) Potential difference between C and D, and
(3) Frequency of incident radiation.

Note: Glass transmits only visible and infra-red lights but not UV light. Quartz transmits UV light.

(1) Effect of Intensity of Incident Radiation: The electrode C, is made positive with respect to D. Keeping the frequency of light and the potentials fixed, the intensity of incident light (I) is varied and the photoelectric current (i) is measured in ammeter. The photoelectric current is directly proportional to the intensity of light. The photoelectric current gives an account of number of photoelectrons ejected per second.



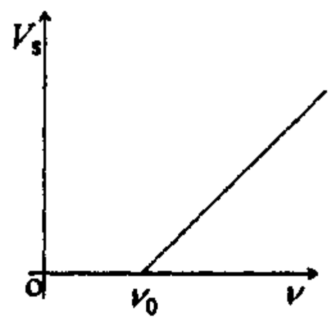
(2) Effect of Potential Difference between C and D: Keeping the intensity and frequency of light constant, the positive potential of C is increased gradually. The photoelectric current increases with increase in voltage till, for a certain positive potential of plate C, the current becomes maximum beyond which it does not increase for any increase in the voltage. This maximum value of the current is called saturation current.



Make the potential of C zero and make it increasingly negative. The photoelectric current decreases as the potential is increasingly made negative till, for a sharply defined negative potential V_S of C, the current becomes zero. The negative potential for which the photoelectric current becomes zero is called the cut-off or stopping potential (V_S).

When light of same frequency is used at higher intensity, the value of saturation current is found to be greater, but the stopping potential remains the same. Hence, the stopping potential is independent of intensity of incident light of same frequency.

(3) Effect of Frequency on Photoelectric Effect: The stopping potential V_S is found to be changing linearly with frequency of incident light, being more negative for high frequency. An increase in frequency of the incident light increases the kinetic energy of the emitted electrons, so greater retarding potential is required to stop them completely. For a given frequency ν, V_S measures the maximum kinetic energy E_{max} of photoelectrons that can reach plate C i.e.,



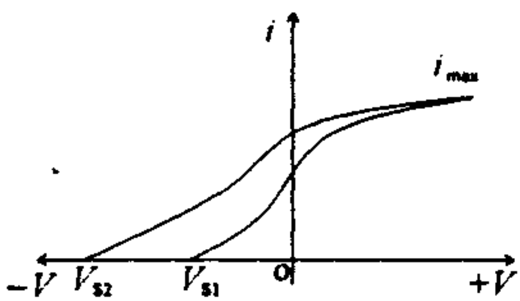
$$eV_S = \frac{1}{2}mv_{max}^2$$

where m is the mass of electron, e is charge of electron, and v_{max} is maximum velocity of electron. This means that the work done by stopping potential must be just equal to maximum kinetic energy of an electron.

The minimum value of frequency ν_0, of incident light, below which the emission stops, however large the intensity of light may be, is called as threshold frequency.

The effect of changing incident frequency ν can also be studied from the plot of photoelectric current versus potential applied across CD, keeping the intensity of incident radiation same.

From the graph shown in the figure, we see that i_{max} is same in all cases (for same intensity) and as frequency ν increases, V_S becomes more negative.



Note: The minimum retarding potential at which photoelectric current becomes zero is called stopping potential.

Laws of Photoelectric Emission:

(1) For a given substance, there is a minimum value of frequency of incident light called threshold frequency below which no photoelectric emission is possible, howsoever, the intensity of incident light may be.

(2) The number of photoelectrons emitted per second (i.e. photoelectric current) is directly proportional to the intensity of incident light provided the frequency is above the threshold frequency.

(3) The maximum kinetic energy of the photoelectrons is directly proportional to the frequency provided the frequency is above the threshold frequency.

(4) The maximum kinetic energy of the photoelectrons is independent of the intensity of the incident light.

(5) The process of photoelectric emission is instantaneous. i.e., as soon as the photon of suitable frequency falls on the substance, it emits photoelectrons in just 10^{-9} s.

(6) The photoelectric emission is one-to-one i.e., for every photon of suitable frequency one electron is emitted.

Note: For the same intensity of light and same potential difference (below the potential for saturation current), the photo electric current increases with the increase in frequency.

Failure of Classical theory to Explain Photoelectric effect:

(1) According to classical theory of electromagnetism, intensity of electromagnetic wave (light) is a function of amplitude of the wave, and the number of photoelectrons and their energy should depend upon intensity of light, which is contrary to the experimental results.

(2) According to wave theory, the transfer of energy from incident wave to the material (electrons) takes time. But as seen from the results, there is hardly any time lag in emission of photoelectrons. Hence emission of photoelectrons cannot be explained on the basis of wave theory of light.

Photon theory of light:

An electromagnetic wave travels in the form of discrete packets or bundles of energy called quanta. One quantum of light radiation is called a photon, which travels with the speed of light. Energy of a photon is

$$E = h\nu = \frac{hc}{\lambda}$$

where h is the Planck's constant, ν is the frequency of the radiation or photon, c is the speed of light (EM wave) and λ is the wavelength.

Properties of photons:

(1) A photon travels at a speed of light c in vacuum.

(2) It has zero rest mass i.e., the photon cannot exist at rest.

(3) The kinetic mass of a photon is, $m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$

(4) The momentum of a photon is, $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{hc}{c\lambda} = \frac{h}{\lambda}$

(5) Photons travel in a straight line.

(6) Energy of a photon depends upon frequency of the photon; so the energy of the photon does not change when photon travels from one medium to another.

(7) Wavelength of the photon changes in different media; so, velocity of a photon is different in different media.

(8) Photons are electrically neutral.
(9) Photons may show diffraction under given conditions.
(10) Photons are not deviated by magnetic and electric fields.

Work function (W_0) of a metal: Work function is defined as the maximum energy of the metal to hold the electron in its surface. It depends on the metal used. Its formula is $W_0 = h\nu_0$

Threshold frequency (ν_0): Threshold frequency is defined as the minimum frequency of EM radiation required to emit an electron from the surface of the metal.

Since $c = \lambda\nu$ then $\nu_0 = \frac{c}{\lambda_0}$.

Threshold wavelength (λ_0): Threshold wavelength is defined as the maximum wavelength of EM radiation required to emit an electron from the surface of the metal.

Einstein's photoelectric equation:

In the photoelectric effect, Einstein summarizes that some of the energy E imparted by a photon is actually used to release an electron from the surface of a metal (i.e., to overcome the binding force) and that the rest appears as the maximum kinetic energy of the emitted electron (photoelectron). It is given by

$$E = K_{max} + W_0$$

Where $E = h\nu$,

$$K_{max} = \frac{1}{2}mv_{max}^2$$

and $W_0 = h\nu_0$

$$\therefore h\nu = \frac{1}{2}mv_{max}^2 + h\nu_0$$

This equation is known as Einstein's photoelectric equation.

The above Einstein's photoelectric equation can also be expressed as

$$\frac{1}{2}mv_{max}^2 = h\nu - h\nu_0 = h(\nu - \nu_0) \text{——}(1)$$

$$\frac{1}{2}mv_{max}^2 = h(\frac{c}{\lambda} - \frac{c}{\lambda_0}) = hc(\frac{1}{\lambda} - \frac{1}{\lambda_0}) \text{——}(2)$$

$$eV_0 = hc(\frac{1}{\lambda} - \frac{1}{\lambda_0}) \text{——}(3)$$

Application of Photoelectric Effect:

(1) Automatic fire alarm
(2) Automatic burglar alarm
(3) Scanners in Television transmission
(4) Reproduction of sound in cinema film
(5) In paper industry to measure the thickness of paper
(6) To locate flaws or holes in the finished goods
(7) In astronomy
(8) To determine opacity of solids and liquids
(9) Automatic switching of street lights
(10) To control the temperature of furnace

(11) Photometry
(12) Beauty meter – To measure the fair complexion of skin
(13) Light meters used in cinema industry to check the light
(14) Photoelectric sorting
(15) Photo counting
(16) Meteorology

Dual Nature of Radiation and Matter:
Wave theory of electromagnetic radiations explained the phenomenon of interference, diffraction and polarization.
Quantum theory of electromagnetic radiations successfully explained the photoelectric effect, Compton effect, black body radiations, X- ray spectra, etc. Thus, radiations have dual nature. i.e., wave and particle nature.

de Broglie wave: According to de Broglie, a wave is associated with every moving particle. These waves are called de Broglie waves or matter waves.

Expression for de Broglie wave:

According to quantum theory, the energy of the photon is $E = h\nu = \frac{hc}{\lambda}$

According to Einstein's theory, the energy of the photon is $E = mc^2$

$$\therefore mc^2 = \frac{hc}{\lambda}$$

Or $$\lambda = \frac{hc}{mc^2} = \frac{h}{mc} = \frac{h}{p}$$

Where $p = mc$ is momentum of a photon.
If instead of a photon, we have a material particle of mass m moving with velocity v, then the equation becomes

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

which is the expression for de Broglie wavelength.

Conclusion:
1) de Broglie wavelength is inversely proportional to the velocity of the particle. If the particle moves faster, then the wavelength will be smaller and vice versa.
2) If the particle is at rest, then the de Broglie wavelength is infinite. Such a wave cannot be visualized.
3) de Broglie wavelength is inversely proportional to the mass of the particle. The wavelength associated with a heavier particle is smaller than that with a lighter particle.
4) de Broglie wavelength is independent of the charge of the particle.
5) Matter waves, like electromagnetic waves, can travel in vacuum and hence they are not mechanical waves.
6) Matter waves are not electromagnetic waves because they are not produced by accelerated charges.
7) Matter waves are probability waves, amplitude of which, gives the probability of existence of the particle at the point.

de Broglie wavelength of an electron:

$$\because \lambda = \frac{h}{mv} = \frac{h}{p}$$

$$K = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} \qquad \Rightarrow p^2 = 2mK \qquad \text{Or } p = \sqrt{2mK}$$

$$\therefore \lambda = \frac{h}{\sqrt{2mK}}$$

$$\text{Also } \lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda_e = \frac{6.63\times10^{-34}}{\sqrt{2\times(9.11\times10^{-31})\times(1.6\times10^{-19})V}} = \frac{12.3\times10^{-10}}{\sqrt{V}} = \frac{12.3}{\sqrt{V}}\,\text{Å}$$

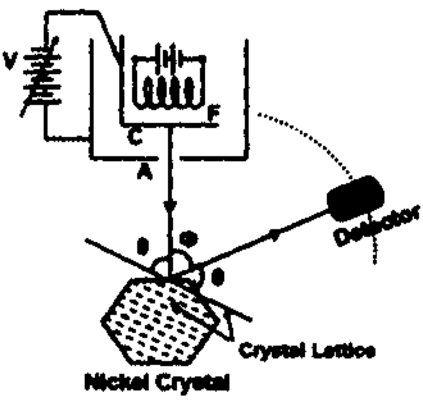
Davisson–Germer Experiment: The first experimental proof of the wave nature of electron was demonstrated in 1927 by two American physicists C. J. Davisson and L. H. Germer. The basis of their experiment was that since the wavelength of an electron is of the order of spacing of atoms of a crystal, a beam of electrons shows diffraction effects when incident on a crystal.

Observation: A beam of electrons emitted by the electron gun is made to fall on Nickel crystal cut along cubical axis at a particular angle.

The scattered beam of electrons is received by the detector which can be rotated at any angle.

The energy of the incident beam of electrons can be varied by changing the applied voltage to the electron gun.

Intensity of scattered beam of electrons is found to be maximum when angle of scattering is 50° and the accelerating potential is 54V.



$\theta + 50^0 + \theta = 180^0$

i.e. $\theta = 65°$

For Ni crystal, lattice spacing $d = 0.91$ Å

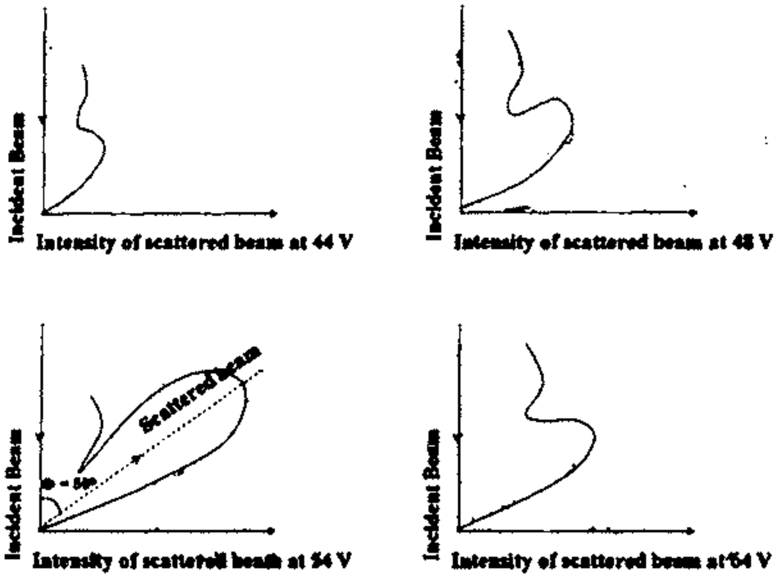
For first principal maximum, $n = 1$

Electron diffraction is similar to X-ray diffraction.

$\therefore$ Bragg's equation $2d\sin\theta = n\lambda$ gives

$2\,(0.91)\sin 65^0 = 1\,\lambda$

$\lambda = 1.65$ Å



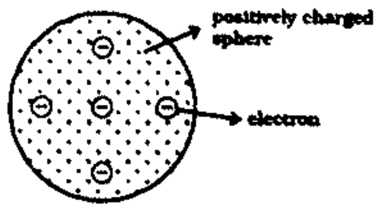
According to de Broglie's hypothesis,

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{12.27}{\sqrt{V}}\ \text{Å}$$

$\therefore$ de Broglie wavelength of moving electron at V = 54 volt is 1.67 Å which is in close agreement with 1.65 Å.

ATOMS AND NUCLEI



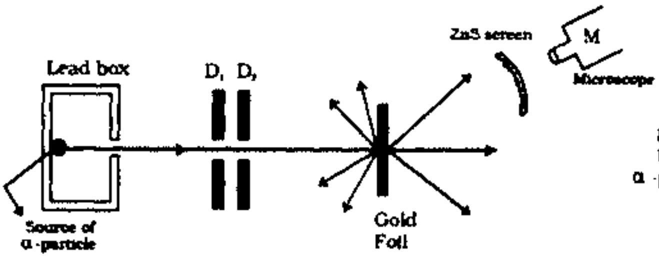
Thomson's model of atom: According to this model, an atom consists of homogenous positively charged sphere with tiny negatively charged electrons embedded throughout the sphere as shown in the figure. This model of the atom is also called 'plum pudding' model.

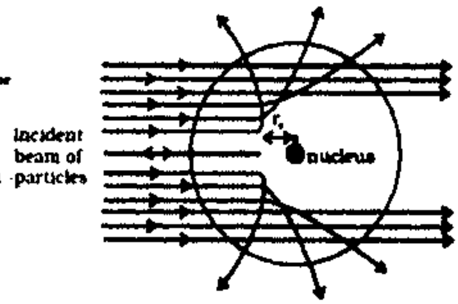
Limitations of Thomson's atomic model:

(1) It could not explain the origin of spectral lines of hydrogen and other atoms.

(2) It could not explain the large angle scattering of alpha particle.

Rutherford's Alpha Scattering Experiment: Alpha-particle is a nucleus of helium atom carrying a charge of '+2e' and mass equal to 4 times that of hydrogen atom. It travels with a speed nearly 10^4 ms^{-1} and is highly penetrating.

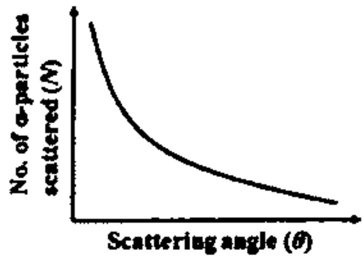


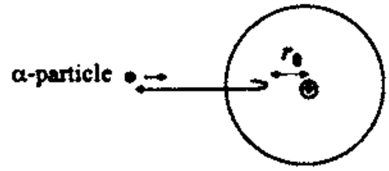


	Rutherford Experiment	Geiger & Marsden Experiment
Source of α-particle	Radon - $_{86}Rn^{222}$	Bismuth - $_{83}Bi^{214}$
Speed of α-particle	10^4 m/s	1.6×10^7 m/s
Thickness of Gold foil	10^{-6} m	2.1×10^{-7} m

Experimental fact

	Observation	Conclusion
1	Most of the α-particles passed straight through the gold foil.	It indicates that most of the space in an atom is empty.
2	Some of the α-particles were scattered by only small angles, of the order of a few degrees.	α-particles being positively charged and heavy compared to electron and could only be deflected by heavy and positive region in an atom. It indicates that all the positive charges and the mass of the atom is concentrated at the centre called 'nucleus'.
3	A few α-particles (1 in 9000) were deflected through large angles (even greater than 90°). Some of them even retraced their path. i.e. angle of deflection was 180°.	α-particles which travel towards the nucleus directly get retarded due to Coulomb's force of repulsion and ultimately comes to rest and then fly off in the opposite direction.





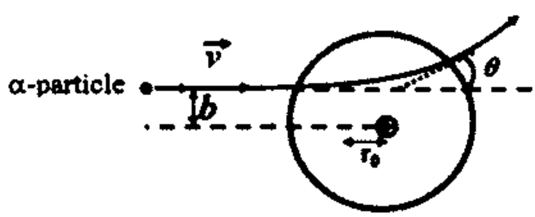
Distance of Closest Approach (Nuclear size): When the distance between α-particle and the nucleus is equal to the distance of the closest approach (r_0), the α-particle comes to rest.
At this point or distance, the kinetic energy of α-particle is completely converted into electric potential energy of the system.

Charge on the α - particle = $+2e$
Charge on a scattering nucleus = $+Ze$

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{r_0} \quad \{\because P.E = V(+2e)\}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0}\frac{4Ze^2}{mv^2}$$

Impact Parameter: The perpendicular distance of the velocity vector of the α-particle from the centre of the nucleus when it is far away from the nucleus is known as impact parameter.



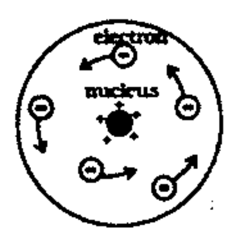
$$b = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2\cot\left(\frac{\theta}{2}\right)}{E} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2\cot\left(\frac{\theta}{2}\right)}{\frac{1}{2}mv^2}$$

(i) For large value of b, $\cot\left(\frac{\theta}{2}\right)$ is large and θ, the scattering angle is small. i.e. α-particles travelling far away from the nucleus suffer small deflections.

(ii) For small value of b, $\cot\left(\frac{\theta}{2}\right)$ is also small and θ, the scattering angle is large.
i.e. α-particles travelling close to the nucleus suffer large deflections.

(iii) For $b = 0$ i.e. α-particles directed towards the centre of the nucleus,
$\cot\left(\frac{\theta}{2}\right) = 0$ or $\frac{\theta}{2} = 90^0$ or $\theta = 180^0$. The α-particles retrace their path.

Rutherford's model of an atom: Based on the results of α-particle scattering experiment, Rutherford suggested the following picture of the atom.

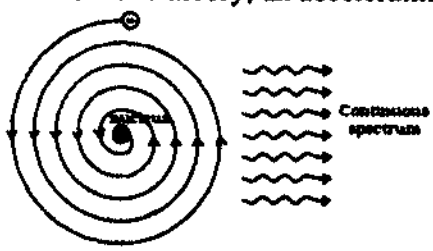


(1) Atom may be regarded as a sphere of diameter $10^{-10}m$, but whole of the positive charge of the atom is concentrated in a small central core called nucleus having diameter of about $10^{-14}m$ as shown in the figure.

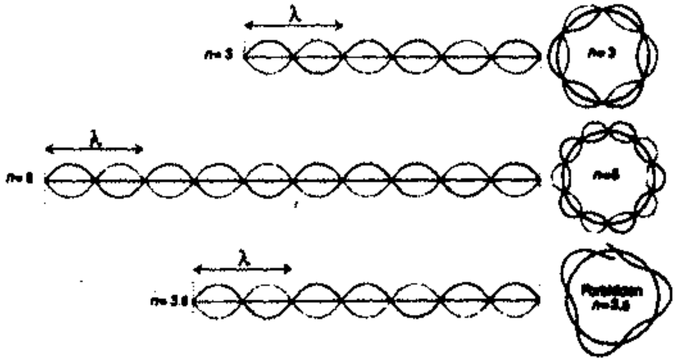
(2) The electrons in the atom were considered to be distributed around the nucleus in the empty space of the atom. If the electrons were at rest, they would be attracted by the nucleus. To overcome this, Rutherford suggested that the electrons are revolving around the nucleus in circular orbits, so that the centripetal force is provided by the electrostatic force of attraction between the electron and the nucleus.

(3) As the atom is electrically neutral, the total positive charge of the nucleus is equal to the total negative charge of the electrons in it.

Drawback of Rutherford's model of an atom: Based on the wave theory, an accelerating charge emits energy. Hence the electrons must emit the EM radiation as they revolve around the nucleus. As a result of the continuous loss of energy, the radii of the electron orbits will be decreased steadily. This would lead the electrons spiral and falls into the nucleus, hence the atom would collapse as shown in the figure.



Bohr's quantisation condition: A circular orbit can be taken to be a stationary energy state only if it contains an integral number of de Broglie wavelengths, i.e., it must have $2\pi r = n\lambda$ where $n = 1,2,3,\ldots$



Bohr's model of hydrogen atom: Neils Bohr in 1913, modified the Rutherford's atom model in order to explain the stability of the atom and the emission of sharp spectral lines. He proposed the following postulates:

(1) The electrons move only in certain circular orbits, called Stationary states or Energy Levels. When it is in one of these orbits, it does not radiate energy.

(2) The angular momentum (L) of the electron in the Stationary orbits is quantised i.e., it is an integral multiple of $\frac{h}{2\pi}$ where h is the plank's constant. Mathematically,

$$L = \frac{nh}{2\pi} \quad \text{and} \quad L = mvr$$

$$mvr = \frac{nh}{2\pi}$$

Where $n = 1, 2, 3,\ldots$ Principal quantum number
r = Radius of the orbit
m = Mass of the electron

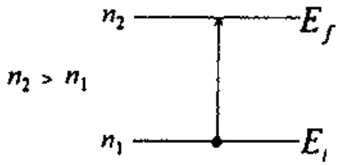
(3) Emission or absorption of radiation occurs only when an electron makes a transition from one orbit to another.
The Energy of the emitted or absorbed radiation is given by

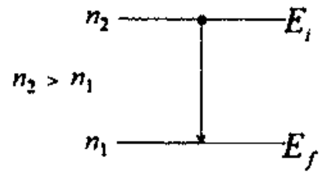
$$h\nu = |E_f - E_i|$$

Where h is the plank's constant, v is the frequency of the emitted or absorbed radiation, E_f is the final energy state and E_i is the initial energy state.

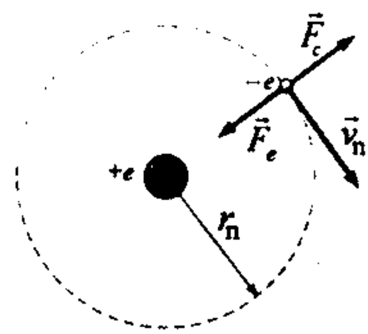
If $E_f > E_i$ $\Rightarrow$ Energy is absorbed

If $E_f < E_i$ $\Rightarrow$ Energy is emitted





Radius of the nth orbit (r_n): Consider one electron of charge $-e$ and mass m moves in a circular orbit of radius r_n around a positively charged nucleus with a velocity v_n as shown in the figure. The electrostatic force between electron and nucleus contributes the centripetal force as given in the relation



$$|\vec{F}_e| = |\vec{F}_c|$$

$$k\frac{e^2}{r_n^2} = \frac{mv_n^2}{r_n} \qquad \text{where } k = \frac{1}{4\pi\varepsilon_0}$$

$$mv_n^2 = k\frac{e^2}{r_n} - - - - - - (1)$$

From the Bohr's second postulate

$$mv_n r_n = \frac{nh}{2\pi}$$

By taking square of both side of the equation, we get

$$m^2 v_n^2 r_n^2 = \frac{n^2 h^2}{4\pi^2} - - - - - - (2)$$

Dividing the equation (2) by equation (1), gives

$$\frac{m^2 v_n^2 r_n^2}{mv_n^2} = \frac{\frac{n^2h^2}{4\pi^2}}{k\frac{e^2}{r_n}} = \frac{n^2h^2}{4\pi^2} \times \frac{r_n}{ke^2}$$

$$mr_n = n^2 \frac{h^2}{4\pi^2 ke^2}$$

$$\text{Or } r_n = n^2 \frac{h^2}{4\pi^2 mke^2}$$

The Bohr's radius a_0 is defined as the radius of the most stable (lowest) orbit or ground state ($n = 1$) in the hydrogen atom

$$a_0 = r_1 = \frac{h^2}{4\pi^2 mke^2} = \frac{(6.63\times10^{-34})^2}{4\times(3.14)^2(9.11\times10^{-31})(9\times10^9)(1.6\times10^{-19})^2} = 5.29\times10^{-11} m$$

or $a_0 = 0.53\,\text{Å}$

$$\because r_n = n^2 \frac{h^2}{4\pi^2 mke^2} = n^2(a_0)$$

$$\therefore r_n = n^2(0.53)\,\text{Å}$$

$r_2 = 2^2(0.53)\,\text{Å} = 2.12\,\text{Å}$ $\qquad$ $r_3 = 3^2(0.53)\,\text{Å} = 4.77\,\text{Å}$

$r_4 = 4^2(0.53)\,\text{Å} = 8.48\,\text{Å}$ $\qquad$ $r_5 = 5^2(0.53)\,\text{Å} = 13.25\,\text{Å}$

Speed of electron in the nth orbit (v_n): From the Bohr's second postulate we have

$$mv_n r_n = \frac{nh}{2\pi} \qquad \text{or } v_n = \frac{nh}{2\pi\, mr_n}$$

$$v_n = \frac{nh}{2\pi\, m\left(n^2 \frac{h^2}{4\pi^2 mke^2}\right)}$$

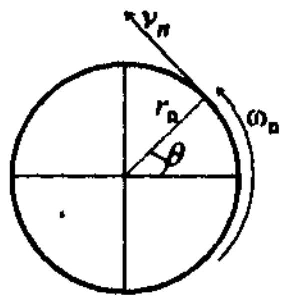
$$v_n = \frac{2\pi ke^2}{nh} = \frac{1}{n}\left(\frac{2\pi ke^2}{h}\right) = \frac{c}{n}\left(\frac{2\pi ke^2}{ch}\right) = \alpha\cdot\frac{c}{n}$$

Where $\alpha = \frac{2\pi ke^2}{ch} = \frac{1}{137}$ is called the fine structure constant.

$$\therefore v_n = \frac{1}{137}\frac{c}{n}$$

$$v_1 = \frac{1}{137}\frac{c}{(1)}, \qquad v_2 = \frac{1}{137}\frac{c}{(2)}, \qquad v_3 = \frac{1}{137}\frac{c}{(3)},\ldots$$

Frequency of electron in the nth orbit (f_n):



$$\because v_n = \omega_n r_n = (2\pi f_n) r_n$$

$$f_n = \frac{v_n}{2\pi r_n} = \frac{2\pi ke^2}{nh(2\pi r_n)}$$

$$\therefore f_n = \frac{ke^2}{nh\, r_n}$$

$$f_1 = \frac{ke^2}{(1)h\, r_1}, \qquad f_2 = \frac{ke^2}{(2)h\, r_2}, \qquad f_3 = \frac{ke^2}{(3)h\, r_3}, \qquad f_4 = \frac{ke^2}{(4)h\, r_4},\ldots$$

Energy of an electron in the nth orbit (E_n): The total energy of the electron is the sum of its potential energy and kinetic energy in its orbit.

The K.E of the electron in the nth orbit $= \frac{1}{2}mv_n^2$

$$KE = \frac{1}{2}k\frac{e^2}{r_n} = \frac{ke^2}{2r_n} \qquad \text{[Using equation (1)]}$$

The P.E of the electron in the nth orbit $= k\dfrac{(+e)}{r_n}(-e)$

$$PE = -k\frac{e^2}{r_n}$$

The total energy of the electron in the nth orbit = K.E + P.E

$$E_n = \frac{ke^2}{2r_n} - \frac{ke^2}{r_n} = -\frac{ke^2}{2r_n}$$

$$E_n = -\frac{ke^2}{2\left(\dfrac{n^2h^2}{4\pi^2mke^2}\right)} = -\frac{2\pi^2mk^2e^4}{n^2h^2}$$

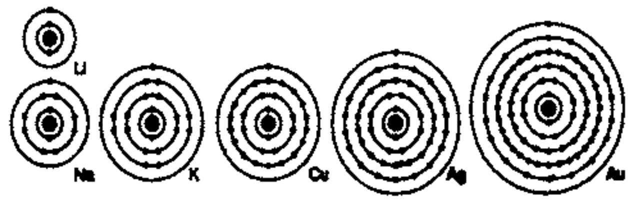
$$E_n = -\frac{2\pi^2mk^2e^4}{h^2}\left(\frac{1}{n^2}\right)$$

For Hydrogen like atom we have

$$r_n = n^2\frac{h^2}{4\pi^2mkZe^2}$$

$$v_n = \frac{2\pi kZe^2}{nh}$$

$$E_n = -\frac{2\pi^2mk^2Z^2e^4}{n^2h^2}$$

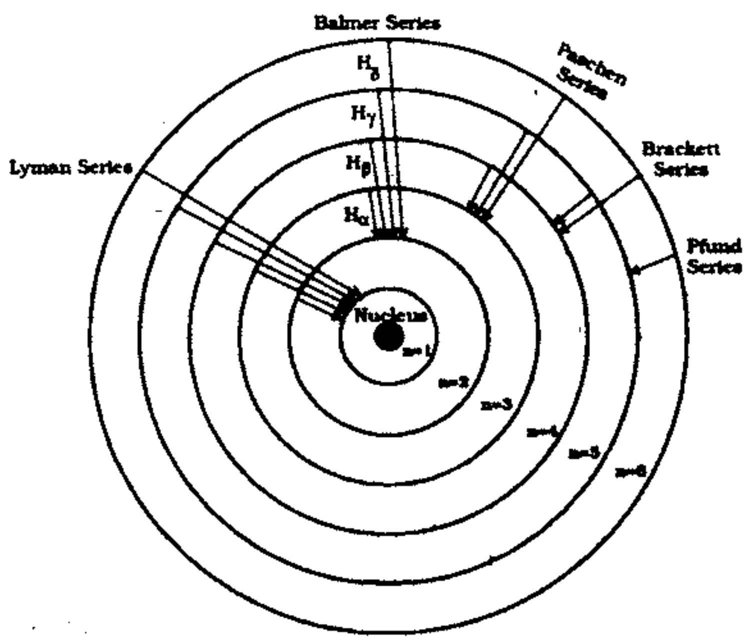


Where Z is the proton number.

Spectral series of hydrogen atom: Whenever an electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference in energies of the two levels is emitted as a radiation of particular wavelength. It is called a spectral line. The following are the spectral series of hydrogen atom.

(1) Lyman series

When the electron jumps from any of the outer orbits to the first orbit, the spectral lines emitted are in the ultraviolet region of the spectrum and they are said to form a series called Lyman series



(2) Balmer series

When the electron jumps from any of the outer orbits to the second orbit, we get a spectral series called the Balmer series. All the lines of this series in hydrogen have their wavelength in the visible region.

(3) Paschen series

This series consists of all wavelengths which are emitted when the electron jumps from outer most orbits to the third orbit. This series is in the infrared region.

(4) Brackett series

This series consists of all wavelengths which are emitted when the electron jumps from outer most orbits to the fourth orbit. This series is in the infrared region.

(5) Pfund series

This series consists of all wavelengths which are emitted when the electron jumps from outer most orbits to the fifth orbit. This series is in the infrared region.

Note: The total number of emission lines from n^{th} (n_2) state to lower state (n_1) are

$$\frac{(n_2-n_1)(n_2-n_1+1)}{2}$$

The total number of emission lines from n^{th} ($n_2 = n$) state to ground state ($n_1 = 1$) are

$$\frac{(n_2-n_1)(n_2-n_1+1)}{2}=\frac{(n-1)(n-1+1)}{2}=\frac{n(n-1)}{2}$$

Frequency of spectral line: Let the electron makes a transition from the higher energy level n_i to the lower energy level n_f, the difference of energy appears in the form of a photon. The frequency ν of the emitted photon is given by

$$h\nu=\left|E_f-E_i\right|$$

$\because E_i > E_f$ i.e., there is an emission of photon

$$h\nu=E_i-E_f=E_{n_i}-E_{n_f}$$

$$h\nu=-\frac{2\pi^2mk^2e^4}{h^2}\left(\frac{1}{n_i^2}\right)+\frac{2\pi^2mk^2e^4}{h^2}\left(\frac{1}{n_f^2}\right)$$

$$h\nu=\frac{2\pi^2mk^2e^4}{h^2}\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$$

$$\nu=\frac{2\pi^2mk^2e^4}{h^3}\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$$

$$\frac{\nu}{c}=\frac{2\pi^2mk^2e^4}{ch^3}\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$$

$\because c=\lambda\nu \qquad \Rightarrow \frac{\nu}{c}=\frac{1}{\lambda}=\bar{\nu}$

$\bar{\nu}$ is called the wave number

$$\bar{\nu}=\frac{2\pi^2mk^2e^4}{ch^3}\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)=R_H\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$$

$R_H=\frac{2\pi^2mk^2e^4}{ch^3}=1.0973\times10^7 m^{-1}$ is called the Rydberg constant

Lyman series $(n_f=1)$ $\quad \Rightarrow \bar{\nu}=\frac{1}{\lambda}=R_H\left(\frac{1}{1^2}-\frac{1}{n_i^2}\right)$ where $n_i=2,3,4,...$

Balmer series $(n_f = 2)$ $\Rightarrow \bar{\nu} = \frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$ where $n_i = 3,4,5,...$

Paschen series $(n_f = 3)$ $\Rightarrow \bar{\nu} = \frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$ where $n_i = 4,5,6,...$

Brackett series $(n_f = 4)$ $\Rightarrow \bar{\nu} = \frac{1}{\lambda} = R_H\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right)$ where $n_i = 5,6,7,...$

Pfund series $(n_f = 5)$ $\Rightarrow \bar{\nu} = \frac{1}{\lambda} = R_H\left(\frac{1}{5^2} - \frac{1}{n_i^2}\right)$ where $n_i = 6,7,8,...$

Energy level diagram for hydrogen atom:

$$\because E_n = -\frac{2\pi^2 mk^2e^4}{h^2}\left(\frac{1}{n^2}\right)$$

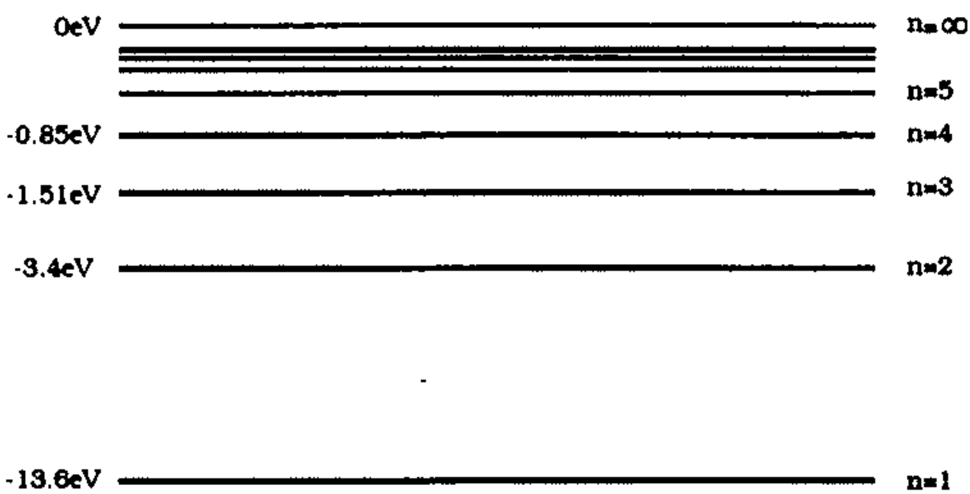
Energy of the electron in the first orbit where n = 1 is

$$E_1 = -\frac{2\pi^2 mk^2e^4}{h^2}\left(\frac{1}{1^2}\right) = -\frac{2(3.14)^2(9.11\times10^{-31})(9\times10^9)^2(1.6\times10^{-19})^4}{(6.63\times10^{-34})^2} = -21.76\times10^{-19}J$$

$$E_1 = -\frac{21.76\times10^{-19}}{1.6\times10^{-19}}eV = -13.6eV$$

$$\therefore E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2}eV$$

$$E_2 = -\frac{13.6}{2^2} = -3.4eV \qquad E_3 = -\frac{13.6}{3^2} = -1.5eV \qquad E_4 = -\frac{13.6}{4^2} = -0.85eV$$



Excitation energy: Excitation energy is defined as the energy required by an electron to jump from the ground state to any one of the excited state.

Examples:

First excitation energy of hydrogen is $\Delta E = E_2 - E_1 = \{-3.4 - (-13.6)\}eV = 10.2eV$

Second excitation energy of hydrogen is $\Delta E = E_3 - E_1 = \{-1.51 - (-13.6)\}eV = 12.09eV$

Ionisation energy: Ionisation energy is defined as the energy required by an electron in the ground state to escape completely from the attraction of the nucleus.
Example:
Ionisation energy of hydrogen is $\Delta E = E_\infty - E_1 = \{0-(-13.6)\}eV = 13.6eV$

Ground state: Ground state is defined as the lowest stable energy state of an atom.

Excited state: Excited state is defined as the energy levels that are higher than the ground state.

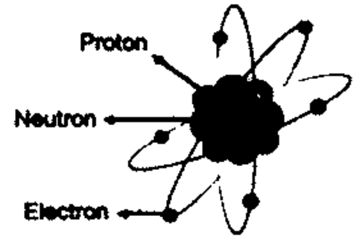
Excitation potential: It is the accelerating potential which gives to a bombarding electron, sufficient energy to excite the target atom by raising one of its electrons from an inner to an outer orbit.
Examples:
First excitation potential of hydrogen is $\Delta E = E_2 - E_1 = \{-3.4-(-13.6)\}V = 10.2V$
Second excitation potential of hydrogen is $\Delta E = E_3 - E_1 = \{-1.51-(-13.6)\}V = 12.09V$

Ionisation potential: It is the accelerating potential which gives to a bombarding electron, sufficient energy to ionise the target atom by knocking one of its electrons completely out of the atom.
Example:
Ionisation potential of hydrogen is $\Delta E = E_\infty - E_1 = \{0-(-13.6)\}V = 13.6V$

Success of Bohr's theory:
(1) It introduced quantum mechanics for the first time.
(2) Bohr's theory made the atom stable.
(3) It can explain the spectral lines of hydrogen atom correctly.

Limitations of Bohr's theory:
(1) The theory could not account for the spectra of atoms more complex than hydrogen.
(2) The theory does not give any information regarding the distribution and arrangement of electrons in an atom.
(3) It does not explain, the experimentally observed variations in intensity of the spectral lines of the element.
(4) When the spectral line of hydrogen atom is examined by spectrometers having high resolving power, it is found that a single line is composed of two or more close components. This is known as the fine structure of spectral lines. Bohr's theory could not account for the fine structure of spectral lines.
(5) It is found that when electric or magnetic field is applied to the atom, each of the spectral line split into several lines. The former effect is called as Stark effect, while the latter is known as Zeeman effect. Bohr's theory could not explain the Stark effect and Zeeman effect.

Composition of a nucleus: A nucleus of an atom is made up of protons and neutrons that are collectively known as nucleons as shown in the figure.
Proton and neutron are characterised by the following properties.



	Proton (p)	Neutron (n)
Charge (coulomb)	$+1.6 \times 10^{-19}$	0
Mass (kg)	1.672×10^{-27}	1.675×10^{-27}

Neutral atom:
For a neutral atom, the number of protons inside the nucleus is equal to the number of electrons orbiting the nucleus. This is because the magnitude of an electron charge equals to the magnitude of a proton charge but opposite in sign.

Nuclei: Nuclei are characterised by the number and type of nucleons they contain as shown in Table.

Number	Symbol	Definition
Atomic number	Z	The number of protons in a nucleus
Neutron number	N	The number of neutrons in a nucleus
Mass (nucleon) number	A	The number of nucleons in a nucleus

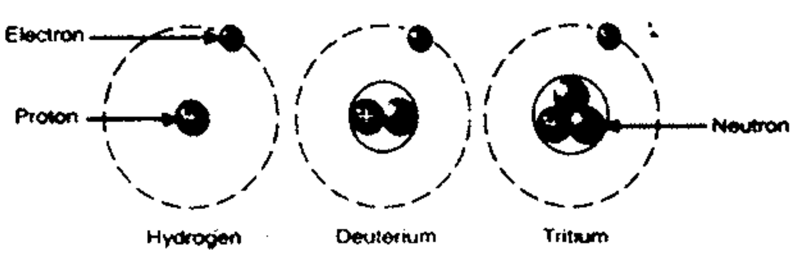
The relation between Z, N and A is $A = Z + N$

Nuclide: Any nucleus of elements in the periodic table called a nuclide is characterised by its atomic number Z and its mass number A. The number of protons Z is not necessary equal to the number of neutrons N

Isotopes: The atoms of the element which have the same atomic number (Z) but different mass number (A) are called isotopes.

Example:
Isotopes of Hydrogen

$$^{1}_{1}H: \quad Z=1, \quad A=1, \quad N=0, \quad \Rightarrow \text{Protium}\ (^{1}_{1}P)$$
$$^{2}_{1}H: \quad Z=1, \quad A=2, \quad N=1, \quad \Rightarrow \text{Deuterium}\ (^{2}_{1}D)$$
$$^{3}_{1}H: \quad Z=1, \quad A=3, \quad N=2, \quad \Rightarrow \text{Tritium}\ (^{3}_{1}T)$$

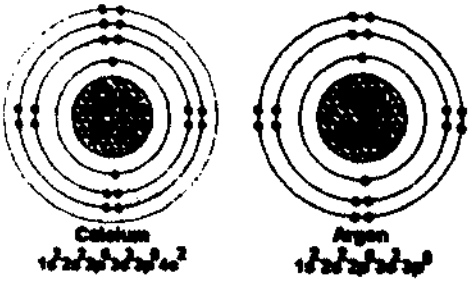


Isobars: The atoms of the element which have the same mass number (A) but different atomic number (Z) are called isobars.

Example:

$^{40}_{20}Ca$: $Z = 20,$ $A = 40,$ $N = 20$

$^{40}_{18}Ar$: $Z = 18,$ $A = 40,$ $N = 22$



Isotones: The atoms of the element which have the same number of neutrons (N) are called isotones.

$^{37}_{17}Cl$: $Z = 17,$ $A = 37,$ $N = 20$

$^{39}_{19}K$: $Z = 19,$ $A = 39,$ $N = 20$

Aids to Remember

	Z	A	N
Isotopes	√	×	×
Isobars	×	√	×
Isotones	×	×	√

Atomic mass unit (*amu* or *u*): One atomic mass unit is defined as $\frac{1}{12}th$ of the actual mass of carbon-12 atom.

$$6.023\times10^{23} \text{ number of } {}^{12}C \text{ atoms} = 12g$$

$$\text{The mass of } 1\ {}^{12}C \text{ atom} = \frac{12}{6.023\times10^{23}}g$$

$$= 1.992678\times10^{-23}g$$

$$= 1.992678\times10^{-26}kg$$

$$1\,amu = \frac{1}{12}\times1.992678\times10^{-26}kg$$

$$\therefore 1\,amu = 1.660565\times10^{-27}kg$$

Note:

$$m_p = 1.0073\,amu = 1.6726\times10^{-27}kg$$

$$m_n = 1.0086\,amu = 1.6749\times10^{-27}kg$$

$$m_e = 1.00055\,amu = 9.11\times10^{-31}kg$$

Mass of Hydrogen atom $m_H = m_p + m_e = 1.0078\,amu$

Electron volt: It is defined as the energy acquired by an electron when it is accelerated through a potential difference of 1 volt and is denoted by eV.

$$1\,eV = 1.602\times10^{-19}J$$

$$1\,MeV = 10^6 eV = 1.602\times10^{-13}J$$

Relation between *amu* and *MeV*:

$$\because E = mc^2$$

$$m = 1.66\times10^{-27}kg$$

$$c = 2.998 \times 10^8\, m/s$$
$$E = (1.66 \times 10^{-27})(2.998 \times 10^8)^2\, J$$
$$= \frac{(1.66 \times 10^{-27})(2.998 \times 10^8)^2}{1.602 \times 10^{-19}}\, eV$$
$$E \approx 931 \times 10^6\, eV \quad \Rightarrow E = 931\, MeV$$
$$\therefore 1\, amu = 931\, MeV$$

Nuclear size: Nucleus does not have a sharp or well-defined boundary. However, the radius of nucleus can be given by $R = R_o A^{\frac{1}{3}}$, where R is the average radius of the nucleus, $R_o = 1.2 \times 10^{-15}\, m$ is a constant and A is the mass (nucleon) number.

Note:

$$10^{-15}\, m = 1 \text{fermi or } fm \qquad \therefore R_o = 1.2\, fm$$
$$R \propto \sqrt[3]{A}$$

Volume of the nucleus:
Let V be the volume of the nucleus

$$\therefore V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_o A^{\frac{1}{3}})^3 = \frac{4}{3}\pi\, R_o^3 A$$
$$\Rightarrow V \propto A$$

Density of the nucleus:
Let m be the average mass of the nucleons
Then mass of the nucleus $M = mA$
The density ρ is given by

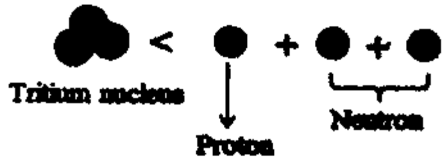
$$\rho = \frac{M}{V} = \frac{mA}{\frac{4}{3}\pi\, R_o^3 A} = \frac{3m}{4\pi\, R_o^3}$$
$$\rho = \frac{3(1.67 \times 10^{-27})}{4(3.14)(1.2 \times 10^{-15})^3}\, kg/m^3 = 2.30 \times 10^{17}\, kg/m^3$$

Properties of nuclear force:
(1) Strongest interaction
(2) Short-range force
(3) Variation with distance (strongest at the separation $\cong 1\, fm$)
(4) Charge independent character
(5) Saturation effect (interact only with neighbouring nucleon)
(6) Spin dependent character (parallel spin is stronger)
(7) Exchange forces (meson theory)
(8) Non-central force

Einstein's mass-energy equivalence: From the theory of relativity, Einstein showed that mass is a form of energy. Mass and energy can be related by the relation $E = mc^2$ where E is the amount of energy, m is the mass and c is the speed of light.
The Energy of 1kg of a substance is

$$E = 1kg \cdot (3 \times 10^8)^2\, m/s = 9 \times 10^{16}\, J$$

Mass defect: The difference between the rest mass of a nucleus and the sum of the rest masses of its constituent nucleons is called its mass defect. The mass of a nucleus (M_A) is always less than the total mass of its constituent nucleons (Zm_p+Nm_n) i.e



$M_A < (Zm_p + Nm_n)$, where M_A is the mass of the nucleus, m_p is the mass of the proton and m_n is the mass of the neutron. The mass defect Δm is

$$\Delta m = (Zm_p + Nm_n) - M_A$$

$$\Delta m = Zm_p + (A - Z)m_n - M_A$$

The reduction in mass arises because the act of combining the nucleons to form the nucleus causes some of their mass to be released as energy.

Example:

The mass defect for tritium nucleus is

$$\Delta m = (Zm_p + Nm_n) - M_A$$

$$\Delta m = (1.0073 + 2.0172)\,amu - 3.0160\,amu$$

$$\Delta m = 3.0245\,amu - 3.0160\,amu$$

$$\Delta m = 0.0085\,amu$$

Binding energy (*BE*): It is the energy required to break up a nucleus into its constituent parts and place them at an infinite distance from one another. Or the energy required to bind the nucleons together in the nucleus is called binding energy.

$$BE = \Delta m\,c^2$$

Or $BE = \{Zm_p + (A - Z)m_n - M_A\}c^2$

Or $BE = \{Zm_p + (A - Z)m_n - M_A\}\,931.5\,MeV$

Or $BE = \{\Delta m\}\,931.5\,MeV$

Binding energy per nucleon: It is the binding energy divided by the total number of nucleons. It is denoted by $\overline{B}$

$$\overline{B} = \frac{BE}{A} = \frac{\Delta m\,c^2}{A}$$

Explanation of binding energy curve:

(1) The binding energy per nucleon increases sharply with mass number A upto 20. It increases slowly after $A = 20$. For $A < 20$, there exists recurrence of peaks corresponding to those nuclei, whose mass numbers are multiples of four and they contain not only equal but also even number of protons and neutrons. Example: $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$, and $_{10}Ne^{20}$. The curve becomes almost flat for mass number between 40 and 120. Beyond 120, it decreases slowly as A increases.

(2) The binding energy per nucleon reaches a maximum of 8.8 MeV at $A = 56$, corresponding to the iron nucleus ($_{26}Fe^{56}$). Hence, iron nucleus is the most stable.

(3) The average binding energy per nucleon is about 8.5 MeV for nuclei having mass number ranging between 40 and 120. These elements are comparatively more stable and non radioactive.

$$\begin{array}{ccccc} {}_{Z}^{A}Y & = {}_{Z-1}^{A}X & + {}_{+1}^{0}e + & \nu_e & + Q \\ (parent) & (daughter) & (\beta^+) & (neutrino) & (energy) \end{array}$$

The emission of β^+ or $({}_{+1}^{0}e)$ is accompanied with the release of neutrino ν_e, a mass less and charge less particle with half integral spin

$${}_{1}^{1}p \to {}_{0}^{1}n + {}_{+1}^{0}e + \nu_e + Q$$

Gamma ray (γ):

(1) Gamma rays are high energy photons (electromagnetic radiation).

(2) Emission of gamma ray does not change the parent nucleus into a different nuclide, since neither the charge nor the nucleon number is changed.

(3) A gamma ray photon is emitted when a nucleus in an excited state makes a transition to a ground state.

(4) It is uncharged (neutral) ray and zero mass.

(5) The difference between gamma-rays and x-rays of the same wavelength is; gamma-rays are a result of nuclear processes, whereas x-rays originate outside the nucleus.

Example of γ decay is: ${}_{81}^{208}Ti^* \to {}_{81}^{208}Ti + \gamma$

Comparison of the properties between alpha particle, beta particle and gamma ray

	Alpha	Beta	Gamma
Charge	+2e	-1e or +1e	0
Deflection by electric and magnetic fields	Yes	Yes	No
Ionization power	Strong	Moderate	Weak
Penetration power	Weak	Moderate	Strong
Ability to affect a photographic plate	Yes	Yes	Yes
Ability to produce fluorescence	Yes	Yes	Yes

Nuclear energy levels: The nucleus, like the atom, has discrete energy levels whose location and properties are governed by the rules of quantum mechanics. These are nuclear stationary states. The stationary state of lowest energy is called the ground state. When a nucleus makes a transition from some higher energy level to a lower energy level, the difference of energy is emitted as a photon in gamma-ray region of the electromagnetic spectrum.

Radioactive decay law: For a radioactive source, the decay rate $\left(-\frac{dN}{dt}\right)$ is directly proportional to the number of radioactive nuclei N remaining in the source.

$$-\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N$$

Negative sign means the number of remaining nuclei decreases with time and λ is the decay constant.

$$\lambda = -\frac{\frac{dN}{dt}}{N} = \frac{decay\ rate}{number\ of\ remaining\ radioactive\ nuclei}$$

Hence the decay constant is defined as the probability that a radioactive nucleus will decay in one second. Its unit is s^{-1}.

Mathematical treatment of decay law: Let N_0 be the number of radioactive atoms present initially and N, the number of atoms at a given instant t. Let dN be the number of atoms undergoing disintegration in a small interval of time dt. Then the rate of disintegration is

$$\frac{dN}{dt} = -\lambda N \text{-----}(1)$$

where λ is a constant known as decay constant or disintegration constant. The negative sign indicates that N decreases with increase in time.

Equation (1) can be written as

$$\frac{dN}{N} = -\lambda dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

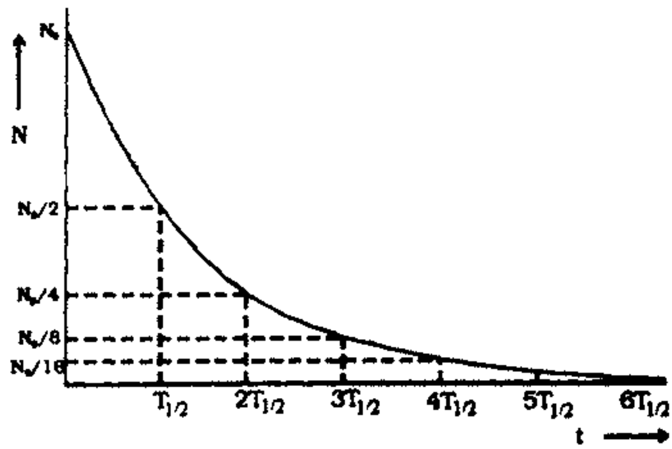
$$[\log N]_{N_0}^{N} = -\lambda [t]_0^t$$

$$\log N - \log N_0 = -\lambda t$$

$$\log \frac{N}{N_0} = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\Rightarrow N = N_0 e^{-\lambda t}$$



This is the exponential law of radioactive decay

From the equation (1), the graph of N, (the number of remaining radioactive nuclei in a sample) against the time t is shown in figure.

Half-life ($T_{1/2}$): Half-life is defined as the time taken for a sample of radioactive nuclides to disintegrate to half of the initial number of nuclei

From the equation $N = N_0 e^{-\lambda t}$

When $t = T_{1/2}$ $\quad N = \frac{N_0}{2}$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \qquad \text{Or} \quad \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$2 = e^{\lambda T_{1/2}} \qquad \text{Or} \quad \ln 2 = \ln e^{\lambda T_{1/2}}$$

$$\ln 2 = \lambda T_{1/2} \qquad \text{Or} \quad T_{1/2} = \frac{\ln 2}{\lambda}$$

$$\Rightarrow T_{1/2} = \frac{0.693}{\lambda}$$

The half-life of any given radioactive nuclide is constant, it does not depend on the number of remaining nuclei.

The units of the half-life are second (s), minute (min), hour (hr), day (d) and year (y). Its unit depends on the unit of decay constant.

Table shows the value of half-life for several isotopes.

Isotope	Half-life
${}_{92}U^{238}$	4.5×10^9 years
${}_{88}Ra^{226}$	1.6×10^3 years
${}_{84}Po^{210}$	138 days
${}_{90}Th^{234}$	24 days
${}_{86}Rn^{222}$	3.8 days
${}_{83}Bi^{214}$	20 minutes

Average-life or mean life (τ): The mean life of a radioactive substance is defined as the ratio of total life time of all the radioactive atoms to the total number of atoms in it.

The mean life calculated from the law of disintegration shows that the mean life is the reciprocal of the decay constant.

$$\tau = \frac{1}{\lambda} = \frac{1}{\frac{0.693}{T_{1/2}}} = \frac{1}{0.693} T_{1/2} = 1.443\ T_{1/2}$$

$$\Rightarrow \tau > T_{1/2}$$

Activity of radioactive sample A: The activity of a radioactive substance is defined as the rate at which the atoms decay. If N is the number of atoms present at a certain time t, the activity A is given by

$$A = -\frac{dN}{dt}$$

The unit of activity is becquerel named after the scientist Henri Becquerel
∴ 1 becquerel = 1 disintegration per second
The activity of a radioactive substance is generally expressed in curie. Curie is defined as the quantity of a radioactive substance which gives 3.7×10^{10} disintegrations per second or 3.7×10^{10} becquerel. This is equal to the activity of one gram of radium.

Natural radioactivity: It is the phenomenon of spontaneous emission of α, β and γ radiations.

Artificial or induced radioactivity: It is the phenomenon of inducing radioactivity in certain stable nuclei by bombarding them by suitable high energy particles.

Nuclear fission: Nuclear fission is defined as a nuclear reaction in which a heavy nucleus splits into two lighter nuclei that are almost equal in mass with the emission of neutrons and energy.

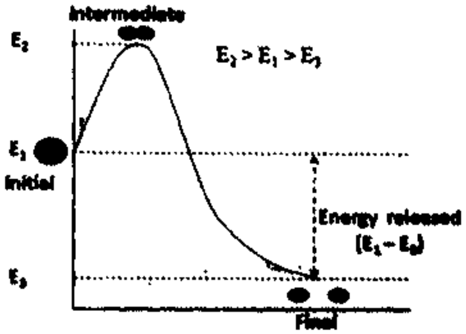
Or

The process of breaking up of the nucleus of a heavier atom into two fragments with the release of large amount of energy is called nuclear fission.

(1) Nuclear fission releases an amount of energy that is greater than the energy released in chemical reaction.
(2) Energy is released because the average binding energy per nucleon of the fission products is greater than that of the parent.

Spontaneous Fission: Some radioisotopes contain nuclei which are highly unstable and decay spontaneously by splitting into two smaller nuclei. Such spontaneous decays are accompanied by the release of neutrons.

Note: Heavy nucleus has larger rest mass energy than that of its two middle-weight fragments. Such processes are called barrier penetration

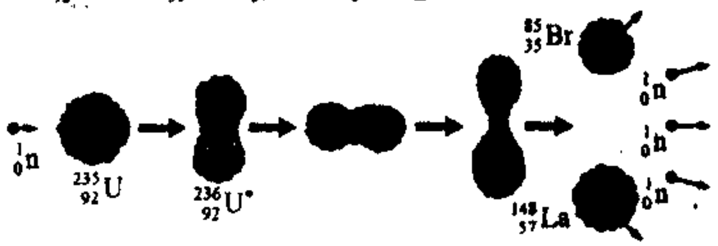


Induced Fission: Fission which takes place only when a nucleus is bombarded with neutron, proton, or other particle (particle-induced fission) or by gamma-ray excitation (photofission).

Example:

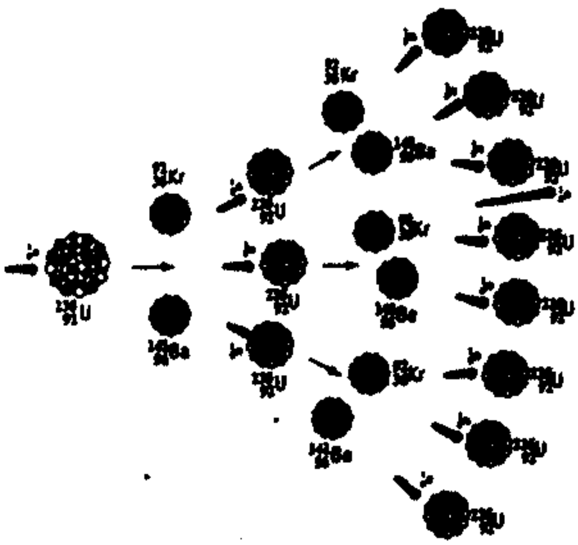
Consider the bombardment of ${}_{92}U^{235}$ by slow neutrons. One of the possible reaction is

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U^{*} \rightarrow ^{86}_{35}Br + ^{148}_{57}La + 3^{1}_{0}n + Q$$



Chain reaction: The nuclear fission which once started continues till all the atoms of the fissionable material are disintegrated is known as chain reaction.

Consider a neutron causing fission in a uranium nucleus producing three neutrons. The three neutrons in turn may cause fission in three uranium nuclei producing nine neutrons. These nine neutrons in turn may produce twenty seven neutrons and so on. A chain reaction is a self propagating process in which the number of neutrons goes on multiplying rapidly almost in a geometrical progression. Two types of chain reactions are possible. In the uncontrolled chain reaction, the number of neutrons multiply indefinitely and the entire amount of energy is released within a fraction of a second. This type of chain reaction takes place in atom bombs.



In the controlled chain reaction the number of fission producing neutron is kept constant and is always equal to one. The reaction is sustained in a controlled manner. Controlled chain reaction is taking place in a nuclear reactor.

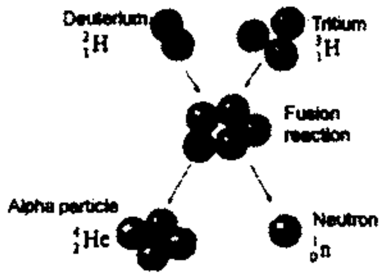
Nuclear fusion: Nuclear fusion is defined as a type of nuclear reaction in which two light nuclei fuse to form a heavier nucleus with the release of large amounts of energy.

(1) The energy released in this reaction is called thermonuclear energy.
Examples of fusion reaction releases the energy are

$$^{2}_{1}H+^{2}_{1}H\rightarrow^{3}_{2}He+^{1}_{0}n+Q$$

$$^{2}_{1}H+^{2}_{1}H\rightarrow^{3}_{1}He+^{1}_{1}H+Q$$

(2) The nuclear fusion reaction can occur in fusion bomb and in the core of a star.
(3) Deuterium-tritium fusion is other example of fusion reaction where it can be represented as shown in the diagram.

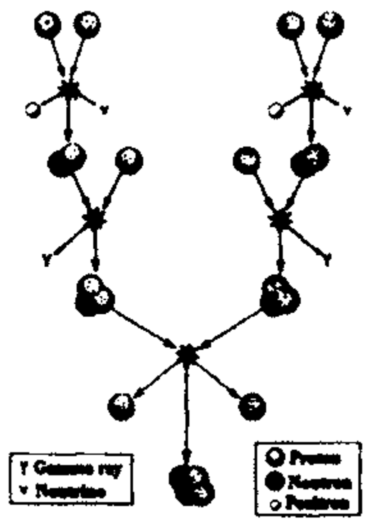


Note:
(1) The two reacting nuclei in fusion reaction above themselves have to be brought into collision.
(2) As both nuclei are positively charged there is a strong repulsive force between them, which can only be overcome if the reacting nuclei have very high kinetic energies.
(3) These high kinetic energies imply temperatures of the order of 10^8 K.

Thermonuclear energy: The energy released during nuclear fusion is known as thermonuclear energy.

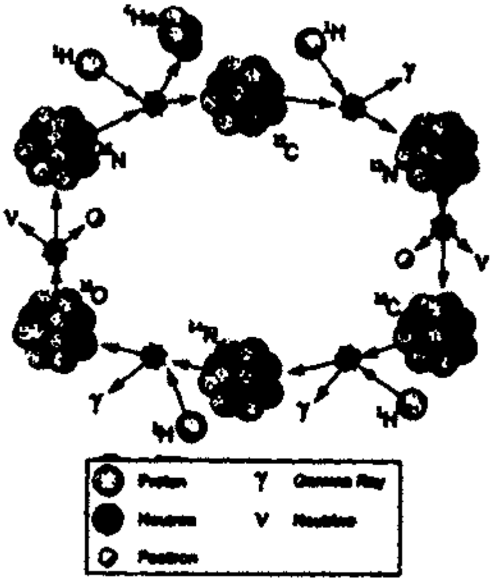
Nuclear fusion in the sun: (proton-proton cycle): This is the nuclear fusion process which fuels the Sun and other stars which have core temperatures less than 15 million Kelvin. A reaction cycle yields about 25 *MeV* of energy.

Note: The sun is a small star which generates energy on its own by means of nuclear fusion in its interior. The fuel of fusion reaction comes from the protons available in the sun. The protons undergo a set of fusion reactions, producing isotopes of hydrogen and also isotopes of helium. However, the helium nuclei themselves undergo nuclear reactions which produce protons again. This means that the protons go through a cycle which is then repeated. Because of this proton-proton cycle, nuclear fusion in the sun can be self sustaining. The set of fusion reactions in the proton-proton cycle can be illustrated by the figure



Nuclear fusion in the sun: CNO (Heavier than the sun): In stars with central temperatures greater than 15 million Kelvin, carbon fusion is thought to take over the dominant role rather than hydrogen fusion. A star like Sirius with somewhat more than twice the mass of the sun derives almost all of its power from the carbon cycle. The carbon cycle yields 26.72 *MeV* per helium nucleus.

Note: The main theme of the carbon cycle is the adding of protons, but after a carbon-12 nucleus fuses with a proton to form nitrogen-13, one of the protons decays with the emission of a positron and a neutrino to form carbon -13. Two more proton captures produce nitrogen-14 and then oxygen-15. Another neutron decay leaves nitrogen-15. Another proton capture produces oxygen-16 which emits an energetic alpha particle to return to carbon-12 to repeat the cycle. This last reaction is the main source of energy in the cycle for the fueling of the star.



Comparison between fission and fusion

Fission	Fusion
Splitting a heavy nucleus into two small nuclei.	Combines two small nuclei to form a larger nucleus.
It occurs at temperature that can be controlled.	It occurs at very high temperature (10^8 K).
Easier to control and sustain.	Difficult to control and a sustain controlled reaction has not yet been achieved.
Neutrons are needed in fission process	Protons are needed in fusion process

Note: The similarity between the fission and fusion reactions is that both reactions produces energy.

<u>**ELECTRONIC DEVICES**</u>

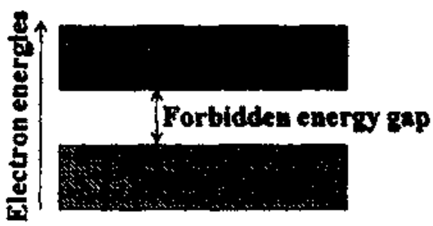
Classification of Solids: Solids can be classified on the basis of (a) conductivity and (b) energy bands.

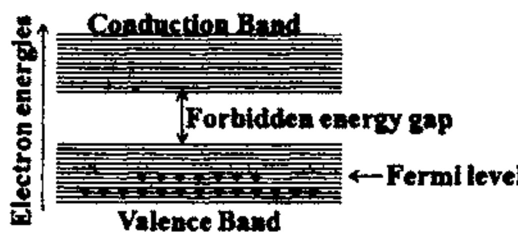
Metals and Insulators: Solids that have very low resistivity and very high electrical conductivity are known as metals and solids that have very high resistivity and very low electric conductivity are known as insulators.

Semiconductors: Solids whose resistivity or conductivity is intermediate between metals and insulators are known as semiconductors.

Compound semiconductors: Semiconductors that have more than one type of atoms as their constituent particles are known as compound semiconductors. Compound semiconductors are further classified as inorganic, organic, and organic polymer semiconductors.

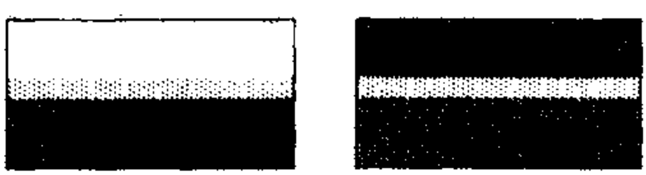
Energy band: The collection of closely spaced energy levels is known as energy band. The lower energy band of filled levels is called the valence band, while the upper energy band of empty levels is called the conduction band. The energy gap between the valence band and the conduction band is called the forbidden energy gap.





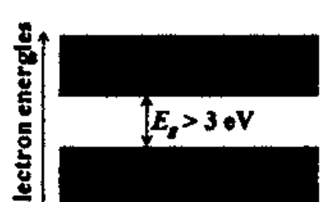
Fermi level: The highest energy level which an electron can occupy in the valence band at $0K$ is called Fermi level.

Metals: In metals, the conduction band is either partially filled (figure a) or overlaps (figure b) the valence band. There is no forbidden energy gap between the valence and conduction bands. Even if a small electric field is applied, free electrons start moving in a direction opposite to the field and hence metals behave as good conductors of electricity.



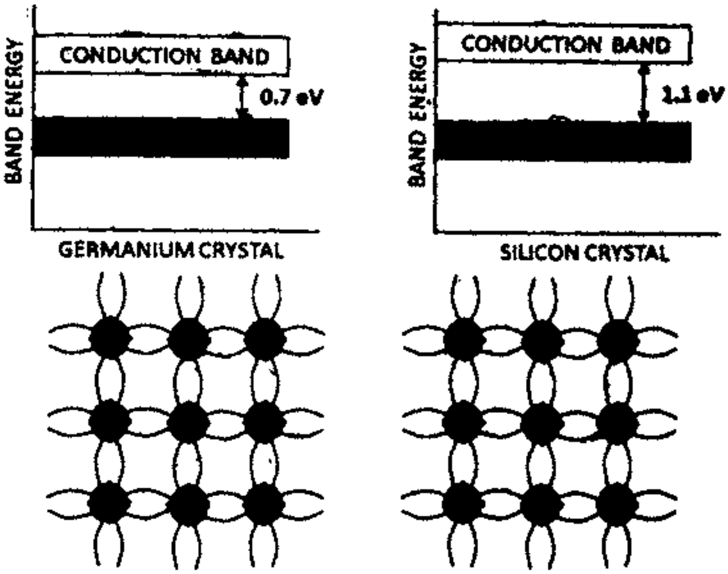
(a) (b)

Insulators: In insulators, the valence band is completely filled, the conduction band is empty, and the forbidden gap is quite large (E_g >3 eV), as shown in the figure. For example, diamond is an insulator and the forbidden energy gap for diamond is E_g = 6 eV. If an electric field is applied across the ends of an insulator, no electron is able to go from the



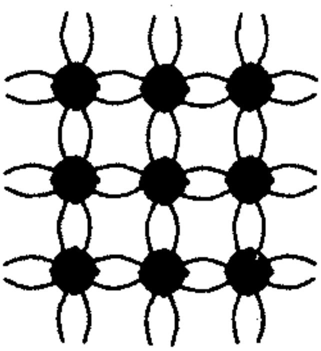
valence band to the conduction band due to the very high band gap. That is why insulators behave as poor conductors of electricity.

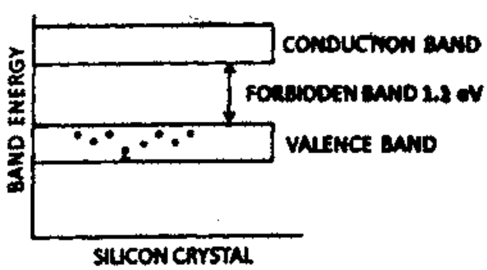
Semiconductors: A semiconductor is a material which has almost filled valence band and nearly empty conduction band with a small energy gap($E_g \approx 1$ eV) separating the two.



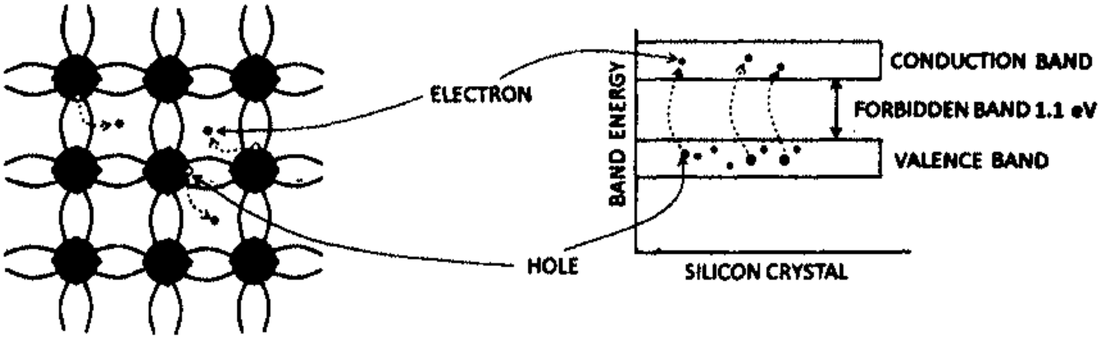
Properties of semiconductors :
(1) They have crystalline structure.
(2) The resistance of the semiconductor decreases with the rise in temperature i.e., they have negative temperature coefficient of resistance.
(3) They are formed by covalent bond.
(4) The number of electrons available for conduction can be increased enormously when suitable metallic impurity (eg, arsenic, gallium etc.) is added to a semiconductor.

Effect of Temperature on Semiconductors :
(1) At absolute zero: At absolute zero, the covalent bonds are very strong and there are no free electrons in the conduction band. Therefore the semiconductor crystal behaves as a perfect insulator.

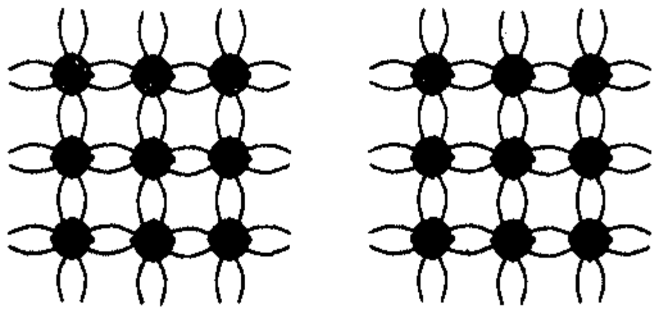




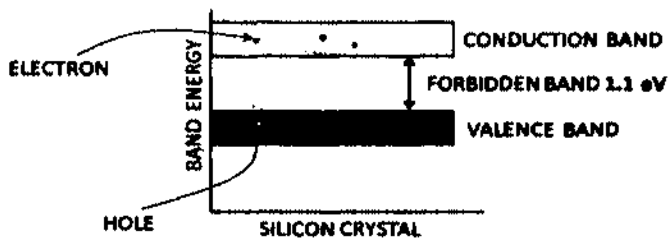
(2) Above absolute zero: Above absolute zero there is a finite probability that an electron in the lattice will be knocked loose from its position, leaving behind an electron deficiency called a "hole". If a voltage is applied, then both the electron and the hole can contribute to a small current flow.



Intrinsic semiconductor: A semiconductor in an extremely pure form is known as an intrinsic semiconductor or an undoped semiconductor or an *i*-type semiconductor.



In an intrinsic semiconductor, the number of free electrons (n_e) in conduction band is exactly equal to the number of holes (n_h) in the valence band



$\therefore\ n_e = n_h = n_i$

or $n_e\, n_h = n_e\, n_e = (n_i)^2$

or $n_e = n_i$

Where n_e = number density of free electrons in conduction band

n_h = number density of holes in the valence band

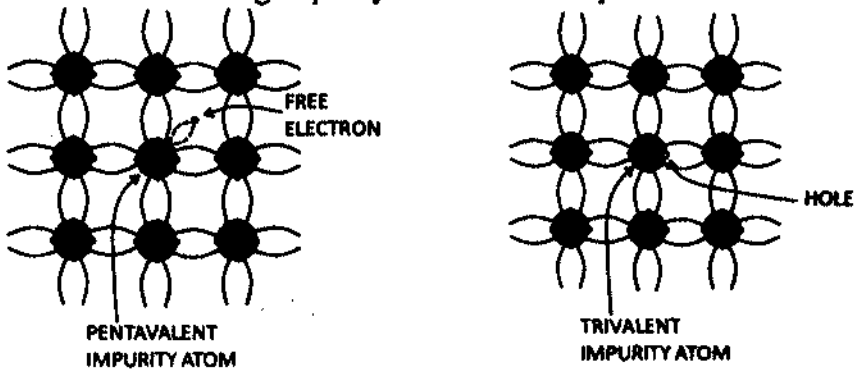
n_i = number density of intrinsic carriers (free electrons or holes)

Doping a Semiconductor:

(1) Doping is the process of deliberate addition of a very small amount of impurity into an intrinsic semiconductor.

(2) The impurity atoms are called 'dopants'.

(3) The semiconductor containing impurity is known as 'impure or extrinsic semiconductor'.



Methods of doping:

(1) Heating the crystal in the presence of dopant atoms.

(2) Adding impurity atoms in the molten state of semiconductor.

(3) Bombarding semiconductor by ions of impurity atoms.

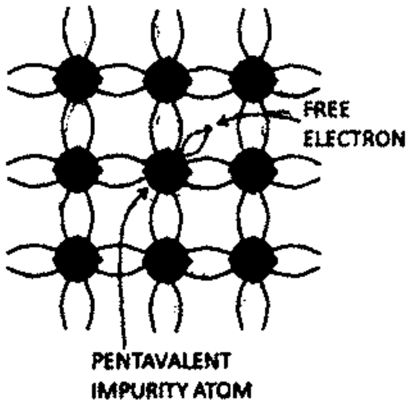
Extrinsic semiconductor:

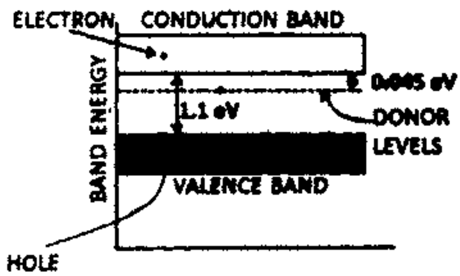
A semiconductor whose conductivity is mainly due to impurity is called the extrinsic semiconductor. Extrinsic semiconductors are of two types.

(1) n-type semiconductor

(2) p-type semiconductor

(1) n-type semiconductor: When a small amount of pentavalent impurity is added to a pure semiconductor, then it is known as n-type Semiconductor.





In n-type semiconductor the following points may be noted:

(i) By the addition of pentavalent impurity many new free electrons are produced.

(ii) Few hole-electron pairs are generated at room temperature due to thermal energy. However, the number of free electrons provided by the pentavalent impurity far exceeds the number of holes.

(iii) For n-type semiconductor $n_e n_h = n_i^2$

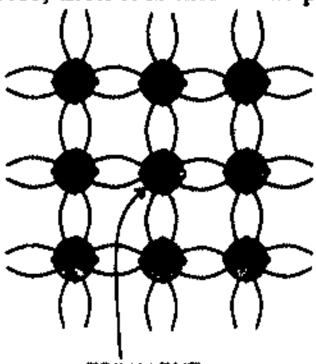
Where n_e = number density of free electrons in conduction band

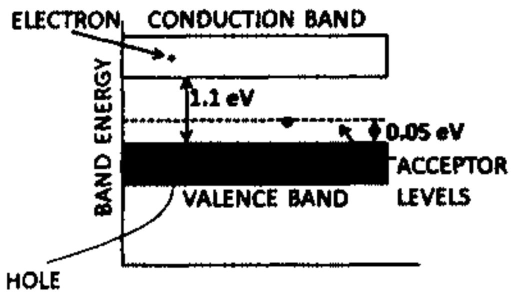
n_h = number density of holes in the valence band

n_i = number density of intrinsic carriers (free electrons or holes)

(iv) In n-type semiconductor, the number density of electrons (n_e) in conduction band is approximately equal to that of donor atoms (N_d) but is very large as compared to the number density of holes (n_h) in the valence band i.e., $n_e \approx N_d >> n_h$

(2) p-type semiconductor: When a small amount of trivalent impurity is added to a pure semiconductor, then it is known as p-type Semiconductor.





In p-type semiconductor the following points may be noted:

(i) By the addition of trivalent impurity many new holes are produced.

(ii) Few hole-electron pairs are generated at room temperature due to thermal energy. However, the number of holes provided by the trivalent impurity far exceeds the number of free electrons.

(iii) For p-type semiconductor $n_e n_h = n_i^2$

Where n_e = number density of free electrons in conduction band
n_h = number density of holes in the valence band
n_i = number density of intrinsic carriers (free electrons or holes)

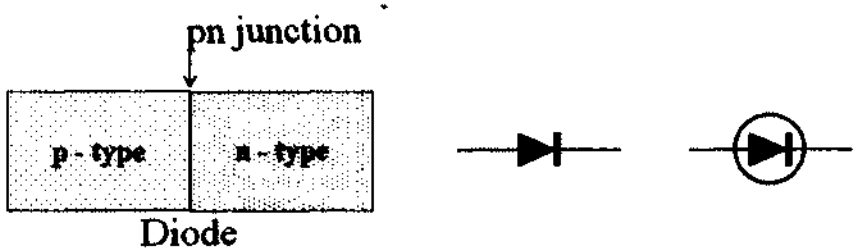
(iv) In p-type semiconductor, the number density of holes (n_h) in valence band is approximately equal to that of acceptor atoms (N_a) but is very large as compared to the number density of electrons (n_e) in the conduction band i.e., $n_h \approx N_a >> n_e$

Majority and Minority Carriers:

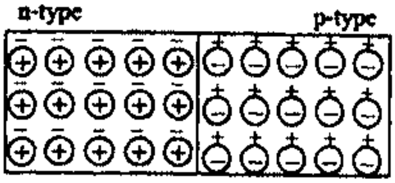
In n-type semiconductor, free electrons are considered to be the majority carriers. Since the majority portion of current in n-type material is by the flow of free electrons, and the holes are the minority carriers.

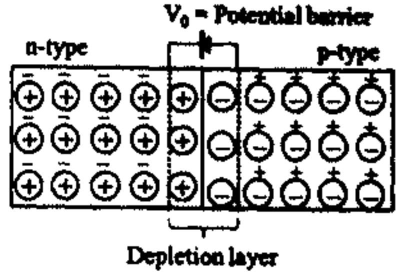
In the p-type semiconductor, holes outnumber the free electrons. Therefore, holes are the majority carriers and free electrons are the minority carriers.

pn junction (Semiconductor diode): A pn junction is a thin region between a p-type and an n-type semiconducting material across which a potential barrier exists.



Potential barrier: This potential prevents further diffusion of holes and electrons across the junction and ensures zero current through the junction. It is also referred to as the barrier potential.





Depletion layer: Since no charge carrier can remain in this region (i.e., the region is depleted of mobile charges), it is also called the depletion layer or depletion region or space charge region.

Note:

(1) As soon as pn junction is formed, potential barrier V_0 appears across the junction and it seems as if some fictitious battery of magnitude V_0 is connected across the junction with its negative terminal connected to p region and the positive terminal to n region as shown in the figure.

(2) The potential barrier does not allow the current carriers (free electrons and holes) to cross the pn junction. Due to the presence of V_0 across the junction, an electron requires an energy eV_0 to cross the junction from n region to p region and an equal amount of energy is required to move a hole from p region to n region to cross the junction.

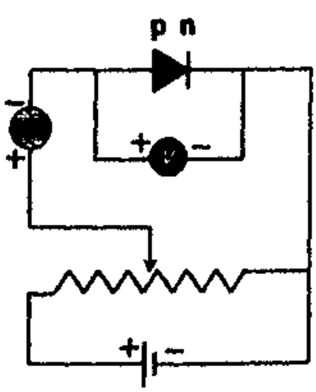
(3) The width of depletion layer is very small ($d \approx 10^{-6}$ m). The value of barrier potential V_0 is about 0.7 V for silicon and about 0.3 V for germanium semiconductor (Average $V_0 \approx 0.5$V). This means that barrier electric field E set up by V_0 across the junction is very high.

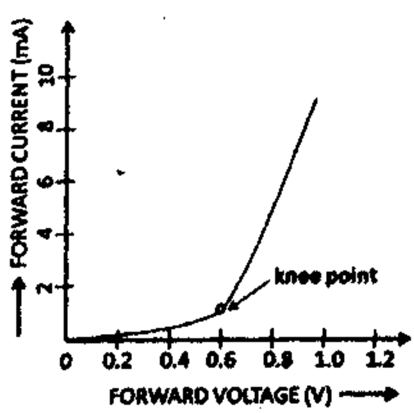
$$E = \frac{V_0}{d} = \frac{0.5}{10^{-6}} = 5 \times 10^5 Vm^{-1}$$

V-I characteristics of a pn junction diode:

The variation of current as a function of applied voltage is known as the V-I characteristics of the pn junction diode.

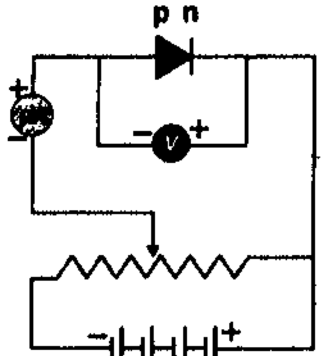
pn junction diode under forward bias and forward characteristics of the pn junction diode: When a pn junction diode is biased such that the positive terminal of the battery is connected to the p-side and negative terminal of the battery is connected to the n-side, the pn junction diode is said to be forward biased.

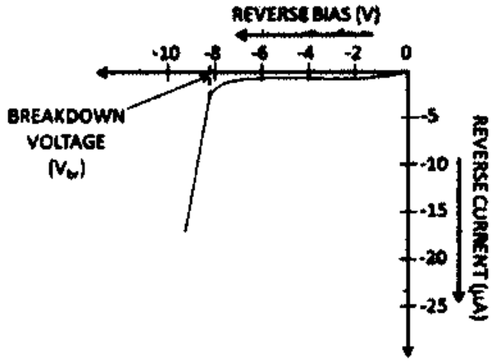




The minimum forward voltage after which the current through the pn junction diode increases rapidly with the voltage is known as threshold voltage or knee voltage.

pn junction diode under reverse bias and reverse characteristics of the pn junction diode: When a pn junction diode is biased such that the positive terminal of the battery is connected to the n-side and negative terminal of the battery is connected to the p-side, the pn junction diode is said to be reverse biased.

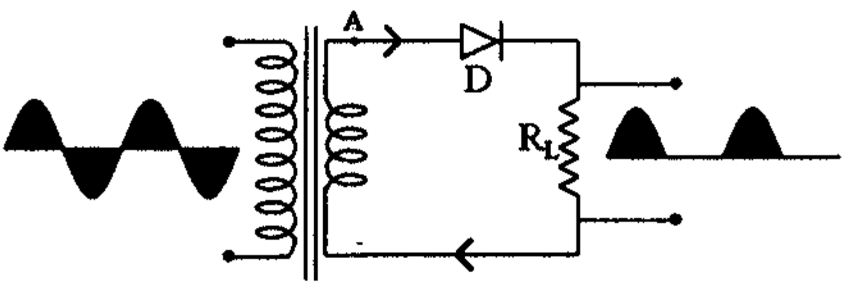




The reverse voltage V_{br} at which the current through the pn junction diode becomes infinite (or very large) and the diode breaks down is known as the breakdown voltage.

pn Junction Diode as Rectifier: The process of converting alternating current into direct current is called 'rectification'. The device used for rectification is called 'rectifier'. The pn junction diode offers low resistance in forward bias and high resistance in reverse bias. Therefore a diode can conduct well only in one direction i.e., when forward bias.

pn Junction Diode as a Half Wave Rectifier: During the first half cycle when a.c voltage at A is positive, the diode D conducts. In the next half cycle when the voltage at A is negative, the diode does not conduct. Therefore during the first half cycle current flows through the load resistance R_L and during the second half cycle there is no output as shown in the figure. Since the voltage across the load appears only during the positive half cycle of the input a.c., this process is called half-wave rectification and the arrangement used is called half-wave rectifier.

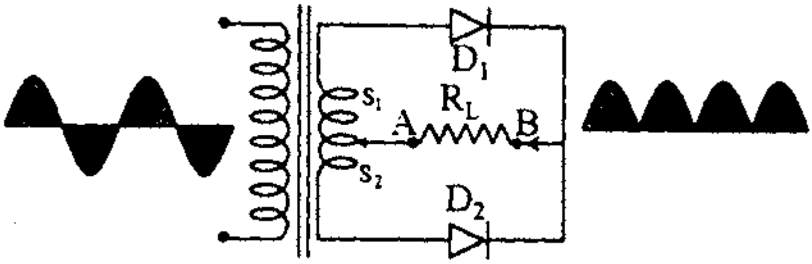


Disadvantages of Half Wave Rectifier:

(1) Filter circuit is required to produce steady direct current.

(2) Circuit delivers a.c output power only for half time period, therefore the output is low.

pn Junction Diode as a Full Wave Rectifier: During the positive half cycle of the secondary voltage, S_1 of the secondary becomes positive and S_2 negative. Diode D_1 becomes forward biased and D_2 becomes reverse biased. Therefore during positive half cycle D_1 conducts and D_2 does not conduct. The current flows through diode D_1, load resistance R_L and upper half of secondary S_1. During the negative half cycle of the secondary voltage, S_1 of the secondary becomes negative and S_2 positive. Diode D_1 becomes reverse biased and D_2 becomes forward biased. Therefore during negative half cycle D_2 conducts and D_1 does not conduct. The current flows through diode D_2, load resistance R_L and lower half of secondary S_2. Since output voltage across the load resistance RL is obtained for both half cycles of input a.c., this process is called full-wave rectification and the arrangement used is called full-wave rectifier.



Disadvantages of Full Wave Rectifier:

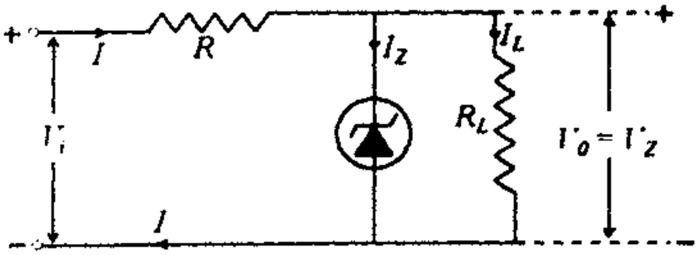
(1) Filter circuit is required to produce steady direct current.

(2) Each diode utilises only one half of the transformer secondary voltage, therefore d.c., output is small.

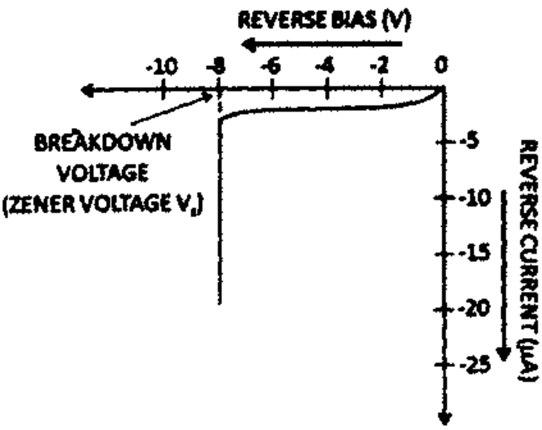
(3) The diode used must have high break down voltage.

Zener diode: A properly doped junction diode which has a sharp breakdown voltage is called Zener diode. Symbol of zener diode is shown in the figure.

Principle of zener diode: The basic principle of Zener diode is the Zener breakdown. When a diode is heavily doped, its depletion region will be narrow. When a high reverse voltage is applied across the junction, there will be very strong electric field at the junction, and the electron hole pair generation takes place. Thus heavy current flows. This is known as Zener break down. A Zener diode in a forward biased condition acts as a normal diode. In reverse biased mode, after the break down of junction, current through diode increases sharply, but the voltage across it remains constant. This principle is used in voltage regulator using Zener diodes.



Zener diode as a voltage regulator: When a Zener diode is operated in the reverse breakdown region, the voltage across it remains practically constant (equal to the breakdown voltage V_Z) for a large change in the reverse current. The use of Zener diode as a d.c. voltage regulator is based on this fact.



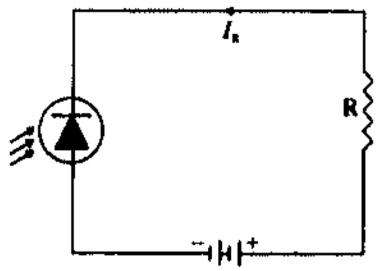
Note:

(1) A Zener diode is like an ordinary diode except that it is properly doped so as to have a sharp breakdown voltage called zener voltage V_Z

(2) A Zener diode is always connected, in the reverse biased.
(3) When connected in forward biased, it behaves like ordinary diode.

Photodiode: An optoelectronic device in which current-carriers are generated by photons from the incident light is known as a photodiode. It is sensitive to low light conditions. A photodiode is designed to respond to photon absorption, and is operated in a reverse biased pn junction mode

Principle:
(1) A reverse biased pn junction diode has a very low reverse current. It is referred as dark current.
(2) When light falls, additional electron-hole pairs are generated in both p and n region. It produces a very large change in minority carrier concentration and hence increases the reverse current through the diode. This current varies almost linearly with the light flux.

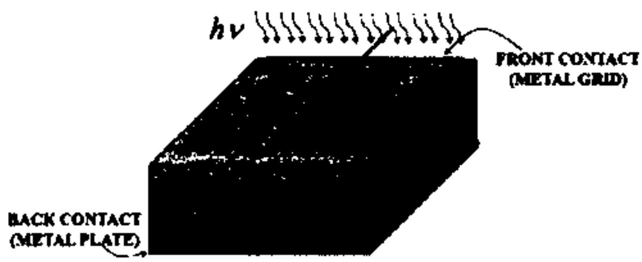


Working:
Under no light condition, a sufficient reverse voltage is applied to photodiode to get constant current, independent of magnitude of reverse bias. This reverse saturation current is called dark current. Light is absorbed in the depletion region (intrinsic region) generates electron-hole pairs, which contribute to the photo current. The photo current is proportional to the incoming light intensity over a wide range of optical powers.

Note: To improve the absorption of light, an intrinsic (undoped) layer is introduced between n and p region. Devices with an intrinsic layer I are called P-I-N photodiodes.

Uses of photodiode:
(1) It can turn ON and OFF in nanoseconds, It is used as ON/OFF switch at a very fast rate.
(2) In modern optical communication system, fast and sensitive photodiodes play important role.
(3) In light detection
(4) In light operated switches.
(5) Reading of computer punched cards.
(6) They find many uses in instrumentation, control and automation.
(7) Depending on applications many variation of photodiodes are developed such as P-I-N photodiodes, velocity-match photodetectors, Avalanche photodiodes, metalsemiconductor-metal photodetectors.

Solar cell: An optoelectronic device which is used to convert sun light into electrical energy is known as a solar cell. The pn junction of a solar cell is not biased with any external voltage source. It works at bright light conditions.



Principle:
When light falls, additional electron-hole pairs are generated in both p and n region. It produces a very large change in minority carrier concentration and hence increases the reverse current through the cell.

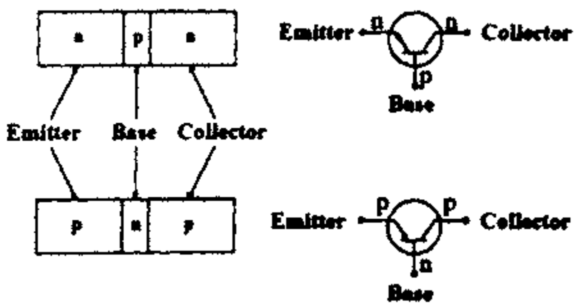
Uses of solar cell: Solar cells are used to supply power to electronic devices, in space vehicles and artificial satellites.

Light Emitting Diode (LED): A light emitting diode (LED) is a forward biased pn junction diode, which emits visible light when energized.

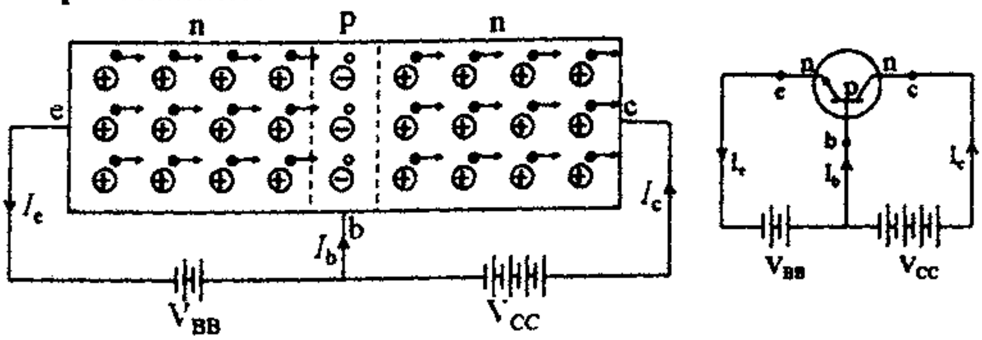
Working: When a junction diode is forward biased, electrons from n-side and holes from p-side move towards the depletion region and recombination takes place. When an electron in the conduction band recombines with a hole in the valence band, energy is released. In the case of semiconducting materials like gallium-arsenide (GaAs), gallium-phosphide (GaP) and gallium-arsenide-phosphide (GaAsP), a greater percentage of energy is given out in the form of light. If the semiconductor material is transluscent, light is emitted and the junction becomes a light source (turned ON). The LED is turned ON, when it is forward biased and it is turned OFF, when it is reverse biased. The colour of the emitted light will depend upon the type of the material used. By using gallium-arsenide-phosphide and gallium-phosphide, a manufacturer can produce LEDs that radiate red, green, yellow and orange. The symbol of LED is shown in the figure.

Uses of LED: LEDs are used for instrument displays, calculators and digital watches.

Junction Transistor: Transistor is a combination of two words 'transfer' and 'resistor' which means that transfer of resistance takes place from input to output section.
It is formed by sandwiching one type of extrinsic semiconductor between other type of extrinsic semiconductor.
n-p-n transistor contains p-type semiconductor sandwiched between two n-type semiconductors.
p-n-p transistor contains n-type semiconductor sandwiched between two p-type semiconductors.



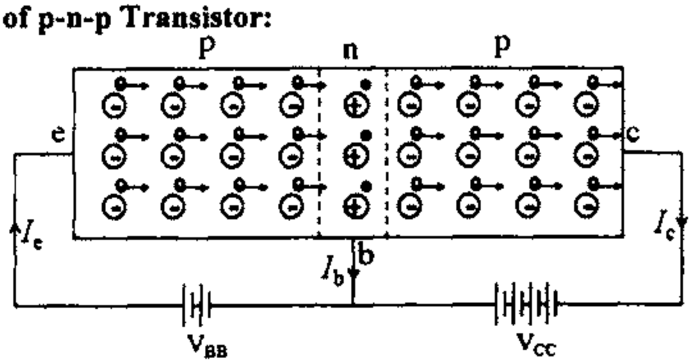
Action of n-p-n Transistor:

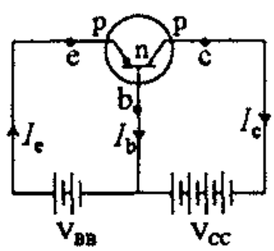


In n-p-n transistor, the arrow mark on the emitter is coming away from the base and represents the direction of flow of current. It is the direction opposite to the flow of electrons which are the main charge carriers in n-type crystal.
The emitter junction is forward-biased with emitter-base battery V_{BB}. The collector junction is reverse biased with collector-base battery V_{CC}.
The forward bias of the emitter-base circuit helps the movement of electrons (majority carriers) in the emitter, and holes (majority carriers) in the base towards the junction between the emitter and the base. This reduces the depletion region at this junction.
On the other hand, the reverse bias of the collector-base circuit forbids the movement of the majority carriers towards the collector-base junction and the depletion region increases.
The electrons in the emitter are repelled by the negative terminal of the emitter-base battery.
Since the base is thin and lightly doped, therefore, only a very small fraction (say, 5%) of the incoming electrons combine with the holes. The remaining electrons rush through the collector and are swept away by the positive terminal of the collector-base battery.
For every electron-hole recombination that takes place at the base region one electron (i.e., only one electron combine with one hole) is released into the emitter region by the negative terminal of the emitter-base battery. The deficiency of the electrons caused due to their movement towards the collector is also compensated by the electrons released from the emitter-base battery.
The current is carried by the electrons both in the external as well as inside the transistor.

$$I_e = I_b + I_c$$

Action of p-n-p Transistor:





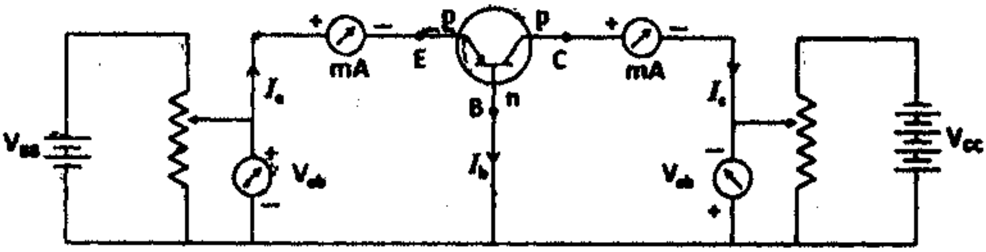
In p-n-p transistor, the arrow mark on the emitter is going into the base and represents the direction of flow of current. It is in the same direction as that of the movement of holes which are main charge carriers in p-type crystal.
The emitter junction is forward-biased with emitter-base battery V_{BB}. The collector junction is reverse biased with collector-base battery V_{CC}.
The forward bias of the emitter-base circuit helps the movement of holes (majority carriers) in the emitter and electrons (majority carriers) in the base towards the junction between the emitter and the base. This reduces the depletion region at this junction.
On the other hand, the reverse bias of the collector-base circuit forbids the movement of the majority carriers towards the collector-base junction and the depletion region increases.
The holes in the emitter are repelled by the positive terminal of the emitter-base battery.
Since the base is thin and lightly doped, therefore, only a very small fraction (say, 5%) of the incoming holes combine with the electrons. The remaining holes rush through the collector and are swept away by the negative terminal of the collector-base battery.
For every electron-hole recombination that takes place at the base region one electron (i.e., only one electron combine with one hole) is released into the emitter region by breaking the covalent bond and it enters the positive terminal of the emitter-base battery. The holes

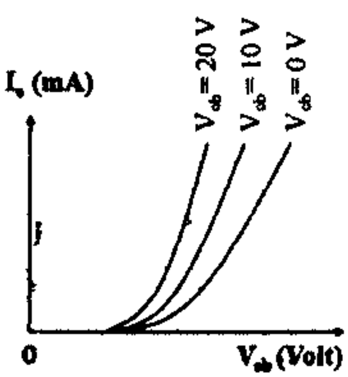
reaching the collector are also compensated by the electrons released from the collector-base battery.

The current is carried by the electrons in the external circuit and by the holes inside the transistor.

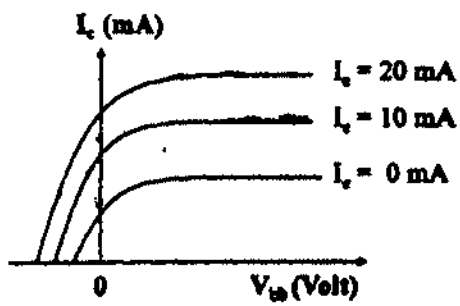
$$I_e = I_b + I_c$$

p-n-p Transistor Characteristics in Common Base (CB) Configuration: The plot between I_e and V_{eb} for fixed V_{cb} is called input characteristics while the plot between I_C and V_{cb} for fixed I_e is called output characteristics for the common base configuration.





Input Characteristics



Output Characteristics

Current Amplification Factor or Current Gain:

(1) Direct current (dc) gain: It is the ratio of the collector current (I_c) to the emitter current (I_e) at constant collector voltage (V_{cb})

$$\alpha_{dc} = \left(\frac{I_c}{I_e}\right) \text{ at constant } V_{cb}$$

(2) Alternating current (ac) gain: It is the ratio of change in collector current (ΔI_c) to the change in emitter current (ΔI_e) at constant collector voltage (V_{cb}).

$$\alpha_{ac} = \left(\frac{\Delta I_c}{\Delta I_e}\right) \text{ at constant } V_{cb}$$

Alternating voltage gain: It is the ratio of change in output voltage (collector voltage ΔV_{cb}) to the change in input voltage (applied signal voltage ΔV_i))

$$A_{V-ac} = \left(\frac{\Delta V_{cb}}{\Delta V_i}\right) = \left(\frac{\Delta I_c \times R_0}{\Delta I_e \times R_i}\right) = \alpha_{ac} \times \text{resistance gain}$$

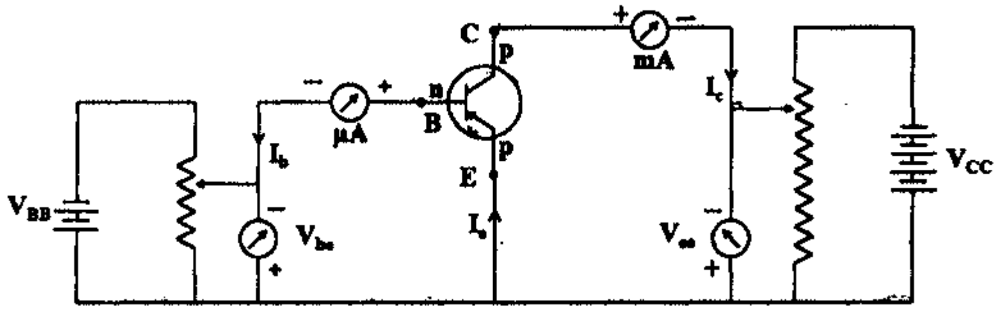
Input resistance (r_i): Ratio of change in base-emitter voltage (ΔV_{be}) to the resulting change in emitter current (ΔI_e) at constant collector base voltage (V_{cb}).

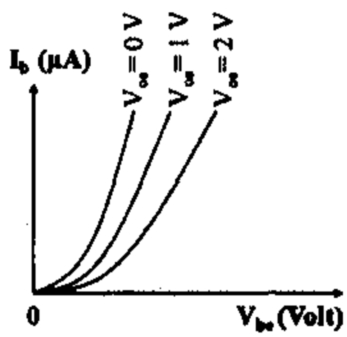
$$r_i = \left(\frac{\Delta V_{be}}{\Delta I_e}\right) \text{ at constant } V_{cb}$$

Output resistance (r_o): The ratio of change in collector-base voltage (ΔV_{cb}) to the change in collector current (ΔI_c) at constant emitter current (I_e).

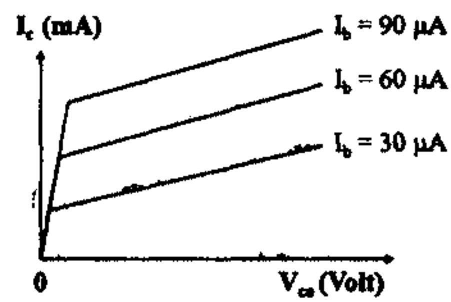
$$r_o = \left(\frac{\Delta V_{cb}}{\Delta I_c}\right) \text{ at constant } I_e$$

p-n-p Transistor Characteristics in Common Emitter (CE) Configuration: The plot between I_b and V_{be} for fixed V_{ce} is called input characteristics while the plot between I_C and V_{ce} for fixed I_b is called output characteristics for the common emitter configuration.





Input Characteristics



Output Characteristics

Current Amplification Factor or Current Gain:

(1) Direct current (dc) gain: It is the ratio of the collector current (I_c) to the base current (I_b) at constant collector voltage (V_{ce})

$$\beta_{dc} = \left(\frac{I_c}{I_b}\right) \text{ at constant } V_{ce}$$

(2) Alternating current (ac) gain: It is the ratio of change in collector current (ΔI_c) to the change in base current (ΔI_b) at constant collector voltage (V_{ce}).

$$\beta_{ac} = \left(\frac{\Delta I_c}{\Delta I_b}\right) \text{ at constant } V_{ce}$$

Alternating voltage gain: It is the ratio of change in output voltage (collector voltage ΔV_{ce}) to the change in input voltage (applied signal voltage ΔV_i).

$$A_{V-ac} = \left(\frac{\Delta V_{ce}}{\Delta V_i}\right) = \left(\frac{\Delta I_c \times R_o}{\Delta I_b \times R_i}\right) = \beta_{ac} \times \text{resistance gain}$$

Input resistance (r_i): Ratio of change in base-emitter voltage (ΔV_{be}) to the resulting change in base current (ΔI_b) at constant collector emitter voltage (V_{ce}).

$$r_i = \left(\frac{\Delta V_{be}}{\Delta I_b}\right) \text{ at constant } V_{ce}$$

Output resistance (r_o): The ratio of change in collector-emitter voltage (ΔV_{ce}) to the change in collector current (ΔI_c) at constant base current (I_b).

$$r_o = \left(\frac{\Delta V_{ce}}{\Delta I_c}\right) \text{ at constant } I_b$$

Show that $\beta > \alpha$:

$$\because \alpha = \left(\frac{I_c}{I_e}\right), \qquad \beta = \left(\frac{I_c}{I_b}\right) \qquad \text{and} \quad I_e = I_b + I_c$$

$$I_e > I_b$$

$$\therefore \beta > \alpha$$

So current amplification in CE configuration is greater than that in CB configuration.

Relation between α and β:

$$\because I_e = I_b + I_c$$

Dividing the above equation by I_c, we get

$$\frac{I_e}{I_c} = \frac{I_b}{I_c} + \frac{I_c}{I_c}$$

$$\text{Or } \frac{I_e}{I_c} = \frac{I_b}{I_c} + 1$$

$$\text{But } \alpha = \left(\frac{I_c}{I_e}\right) \qquad \text{and } \beta = \left(\frac{I_c}{I_b}\right)$$

$$\frac{1}{\alpha} = \frac{1}{\beta} + 1 = \frac{1+\beta}{\beta}$$

$$\therefore \alpha = \frac{\beta}{1+\beta} \quad \text{and } \beta = \frac{\alpha}{1-\alpha}$$

p-n-p transistor as Common Emitter Amplifier:

Input section is forward biased and output section is reverse biased with biasing batteries V_{BB} and V_{CC}.

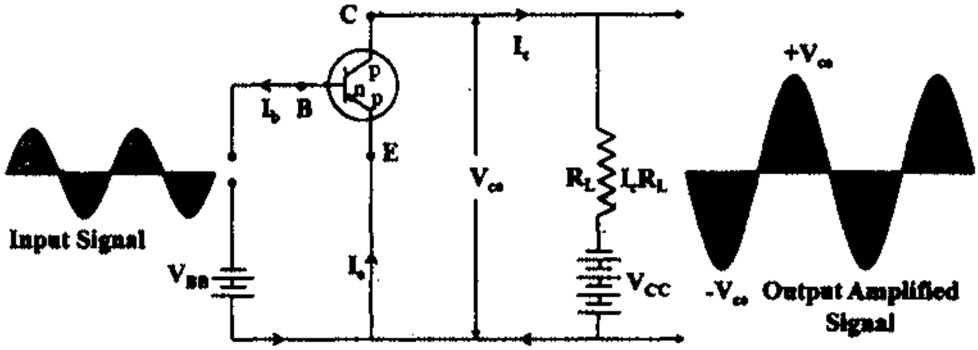
The currents I_e, I_b and I_c flow in the directions shown such that

$$I_e = I_b + I_c \quad \text{-------(1)}$$

I_cR_L is the potential drop across the load resistor R_L.

By Kirchhoff's rule,

$$V_{ce} = V_{CC} - I_c R_L \quad \text{-------(2)}$$



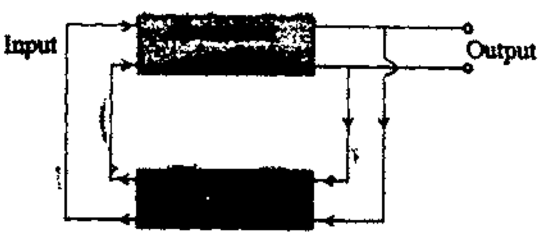
Positive Half cycle:
During the positive half cycle of the input sinusoidal signal, forward-bias of base and emitter decreases (since n-type base becomes less negative and p-type emitter becomes less positive). This decreases the emitter current and hence the collector current. In consequence, the voltage drop across the load resistance R_L decreases. From equation (2), it follows that V_{ce} increases. But, since p-type collector is negatively biased, therefore, increase means that the collector becomes more negative w.r.t. base and the output goes below the normal value. So, the output signal is negative for positive input signal.

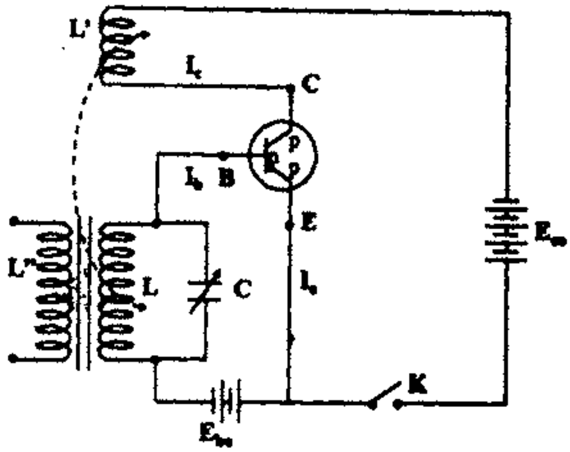
Negative Half cycle:
During the negative half cycle of the input sinusoidal signal, forward-bias of base and emitter increases. This increases the emitter current and hence the collector current. In consequence, the voltage drop across the load resistance R_L increases. From equation (2), it follows that V_{ce} decreases. But, since p-type collector is negatively biased, therefore, decrease means that the collector becomes less negative w.r.t. base and the output goes above the normal value. So, the output signal is positive for negative input signal.

Note: Input and output are out of phase by 180°.

Barkhausen conditions for oscillations: The gain of the amplifier with positive feedback is given by $A_f = \frac{A}{1 - A\beta}$, where A is the voltage gain without feedback, β is the feedback ratio and $A\beta$ is the loop gain. When $A\beta = 1$, then $A_f \to \infty$. This means that output voltage is obtained even if input voltage is zero, i.e., it becomes an oscillator. The essential condition for the maintenance of oscillation is
(i) The loop gain $A\beta = 1$
(ii) There must be positive feedback i.e., the net phase shift round the loop is 0° or integral multiples of 2π.
(iii) Initially, the loop gain ($A\beta$) must be greater than unity.
These are called the Barkhausen conditions for oscillations.

Transistor as an Oscillator (p-n-p): An oscillator is a device which can produce undamped electromagnetic oscillations of desired frequency and amplitude. It is a device which delivers a.c. output waveform of desired frequency from d.c. power even without input signal excitation.





Tank circuit containing an inductance L and a capacitance C connected in parallel can oscillate the energy given to it between electrostatic and magnetic energies. However, the oscillations die away since the amplitude decreases rapidly due to inherent electrical resistance in the circuit.

In order to obtain undamped oscillations of constant amplitude, transistor can be used to give regenerative or positive feedback from the output circuit to the input circuit so that the circuit losses can be compensated.

When key K is closed, collector current begins to grow through the tickler coil L'. Magnetic flux linked with L' as well as L increases as they are inductively coupled. Due to change in magnetic flux, induced EMF is set up in such a direction that the emitter-base junction is forward biased. This increases the emitter current and hence the collector current.

With the increase in collector current, the magnetic flux across L' and L increases. The process continues till the collector current reaches the saturation value. During this process the upper plate of the capacitor C gets positively charged.

At this stage, induced EMF in L becomes zero. The capacitor C starts discharging through the inductor L.

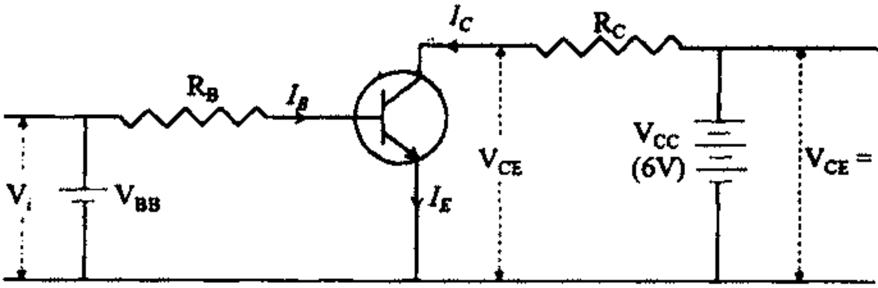
The emitter current starts decreasing resulting in the decrease in collector current. Again the magnetic flux changes in L' and L but it induces EMF in such a direction that it decreases the forward bias of emitter-base junction.

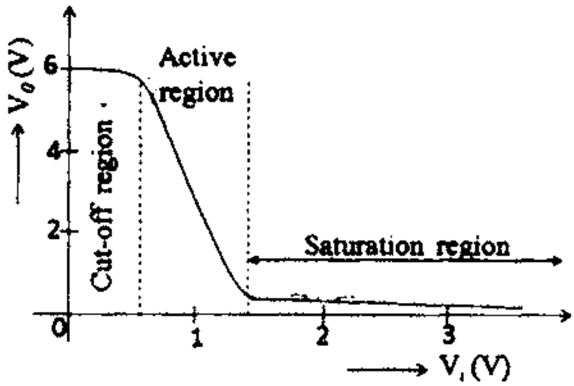
As a result, emitter current further decreases and hence collector current also decreases. This continues till the collector current becomes zero. At this stage, the magnetic flux linked with the coils become zero and hence no induced EMF across L. However, the decreasing current after reaching zero value overshoots (goes below zero) and hence the current starts increasing but in the opposite direction. During this period, the lower plate of the capacitor C gets positively charged.

This process continues till the current reaches the saturation value in the negative direction. At this stage, the capacitor starts discharging but in the opposite direction (giving positive feedback) and the current reaches zero value from negative value. The cycle again repeats and hence the oscillations are produced. The output is obtained across L''. The frequency of oscillations is given by

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

Transistor as a switch: An n-p-n silicon transistor is connected in a common-emitter mode with a load resistance R_C in the collector circuit. Here R_B is the current limiting resistor to keep I_B below the maximum allowed value.





(1) When V_i increases from 0 to about 0.6 V, the base current I_B and the collector current I_C are both zero, and the output voltage V_0 remains equal to the battery voltage V_{CC} (6V). Here the transistor is not conducting and is said to be cut-off or switch off.

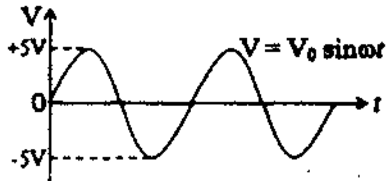
$$\because V_{CC} = V_{CE} + I_C R_C$$
$$\text{Or } V_{CE} = V_{CC} - I_C R_C$$
$$\text{Or } V_0 = V_{CC} - I_C R_C$$
$$\therefore V_0 = V_{CC} - (0 \times R_C) = V_{CC} = 6\,V$$

(2) When V_i increases from about 0.6 V to 1.4 V, I_B and I_C increases rapidly from zero while V_0 falls rapidly.

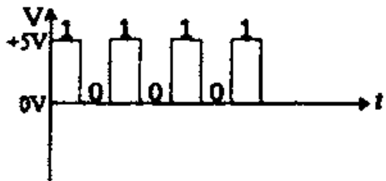
(3) When V_i increases from 1.4 V to 6 V, I_B goes on increasing but soon I_C reaches the maximum and V_0 falls to nearly zero. In this case the transistor is said to be saturated or switch on.

$$\because V_0 = V_{CC} - I_C R_C$$
$$\therefore V_0 = V_{CC} - \{(I_C)_{max} \times R_C\} = V_{CC} \approx 0 \qquad \left(\because (I_C)_{max} \approx \frac{V_{CC}}{R_C}\right)$$

Analogue signal: A continuous signal value which at any instant lies within the range of a maximum and a minimum value.



Analogue signal

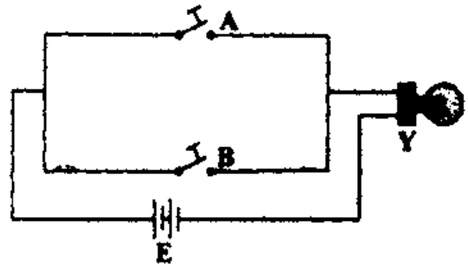


Digital signal

Digital signal: A discontinuous signal value which appears in steps in pre-determined levels rather than having the continuous change.

Digital Circuit:
An electrical or electronic circuit which operates only in two states (binary mode) namely ON and OFF is called a Digital Circuit.
In digital system, high value of voltage such as +10 V or +5 V is represented by ON state or 1 (state) whereas low value of voltage such as 0 V or -5V or -10 V is represented by OFF state or 0 (state).

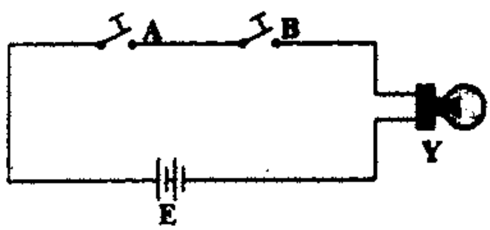
OR Operation:
OR operation is represented by (+).
Its Boolean expression is Y = A + B
It is read as "Y equals A or B".
It means that "if A is true or B is true, then Y will be true".



Truth table

Switch A	Switch B	Bulb Y
OFF	OFF	OFF
OFF	ON	ON
ON	OFF	ON
ON	ON	ON

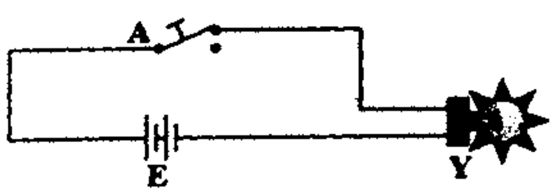
AND Operation:
AND operation is represented by (·)
Its Boolean expression is Y = A · B
It is read as "Y equals A and B".
It means that "if both A and B are true, then Y will be true".



Truth table

Switch A	Switch B	Bulb Y
OFF	OFF	OFF
OFF	ON	OFF
ON	OFF	OFF
ON	ON	ON

NOT Operation:
NOT operation is represented by (′) or (¯).
Its Boolean expression is Y = A′ or $\bar{A}$
It is read as "Y equals not A".
It means that "if A is true, then Y will be false".



Truth table

Switch A	Bulb Y
OFF	ON
ON	OFF

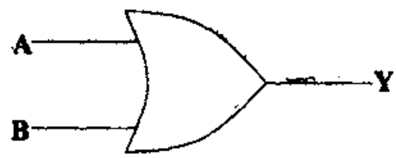
Logic Gates: The digital circuit that can be analysed with the help of Boolean algebra is called logic gate or logic circuit. A logic gate can have two or more inputs but only one output. There are 3 fundamental logic gates namely OR gate, AND gate and NOT gate.

Truth Table: The operation of a logic gate or circuit can be represented in a table called the truth table which contains all possible inputs and their corresponding outputs. If there are n inputs in any logic gate, then there will be n^2 possible input combinations.

Digital OR gate:
An OR gate can have any number of inputs but only one output. It gives high output (1) if either input A or B or both are high (1), otherwise the output Y is low (0).
Boolean expression is Y = A + B

Truth table

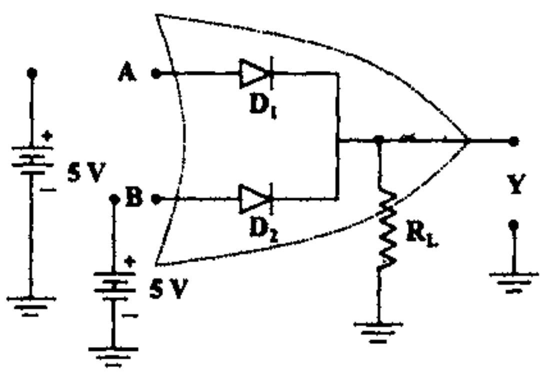
A	B	Y = A + B
0	0	0
0	1	1
1	0	1
1	1	1



Logic Symbol

Realisation of OR gate:
The positive voltage (+5 V) corresponds to high input i.e., 1 (state).
The negative terminal of the battery is grounded and corresponds to low input i.e., 0 (state).

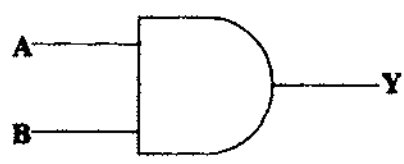
Case 1: Both A and B are given 0 input and the diodes do not conduct current. Hence no output is across R_L. i.e., Y = 0
Case 2: A is given 0 and B is given 1. Diode D_1 does not conduct current (cut-off) but diode D_2 conducts. Hence output (5 V) is available across R_L. i.e., Y = 1
Case 3: A is given 1 and B is given 0. Diode D_1 conducts current but diode D_2 does not conduct. Hence output (5 V) is available across R_L. i.e., Y = 1
Case 4: A and B are given 1. Both the diodes conduct current. However output (only 5 V) is available across R_L. i.e., Y = 1



Digital AND gate:
An AND gate can have any number of inputs but only one output. It gives high output (1) if inputs A and B are both high (1), otherwise the output Y is low (0).
Boolean expression is Y = A · B

Truth table

A	B	Y = A · B
0	0	0
0	1	0
1	0	0
1	1	1



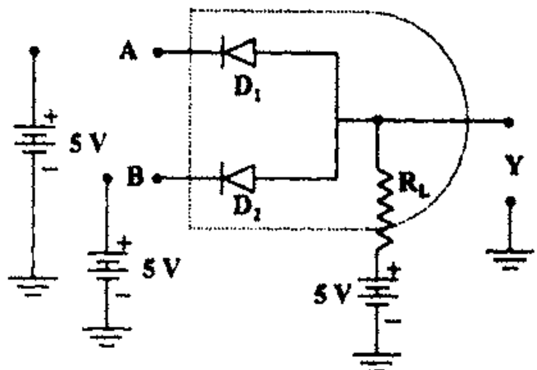
Logic Symbol

Realisation of AND gate:

The positive voltage (+5 V) corresponds to high input i.e., 1 (state).

The negative terminal of the battery is grounded and corresponds to low input i.e., 0 (state).

Case 1: Both A and B are given 0 input and the diodes conduct current (Forward biased). Since the current is drained to the earth, hence, no output across R_L. i.e., Y = 0



Case 2: A is given 0 and B is given 1. Diode D_1 being forward biased conducts current but diode D_2 does not conduct. However, the current from the output battery is drained through D_1. So, Y = 0

Case 3: A is given 1 and B is given 0. Diode D_1 does not conduct current but diode D_2 being forward biased conducts. However, the current from the output battery is drained through D_2. Hence, no output is available across R_L. i.e., Y = 0

Case 4: A and B are given 1. Both the diodes do not conduct current. The current from the output battery is available across R_L. Hence, there is voltage drop (5 V) across R_L. i.e., Y = 1

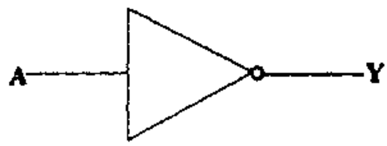
Digital NOT gate:

A NOT gate is the simplest gate, with one input and one output. It gives a high output (1), if the input A is low (0), and vice versa. Whatever the input, the NOT gate inverts it.

Boolean expression is $Y = \bar{A}$

Truth table

A	$Y = \bar{A}$
0	1
1	0

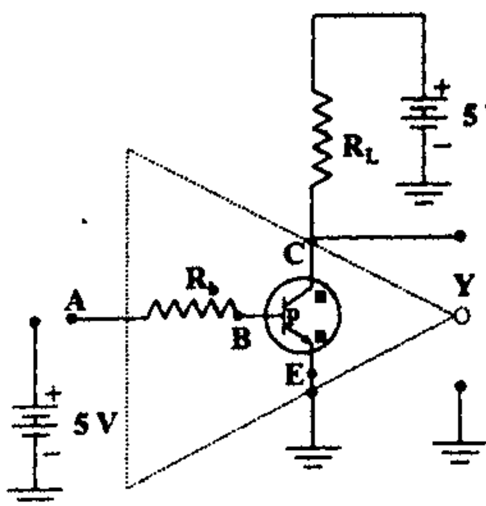


Logic Symbol

Realisation of NOT gate:

n-p-n transistor is connected to biasing batteries through Base resistor (R_b) and Collector resistor (R_L). Emitter is directly earthed. Input is given through the base and the output is tapped across the collector.

Case 1: A is given 0 input. In the absence of forward bias to the p-type base and n-type emitter, the transistor is in cut-off mode (does not conduct current). Hence, the current from the collector battery is available across the output unit. Therefore, voltage drop of 5 V is available across Y. i.e., Y= 1



Case 2: A is given 1 input by connecting the positive terminal of the input battery. p-type base being forward biased makes the transistor in conduction mode. The current supplied by the collector battery is drained through the transistor to the earth. Therefore, no output is available across Y. i.e., Y = 0

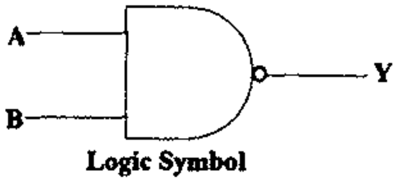
NAND gate:

A NAND gate is the combination of an AND and a NOT gate. It is obtained by connecting the output of an AND gate to the input of a NOT gate.

Boolean expression is $Y = \overline{A \cdot B}$

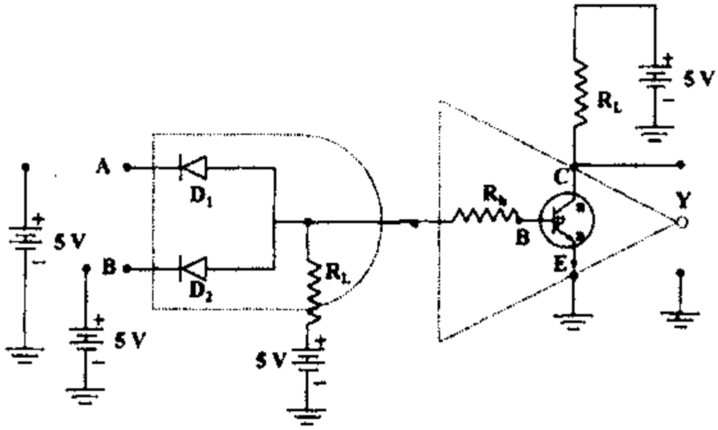
Truth table

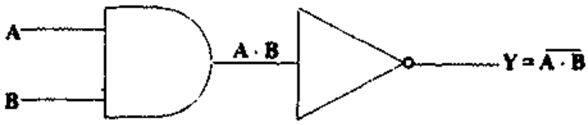
A	B	$A \cdot B$	$Y = \overline{A \cdot B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0



Logic Symbol

Circuit:





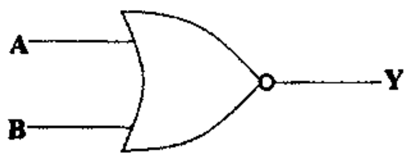
NOR gate:

A NOR gate is the combination of an OR and a NOT gate. It is obtained by connecting the output of an OR gate to the input of a NOT gate.

Boolean expression is $Y = \overline{A + B}$

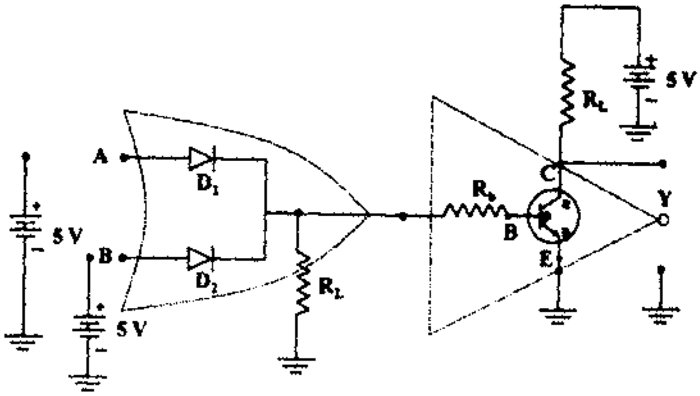
Truth table

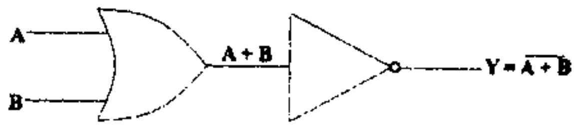
A	B	$A + B$	$Y = \overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0



Logic Symbol

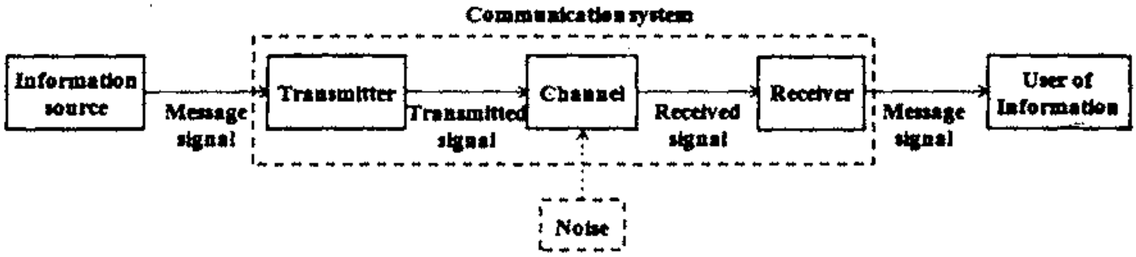
Circuit:



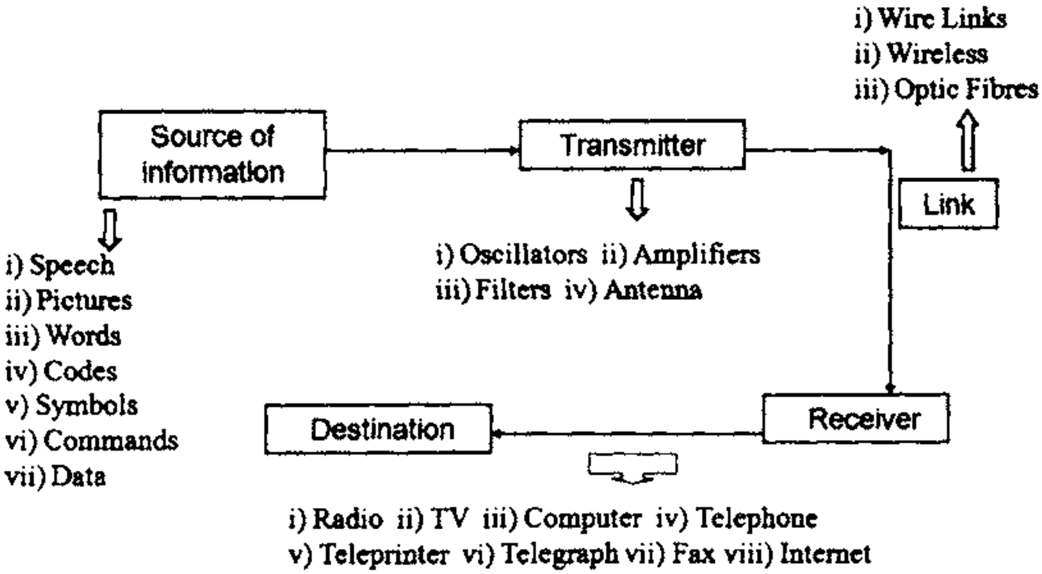


COMMUNICATION SYSTEM

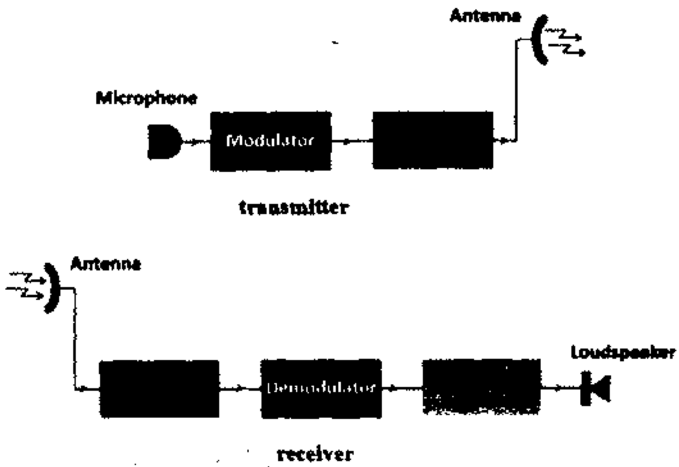
Elements of a communication system: The communication system has three essential elements:
(1) Transmitter
(2) Communication channel
(3) Receiver



Basics of communication:
(1) Communication: Processing, sending and receiving of information
(2) Information: Intelligence, signal, data or any measurable physical quantity



Basic components of a transmitter and a receiver:



Basic terminology used in electronic communication systems:

Transducer: Any device which converts energy from one form to another is called a transducer. For example, a microphone converts a sound signal into an electrical signal.

Signals: It is the electrical analog of the information produced by the source. It may be defined as the single-valued function of time (that conveys the information) and which, at every instant of time, takes a unique value. Signals can be analog or digital.

Noise: The unwanted electrical signals which get interfered with the information signal during its propagation through a transmission medium constitute noise.

Transmitter: It is a device which processes a message signal into a form suitable for transmission and then transmits it into the receiving end through a transmission medium.

Receiver: A device that extracts the original signal from the modulated signal is known as receiver.

Attenuation: The loss of strength of a signal during its propagation through the transmission medium is called attenuation.

Amplification: It is the process of increasing the amplitude and hence the strength of an electrical signal by using an electric circuit (consisting of at least one transistor) called the amplifier

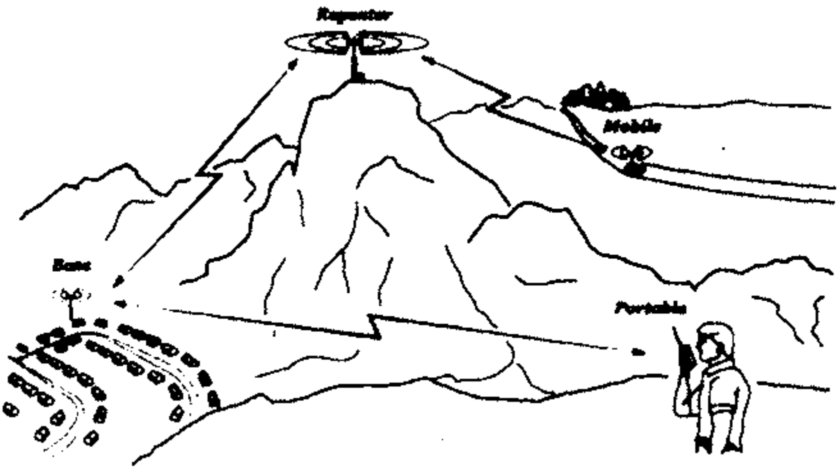
Range: It is defined as the largest distance between the source and the destination upto which a signal can be received with sufficient strength.

Bandwidth: The range over which frequencies in an information signal vary is called bandwidth. It is equal to the difference between the highest and lowest frequencies present in the signal.

Modulation: Modulation is the process in which some characteristic, usually the amplitude, frequency or phase angle of a high frequency carrier wave, is varied in accordance with the instantaneous value of the low frequency information signal, called the modulating signal.

Demodulation: The process of recovering the original information signal from the modulated wave at the receiver end is called demodulation or detection.

Repeater: A repeater is a combination of a transmitter, an amplifier and a receiver which picks up a signal from the transmitter, amplifies and retransmits it to the receiver sometimes with a change of carrier frequency.



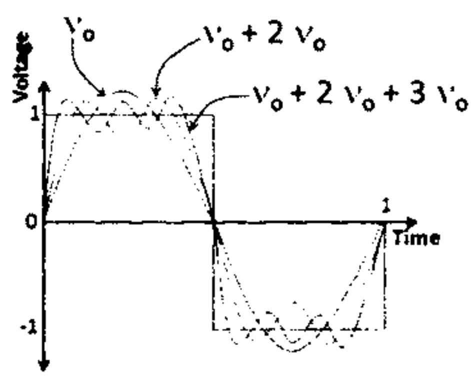
Bandwidth of signals:

Speech signals: Speech signals contain frequencies between 300Hz to 3100Hz. Such signals require a bandwidth of 2800Hz (3100Hz - 300Hz) for telephonic transmission.

Music signals: Music signals have frequencies between 20Hz to 20kHz. Such signals require a bandwidth of about 20kHz (20kHz - 20Hz)

Video signals: The bandwidth of 4.2 MHz is required for the transmission of pictures.

TV signals: The bandwidth required for transmission of TV signals which contain both audio and video is 6 MHz.



Digital signals: If the signals are digital, they are rectangular in shape. They can be expressed as a superposition of sinusoidal waves of frequencies ν_0, $2\nu_0$, $3\nu_0$, $4\nu_0$... $n\nu_0$ where ν_0 is the fundamental frequency and n is an integer extending to infinity. The bandwidth in this case is infinite

Note: Information of a digital signal is conveyed by the pattern of the pulses, not by their shape.

Some important wireless cômmunication frequency bands:

Service	Frequency bands	Comments
Standard AM broadcast	540 – 1600 kHz	
FM broadcast	88 – 108 MHz	
Television	54 – 72 MHz 76 – 88 MHz 174 – 216 MHz 420 – 890 MHz	VHF (very high frequency) TV UHF (ultra high frequency) TV
Cellular Mobile Radio	896 – 901 MHz 840 – 935 MHz	Mobile to base station Base station to mobile
Satellite Communication	5.925 – 6.425 MHz 3.7 – 4.2 MHz	Uplink Downlink

Propagation of Electromagnetic Waves:
Depending on the frequency, radio waves and micro waves travel in space in different ways. Due to the behaviour of these waves w.r.t. the earth and the atmosphere, then we have
(1) Ground or surface wave propagation
(2) Sky or ionospheric wave propagation
(3) Space or tropospheric wave propagation

(1) Ground wave propagation: The mode of propagation of EM waves in which there is a strong influence of the ground on the propagation of signal waves from the transmitting to the receiving antenna is known as ground or surface wave propagation. It is suitable for low and medium frequency upto 2 MHz. It is used for local broad casting.

This is used when the transmitting and receiving antenna are located close to the surface of the Earth. The field component of the wave gets vertically polarized and induces charges on the Earth's surface. These constitute a current in the Earth's surface. This results in energy dissipation and the wave weakens. Due to this, ground waves are not suitable for very long-range communication. The energy losses increase for higher frequencies (the impedance of ground will increase). So it is not suitable for high frequencies. It can be sustained only at low frequencies (~500–2000 kHz) or for radio broadcast at long wavelength.

(2) Sky or ionospheric wave propagation: The mode of propagation of waves in which the radio waves emitted from the transmitting antenna reach the receiving antenna after being reflected from the ionosphere is known as sky wave propagation. It is suitable for frequency between 2 MHz to 30 MHz. It is used for long distance radio communication.

The UV and other high-energy radiations coming from the Sun are absorbed by the air molecules that get ionized and form an ionized layer of electrons and ions around the Earth. The ionosphere extends from a height of ~80–300 km above the Earth's surface. The effective dielectric constant ε' and the corresponding refractive index n' in the ionosphere are related to ε_0 and n_0 as

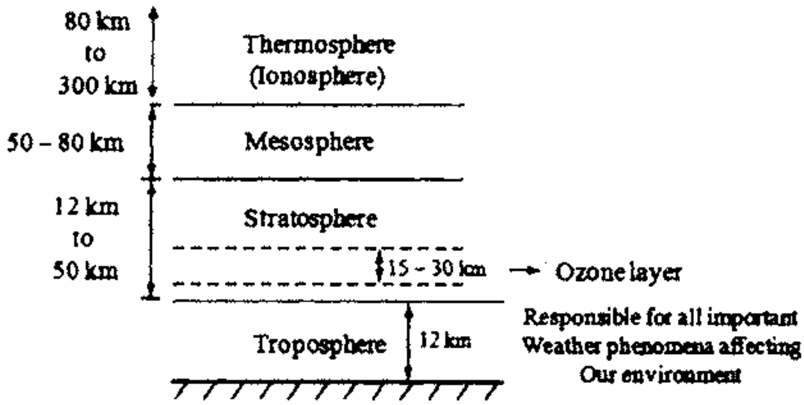
$$n' = n_0[1-(Ne^2/\varepsilon_0 m\omega^2)]^{1/2}$$

where e is the electronic charge, m is the mass of electron, and N is the electron density in the ionosphere. Refractive index of ionosphere is less than its free space value of n_0 and goes on decreasing as we go away from Earth. So the waves entering the ionosphere are refracted away from the normal till they are totally internally reflected back toward the Earth. Different frequencies ω are reflected from different regions of ionosphere having different values of N. The maximum frequency, which is just reflected back to Earth, is called critical frequency f_c.

$$f_c = 9(N_{max})^{1/2}$$

Where N_{max} is the maximum electron density of the ionosphere. It should be noted that f_c is of the order of 5–10 MHz. For frequencies greater than f_c, waves cross the ionosphere do not return back.

Atmosphere: Except for the layer in upper atmosphere, called ionosphere, which is composed of electrons and positive ions, the rest of the atmosphere is composed mostly of neutral molecules. Atmosphere is transparent to visible radiation, infrared is unable to pass through it, and ultraviolet (UV) radiation is blocked by ozone layer.



Important Terms used in Sky wave propagation:

(i) Critical frequency (f_c): The highest frequency above which the ionosphere no longer returns the sky wave back to earth when transmitted in vertical direction is called critical frequency.

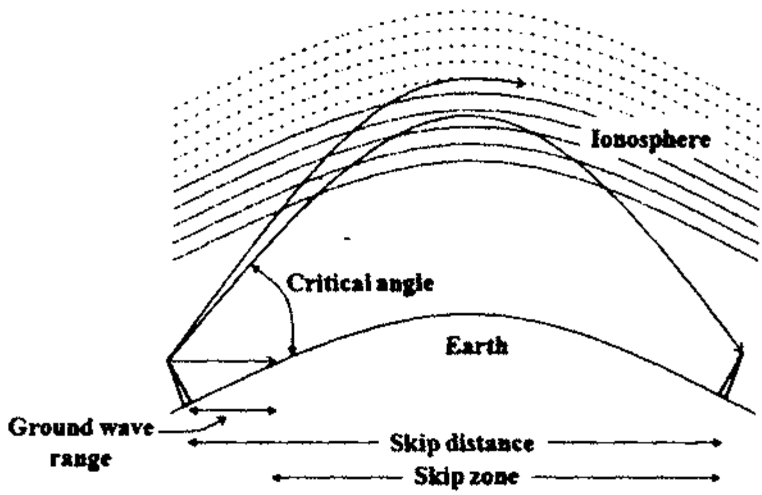
(ii) Critical angle: For a given frequency, the vertical angle above which the sky wave no longer returns to earth but travels outward into space is called critical angle.

(iii) Skip distance (D_{skip}): The distance between the transmitting aerial and the point where the sky wave is first received after returning to earth is called skip distance.

$$D_{skip} = 2h\left(\frac{f_{max}}{f_c}\right)^2 - 1$$

where h is the height of reflecting layer of atmosphere, f_{max} is the maximum frequency of electromagnetic waves and f_c is the critical frequency.

(iv) Skip zone: A region in the ground where no signal can be picked up is called the skip zone.



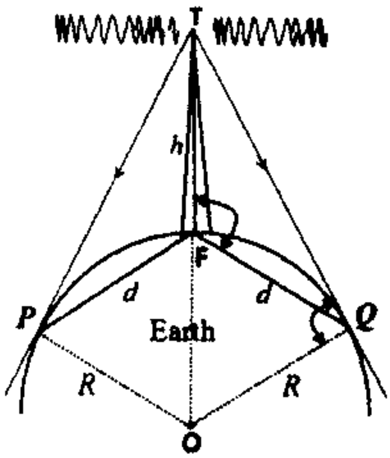
(3) Space or tropospheric wave propagation: Space waves travel in (more or less) straight lines. But they depend on line-of-sight conditions. So, they are limited in their propagation by the curvature of the earth. They propagate very much like electromagnetic waves in free space. It is suitable for 30 MHz to 300 MHz. It is used in television communication and radar communication. It is also called line-of-sight communication.

This mode is forced on the waves because their wavelengths are too short for reflection from the ionosphere, and because the ground wave disappears very close to the transmitter, owing to tilt.

The transmitted waves travelling in a straight line directly reach the receiver end and are then picked up by the receiving antenna. The effective range of the broadcast is essentially the region from P to Q, which is covered by the line-of-sight.

$$d = \sqrt{2Rh}$$

where R is the radius of the Earth, h is the height of antenna above the Earth's surface as shown in the figure. For far-away stations, either a repeater is used or h is increased (by locating the transmitter on a satellite)



Height of TV Transmitting Antenna:

Let h be the height of the transmitting antenna, d be the distance (radius) of coverage from the foot of the tower and R be the radius of the earth.

$OT^2 = OQ^2 + QT^2$

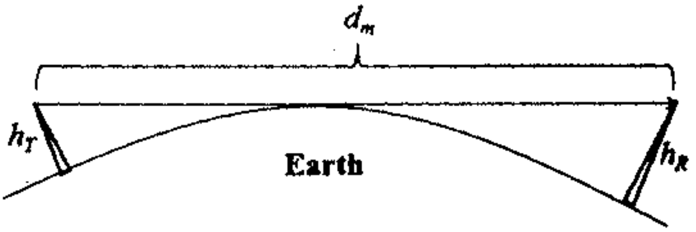
$(R + h)^2 = R^2 + (h^2 + d^2)$ (Note: $\angle$TFQ is a right angle)

$R^2 + h^2 + 2Rh = R^2 + h^2 + d^2$

$d \approx \sqrt{2Rh}$

The maximum line-of-sight distance d_m between the transmitting and receiving antennas is

$$d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$



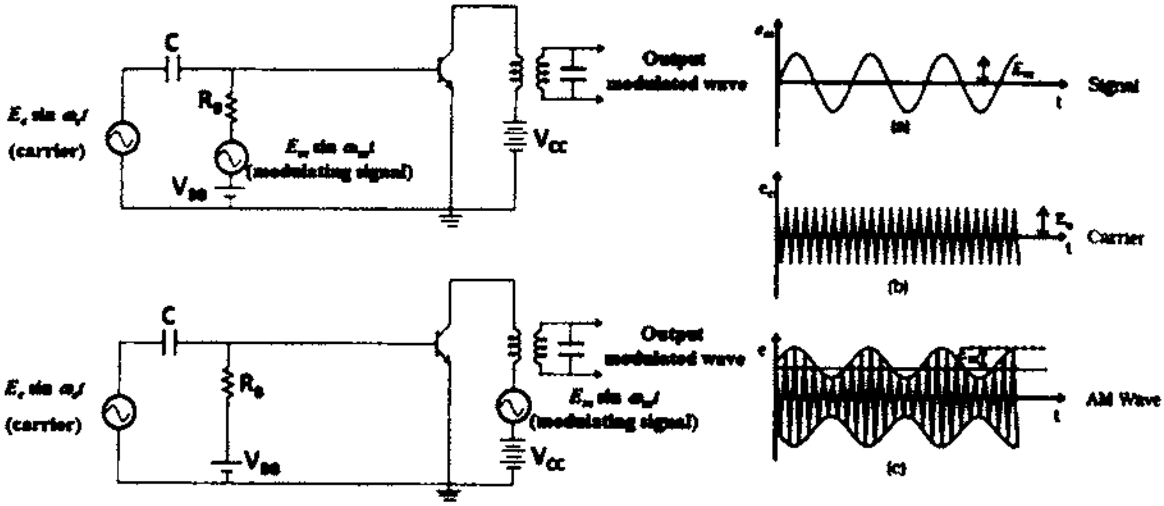
Modulation: Modulation is the process in which some characteristic, usually the amplitude, frequency or phase angle of a high frequency carrier wave, is varied in accordance with the instantaneous value of the low frequency information signal, called the modulating signal.

Types of Modulation:

1) Amplitude Modulation
2) Frequency Modulation
3) Pulse Modulation
4) Phase Modulation

Production of Amplitude modulation (AM) wave: When the amplitude of high frequency carrier wave is changed in accordance with the amplitude of the signal, it is called amplitude modulation.

Let $e_c = E_c \sin \omega_c t$ represent a carrier wave of amplitude E_c and $e_m = E_m \sin \omega_m t$ represent the modulating signal of amplitude E_m.



In amplitude modulation (AM), the amplitude of the carrier wave E_c is varied in accordance with the amplitude of the modulating signal. So the modulated wave can be represented as

$$e = (E_c + K_a E_m \sin \omega_m t) \sin \omega_c t$$

K_a is proportionality factor which determines the maximum variation in amplitude for a given signal voltage e_m

$$e = E_c(1 + \frac{K_a E_m}{E_c} \sin \omega_m t) \sin \omega_c t$$

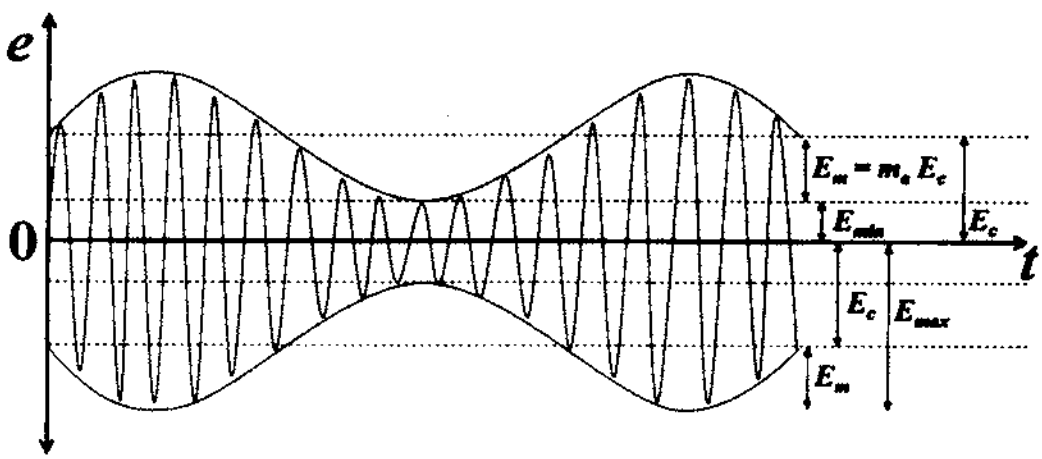
$$e = E_c(1 + m_a \sin \omega_m t) \sin \omega_c t$$

$m_a = \frac{K_a E_m}{E_c}$ is known as modulation index or modulation factor

Modulation factor $m_a = \frac{\textit{Amplitude change of carrier wave}}{\textit{Normal carrier wave}}$

If $K_a = 1$, $\quad m_a = \frac{E_m}{E_c}$

Modulation factor:



$$E_{max} = E_c + m_a E_c = E_c + E_m$$
$$E_{min} = E_c - m_a E_c = E_c - E_m$$

$$m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

$$m_a = \frac{E_c + E_m - E_c + E_m}{E_c + E_m + E_c - E_m} = \frac{2E_m}{2E_c} = \frac{E_m}{E_c}$$

Examples: (1)

Amplitude of carrier wave $E_c = A\,(say)$

Amplitude of modulating signal $E_m = \frac{1}{2}A = 0.5A$

Find m_a in percentage ?

Sol[n]: $E_{max} = E_c + E_m = A + 0.5A = 1.5A$

$$E_{min} = E_c - E_m = A - 0.5A = 0.5A$$

$$m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} = \frac{1.5A - 0.5A}{1.5A + 0.5A} = \frac{A}{2A} = \frac{1}{2}$$

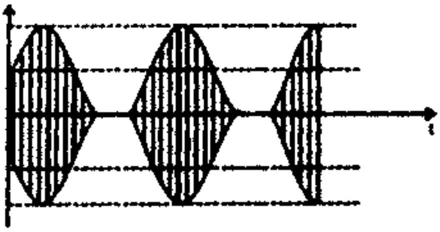
$$\therefore m_a = \frac{1}{2} \times 100\% = 50\% \qquad (i.e., m_a < 100\%)$$

Or

$$m_a = \frac{E_m}{E_c} = \frac{\frac{1}{2}A}{A} = \frac{1}{2}$$

$$\therefore m_a = \frac{1}{2} \times 100\% = 50\%$$

Examples: (2)



Amplitude of carrier wave $E_c = A\,(say)$

Amplitude of modulating signal $E_m = 1\frac{1}{2}A = 1.5A$

Find m_a in percentage ?

Sol[n]: $E_{max} = E_c + E_m = A + 1.5A = 2.5A$

$$E_{min} = E_c - E_m = A - 1.5A = -0.5A$$

$$m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} = \frac{2.5A - (-0.5A)}{2.5A + (-0.5A)} = \frac{3A}{2A} = 1.5$$

$$\therefore m_a = 1.5 \times 100\% = 150\% \qquad (i.e., m_a > 100\%)$$

Or

$$m_a = \frac{E_m}{E_c} = \frac{1\frac{1}{2}A}{A} = \frac{1.5A}{A} = 1.5$$

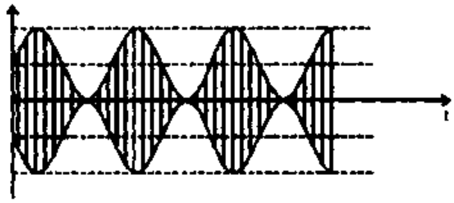
$$\therefore m_a = 1.5 \times 100\% = 150\%$$

Examples: (3)

Amplitude of carrier wave $E_c = A\,(say)$

Amplitude of modulating signal $E_m = A$

Find m_a in percentage ?



$$\text{Sol}^n\text{:}\quad E_{max} = E_c + E_m = A + A = 2A$$

$$E_{min} = E_c - E_m = A - A = 0$$

$$m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} = \frac{2A - 0}{2A + 0} = \frac{2A}{2A} = 1$$

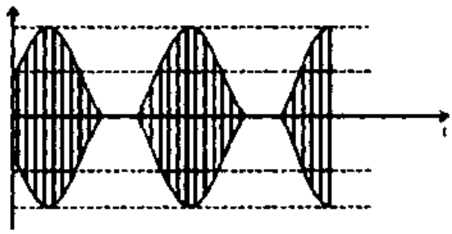
$$\therefore m_a = 1 \times 100\% = 100\% \qquad (i.e., m_a = 100\%)$$

Or

$$m_a = \frac{E_m}{E_c} = \frac{A}{A} = 1$$

$$\therefore m_a = 1 \times 100\% = 100\%$$

Importance of modulation factor: Modulation factor is very important since it determines the strength and quality of the transmitted signal. In an AM wave, the signal is contained in the variations of the carrier amplitude. When the carrier is modulated to a small degree i.e., small m_a, the amount of carrier amplitude variation is small. Consequently, the audio signal being transmitted will not be very strong. The greater the degree of modulation i.e., m_a, the stronger and clearer will be the audio signal. But if the carrier is over modulated i.e., $m_a > 100\%$ or $m_a > 1$, distortion will occur during reception.



Need for modulation:

(1) Size of the antenna or aerial: For transmitting a signal, we need an antenna or an aerial. This antenna should have a size comparable to the wavelength of the signal (at least l/4 in dimension) so that the antenna properly senses the time variation of the signal. For an electromagnetic wave of frequency 20 kHz, the wavelength λ is

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{20\times10^3} = 15000m$$

This means that for the transmission of this signal, the antenna to be used should have a length of $\frac{\lambda}{4} = \frac{15000}{4} = 3750m = 3.75km$ which is not possible to construct and operate. Hence direct transmission of such baseband signals is not practical. We can obtain transmission with reasonable antenna lengths if transmission frequency is high (for example, if ν is 1 *MHz* or 10^6 *Hz*, then λ is 300 *m* and the length of the antenna is 75*m*). Therefore, there is a need of translating the information contained in our original low frequency baseband signal into high or radio frequencies before transmission.

Or

The antenna size should be at least $\lambda/4$ in dimension where λ is the wavelength of the signal.

(2) Effective power radiated by an antenna: The power radiated from a linear antenna obey the relation

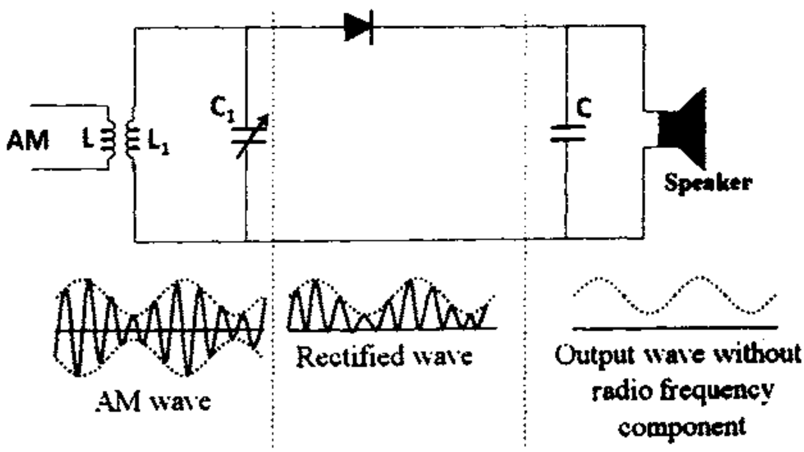
$$P \propto \left(\frac{l}{\lambda}\right)^2$$

This implies that for the same antenna length *l*, the power radiated increases with decreasing λ, i.e., increasing frequency. Hence, the effective power radiated by a long wavelength baseband signal would be small. For a good transmission, we need high powers and hence this also points out to the need of using high frequency transmission.

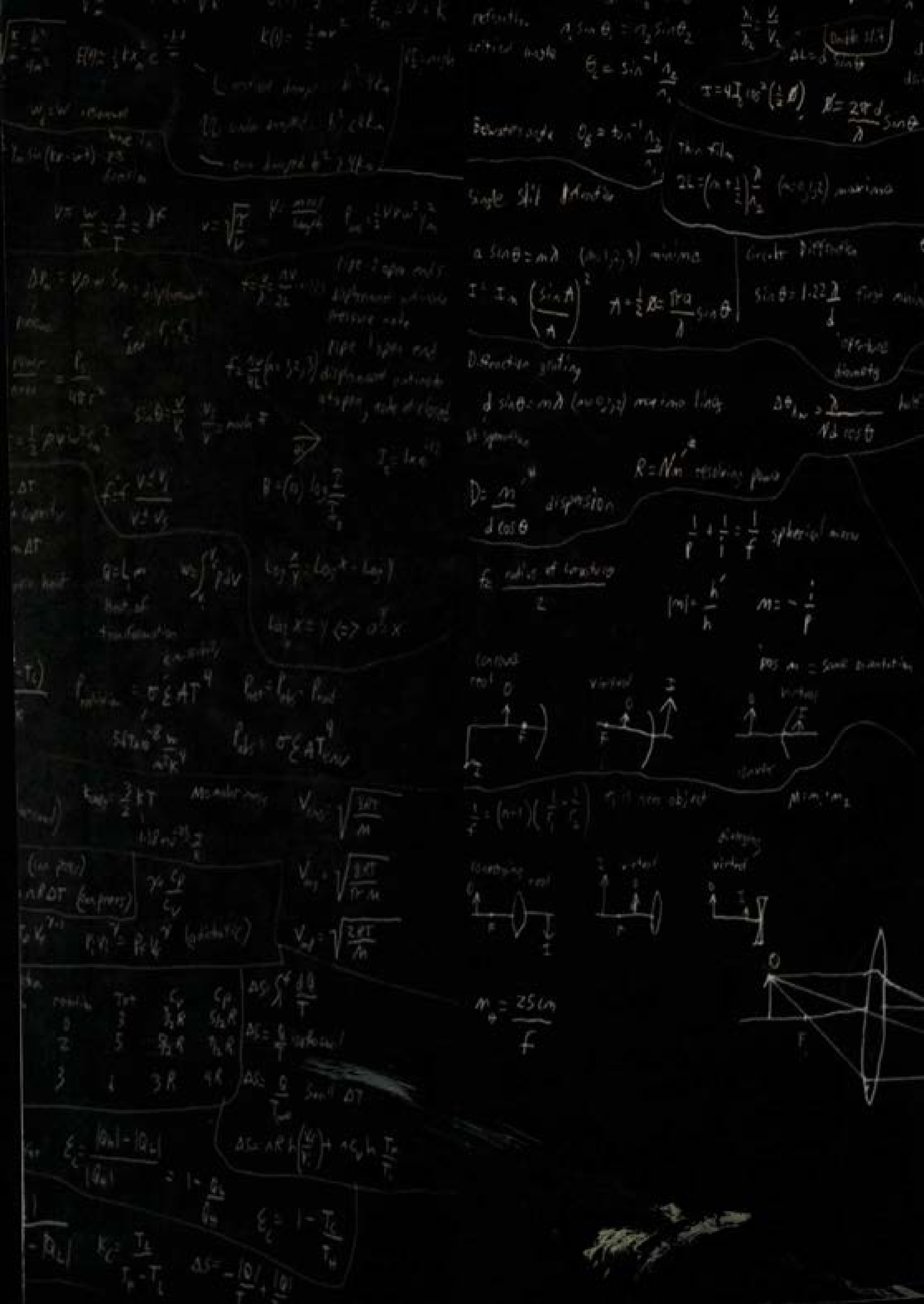
Or

Effective power radiated by an antenna is directly proportional to $\left(\frac{l}{\lambda}\right)^2$ where *l* is the length of antenna. So for a fixed antenna length, power radiated by small wavelength or high frequencies would be large.

Detection of Amplitude Modulated (AM) Wave: The process of recovering the original information signal from the modulated wave at the receiver end, is called demodulation or detection.



The figure shows a simple detector circuit. The modulated wave of desired frequency is selected by the parallel tuned L_1C_1 and is applied to a crystal diode. As a result, the output of the diode consists of positive half cycle of modulated wave as shown. The rectified modulated wave is filtered by the capacitor C shunted across the speaker. The capacitor C acts as a by-passed for carrier waves and the audio frequency voltage is fed to the speaker for sound reproduction.


Thin Film
Single Slit Diffraction
Diffraction grating
dispersion
spherical mirror
virtual
real